Computing Inconsistency Using Logical Argumentation

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Abstract: Measuring the degree of conflict of a knowledge base can help us to deal with inconsistencies. Several semantic and syntax based approaches have been proposed separately. In this paper, we use logical argumentation as a field to compute the inconsistency measure for propositional formulae. We show using the complete argumentation tree that our family of measures is able to express finely the inconsistency of a formula following their context and allows us to distinguish between formulae. We extend our measure to quantify the degree of inconsistency of set of formulae and give a general formulation of the inconsistency using some logical properties.

1 INTRODUCTION

Inconsistencies arise naturally when working with logic-based knowledge bases; they can come from ontology learning, merging of several knowledge bases, decisions making, multi-agent system, or belief revision.

The need for handling inconsistencies in knowledge bases has been well recognized in recent years. Recently, the field of inconsistency measurement has gained some attention for knowledge representation formalisms. Therefore, reasoning under inconsistency is an important field in Computer Science (Bertossi et al., 2005) and in Artificial Intelligence in particular and there are many logic-based proposals for analysing inconsistent information. Then, interest in quantifying inconsistency for knowledge bases has grown rapidly in last years. This is because it has been shown that measuring inconsistency is helpful to compare different knowledge bases and evaluate their quality of information. For instance, if given the opportunity to choose between different knowledge bases, we may try to choose the one that is least inconsistent. Already, measuring inconsistency has been seen to be useful and attractive in diverse applications including e-commerce protocols (Chen et al., 2004), software specifications (Martinez et al., 2004), belief merging (Qi et al., 2005), news reports (Hunter, 2006), requirements engineering (Hunter and Konieczny, 2006), integrity constraints (Grant and Hunter, 2006), databases (Martinez et al., 2007), ontologies (Zhou et al., 2009), semantic web (Zhou et al., 2009), network intrusion detection (McAreavey et al., 2011), and multi-agent systems (Hunter et al., 2014).

To tackle this problem, a range of logic-based proposals for analyzing and measuring the amount of inconsistency of knowledge base have been presented in literature, including the maximal n-consistency (Knight, 2002), measures based on variables or via multi-valued models (Grant, 1978; Hunter, 2002; Oller, 2004; Hunter, 2006; Grant and Hunter, 2008; Ma et al., 2010; Xiao et al., 2010; Ma et al., 2011), n-consistency and n-probability (Doder et al., 2010), measures based on minimal inconsistent subsets (Hunter and Konieczny, 2008; Mu et al., 2011a; Mu et al., 2012; Xiao and Ma, 2012), the Shapley inconsistency value (Hunter and Konieczny, 2010), inconsistency measurement based on minimal proofs (Jabbour and Raddaoui, 2013), partitioning based inconsistency measures (Jabbour et al., 2014a), and recently inconsistency characterization using prime implicates (Jabbour et al., 2014c; Jabbour et al., 2014b).

These proposals for measuring inconsistency can be roughly divided into the two following fundamentally categories. The complete comparison of them is challenging. The first one, called *semantic measures*, aims to compute the proportion of language that is affected by the inconsistencies. The measures belonging to this first class are often based on some paraconsistent semantics because we can still find paraconsistent models for inconsistent knowledge bases. The second approach, called *syntactic measures*, involves counting the minimal number of formulae which are

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responsible for the conflict. Viewing minimal inconsistent subsets as the purest form of inconsistency, it is natural to derive syntax sensitive inconsistency measures for a knowledge base from the minimal inconsistent subsets of that base. The inconsistency measures considered in this work are defined in terms of minimal inconsistent subsets and belong to the second class.

In this paper, we consider an argumentation-based framework that uses classical logic as the underlying formalism, which offers a more reasoned way to compute the degree of inconsistency in knowledge bases. Argumentation is an important cognitive process for dealing with conflicting information by generating alternative sets of arguments. It has been established as an Artificial Intelligence keyword for the last fifteen years, especially for handling inconsistency in knowledge bases. There are several approaches to formalize argumentation. Among them is the so-called abstract argumentation system by Dung (Dung, 1995), which consists of a set of arguments and a binary relation between them. A second approach is the deductive or logic-based argumentation (Besnard and Hunter, 2001). We consider here the latter approach in which an argument is a pair (support-conclusion) where the support is a minimal consistent set of formulae that entails the claim. Logical argumentation theory has been exploited as a way to support the comparison and selection of statements. Statements are represented as arguments and argumentation frameworks support the reasoning on their acceptability.

The remainder of this paper is structured as follows: Section 2 introduces the argumentation approach for classical propositional logic, as proposed by (Besnard and Hunter, 2001). Then, we recall several inconsistency measures based on minimal inconsistent subsets and maximal consistent subsets. In section 3, our new framework for measuring inconsistency based on complete argumentation trees is presented. Next, we provide a generalization of our approach to evaluate the inconsistency of a subset of formulae. Then, we study the logical properties of the proposed measures. Finally, we conclude and give some perspectives of this work.

2 FORMAL PRELIMINARIES

2.1 Propositional Logic

We assume a propositional language $\mathcal{L}$ built from a finite set of propositional symbols $\mathcal{P}$ under the conventional Boolean operators $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$, as well as the truth constants $\top$ (for truth) and $\bot$ (for falsity). We will use lower case Roman letters a and b to denote propositional variables. We use Greek letters α and β for propositional formulae and Φ and Ψ for sets of formulae.

A *knowledge base* K is a finite set of propositional formulae. We further assume a distinguished enumeration for every subset of K, its *canonical enumeration*. Importantly, it only serves to provide the total order in which the formulae of any subset of K are to be conjoined to yield a formula logically equivalent to this subset. Therefore, no other constraint is imposed on K, particularly K is not expected to be consistent. It needs not even be the case that individual formulae in K are consistent. We let $\vdash$ denote the classical consequence relation. We write $K \vdash \bot$ to denote that K is inconsistent.

If K is inconsistent, then one can define the notion of *minimal inconsistent subset* as an unsatisfiable set of formulae M in K that is such that any of its subsets is satisfiable, i.e.:

Definition 1. *Let K be a knowledge base and $M \subseteq K$. M is a minimal unsatisfiable (inconsistent) subset (MUS) of K iff:*

1. $M \vdash \bot$
2. $\forall M' \subsetneq M, M' \nvdash \bot$

Therefore, the set of all minimal inconsistent subsets of K, denoted as $MUSes(K)$, is defined as $MUSes(K) = \{M \subseteq K \mid M \text{ is a } MUS \text{ of } K\}$.

A formula α that is not involved in any MUS of K is called *free formula*. The class of free formulae of K is written $free(K) = \{\alpha \mid \nexists M \in MUSes(K), \alpha \in M\}$. When a MUS is singleton, the single formula in it is called a self-contradictory formula. We denote by $SelfC(K)$ the set of self-contradictory formulae in K.

2.2 Logical Argumentation

In the present subsection, we focus on the logical argumentative model developed by Besnard and Hunter (Besnard and Hunter, 2001). This framework adopts a very common intuitive notion of an argument. Essentially, an argument is a set of relevant formulae that can be used to classically prove some point, together with that point. Each point is represented by a formula.

Definition 2. *An argument A is a pair $\langle \Phi, \alpha \rangle$ s.t.:*

1. $\Phi \subseteq K$
2. $\Phi \nvdash \bot$
3. $\Phi \vdash \alpha$
4. $\forall \Phi' \subset \Phi, \Phi' \nvdash \alpha$

A is said to be an argument for α. The sets Φ and α denote the *support*, i.e. $Sup(A) = \Phi$, and the *conclusion* of A, i.e. $Conc(A) = \alpha$, respectively.

An argument A is said to be an *atomic argument* if its support is rooted by only one formula, i.e., $Sup(A) = \alpha$.

Example 1. *Let $K = \{a \to b, \neg b \vee c, a, \neg a \vee \neg c, b \wedge d\}$. In view of K, some arguments are:*

$$\langle\{a, a \to b, \neg b \vee c\}, c\rangle$$
$$\langle\{a, \neg a \vee \neg c\}, \neg c\rangle$$
$$\langle\{b \wedge d\}, c \to d\rangle$$
$$\langle\{a, a \to b\}, a \wedge b\rangle$$
$$\langle\{\neg b \vee c\}, \neg(b \wedge \neg c)\rangle.$$

Also, $\langle\{b \wedge d\}, c \to d\rangle$, and $\langle\{\neg b \vee c\}, \neg(b \wedge \neg c)\rangle$ are examples of atomic arguments.

Proposition 1. *Let K be a knowledge base and $\Phi \subseteq K$. $\langle\Phi, \alpha\rangle$ is an argument iff $\Phi \cup \{\neg\alpha\}$ is a MUS of $K \cup \{\neg\alpha\}$.*

Definition 3. *Let K be a knowledge base. We say that $\langle\Phi, \alpha\rangle$ is a free argument if and only if $\Phi \subseteq K \setminus \bigcup_{S \in MUSes(K)} S$.*

Proposition 2. *Let K be a knowledge base. $\alpha \in free(K)$ if and only if there exists a free argument $\langle\Phi, \beta\rangle$ s.t. $\Phi \subseteq K$ and $\alpha \in \Phi$.*

Arguments are not independent in the sense that an argument can implicitly contain another. The following definition introduces a notion of subsumption among arguments.

Definition 4. *An argument $\langle\Phi, \alpha\rangle$ is* more conservative *than an argument $\langle\Psi, \beta\rangle$ if and only if $\Phi \subset \Psi$ and $\beta \vdash \alpha$.*

That is if $\langle\Phi, \alpha\rangle$ is an atomic argument, then there exists no argument $\langle\Psi, \beta\rangle$ s.t. $\langle\Psi, \beta\rangle$ is more conservative than $\langle\Phi, \alpha\rangle$, unless $\Psi = \emptyset$ and $\beta = \top$.

Example 2. *The argument $\langle\{a\}, a \vee b\rangle$ is more conservative than the argument $\langle\{a, a \to b\}, a \wedge b\rangle$.*

It may happen that some arguments directly oppose the support of other arguments. This leads to the notion of attacks, a major component of an argumentation system. In (Besnard and Hunter, 2001), Besnard and Hunter capture a relation of attack between arguments as stated by the following definition.

Definition 5. *An* undercut *of an argument $\langle\Phi, \alpha\rangle$ is an argument $\langle\Psi, \neg(\beta_1 \wedge \ldots \wedge \beta_n)\rangle$ s.t. $\{\beta_1, \ldots, \beta_n\} \subseteq \Phi$.*

Example 3. *Let $K = \{a, \neg a \vee b, c, \neg c \vee \neg a\}$. Then, $\langle\{c, \neg c \vee \neg a\}, \neg(a \wedge (\neg a \vee b))\rangle$ is an undercut for $\langle\{a, \neg a \vee b\}, b\rangle$. A less conservative undercut for $\langle\{a, \neg a \vee b\}, b\rangle$ is $\langle\{c, \neg c \vee \neg a\}, \neg a\rangle$.*

Definition 6. *$\langle\Psi, \beta\rangle$ is a* maximal conservative undercut *of an argument $\langle\Phi, \alpha\rangle$ iff $\langle\Psi, \beta\rangle$ is an undercut of $\langle\Phi, \alpha\rangle$ such that no other undercut for $\langle\Phi, \alpha\rangle$ is strictly more conservative than $\langle\Psi, \beta\rangle$.*

In other words, $\langle\Psi, \beta\rangle$ is a maximal conservative undercut of an argument $\langle\Phi, \alpha\rangle$ iff for all undercuts $\langle\Psi', \beta'\rangle$ of $\langle\Phi, \alpha\rangle$, if $\Psi' \subseteq \Psi$ and $\beta \vdash \beta'$ then $\Psi \subseteq \Psi'$ and $\beta' \vdash \beta$.

The value of the next definition of canonical undercut is that we only need to take the *canonical undercuts* into account. These arguments are identified by Besnard and Hunter as of relevant focus for generating counter-arguments. This means that we can justifiably ignore the potentially very large number of non-canonical undercuts.

Definition 7. *$\langle\Psi, \neg(\beta_1 \wedge \cdots \wedge \beta_n)\rangle$ is a canonical undercut of $\langle\Phi, \alpha\rangle$ iff $\langle\Psi, \neg(\beta_1 \wedge \ldots \wedge \beta_n)\rangle$ is a maximal conservative undercut of $\langle\Phi, \alpha\rangle$ and $\langle\beta_1, \ldots, \beta_n\rangle$ is the canonical enumeration of Φ.*

Now, in order to obtain a structure gathering arguments and counter-arguments for/against a specific claim, Besnard and Hunter define the so-called argumentation trees that collate such arguments and counter-arguments.

Definition 8. *An* argumentation tree *for α is a tree whose nodes are arguments such that:*

1. *The root is an argument for α*
2. *For every node $\langle\Psi, \beta\rangle$ whose ancestor nodes are $\langle\Psi_1, \beta_1\rangle, \ldots, \langle\Psi_n, \beta_n\rangle$, there exists $\gamma \in \Psi$ such that for $1 \leq i \leq n$, $\gamma \notin \Psi_i$*
3. *Each child node is a canonical undercut of its parent node.*

An argumentation tree aims at capturing the way counter-arguments can take place as the dispute develops. Condition 2 insists that each counter-argument involves extra information thereby precluding cycles. These trees have noticeable properties. As Δ is a finite set of formulae, it can be proved (Besnard and Hunter, 2001) that there are only finitely several argumentation trees for α and each of them is finite.

As several different argumentation trees for a given formula α can co-exist, the following *complete argumentation tree* concept aims to represent them in a global manner by considering all possible attacks and consequently all canonical undercuts.

Definition 9. *A* complete argumentation tree *for α, denoted as $\mathcal{T}(\alpha)$, is an argumentation tree for α such that the children nodes of a node A consist of all the canonical undercuts of A that satisfy condition 2 of Definition 8.*

Notation. To simplify the notation, from now on, the conclusion of a canonical undercut is denoted $\diamondsuit$, obviously, there is no ambiguity as to which formula it stands for.

Example 4. *Let* $K = \{a \wedge b, \neg b \vee \neg c, \neg a \wedge \neg d, c, \neg c, d, \neg d \vee c, e \rightarrow f\}$.

The complete argumentation tree for the formula b *is visualised bellow.*

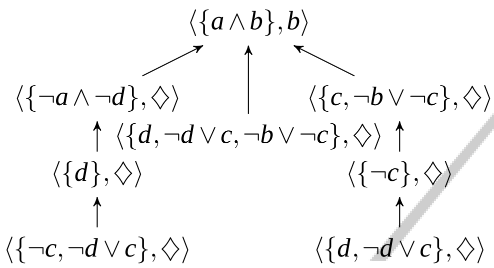

Figure 1: Complete argumentation tree for b.

2.3 Inconsistency Measures

In this section, we describe some inconsistency measures defined through minimal inconsistent subsets and the properties usually used for their characterization. We limit our presentation to the most important and related syntax based measures to the one proposed in this paper.

Reasoning with minimal inconsistent sets is a widely studied concept that gives rise to several measures of inconsistency of an entire knowledge base or one of these formulae. These measures are mostly based on two criteria which are the number of MUSes and their size. In (Hunter, 2004), the authors define the degree of inconsistency of a formula as the number of MUSes containing it. Extended to the entire knowledge base, this measure has resulted in inconsistency measure which is defined to be the number of MUSes. Furthermore, in (Hunter and Konieczny, 2008) the authors introduced a family of inconsistency measures called MinInc inconsistency values MIV. For instance, the MIV_D measure is a basic one that assigns 1 if the formula belongs to a MUS and 0, otherwise. When $MIV_\#$ value is identical to MIV_D, and which associates a formula α the number of MUSes to which it belongs. Finally MIV_C measure, defined as $MIV_C(K,\alpha) = \sum \frac{1}{|M|}$ such that $M \in MUSes(K)$ and $\alpha \in M$, is a generalization of the framework $MIV_\#$, since it takes into account the size of each MUS containing α.

In contrast with the semantic measures, the approaches based on minimal inconsistent subsets have some gaps. Indeed, such syntactic approaches do not make a distinction in the degree of inconsistency between two different knowledge bases with exactly the same size and the same number of minimal inconsistent subsets, what motivated the new measure introduced in (Mu et al., 2011b). This approach combines both the minimal inconsistent subsets and the maximal consistent subsets in order to give an inconsistency degree of the whole knowledge base. Another approach that combines semantic and syntax based approaches have been introduced in (Xiao and Ma, 2012). It is based on counting the variables of minimal inconsistent subsets and the minimal correction subsets (Reiter, 1987).

However, while analyzing deeply the divergence between the different approaches dealing with minimal inconsistent subsets, we notice that an important point has not been taken into account in these approaches. More precisely, the correlation between MUSes have to be taken into account during the evaluation of the inconsistency degrees in the knowledge bases.

In order to give the intuition behind the introduction of our new measures based on logical argumentation, let us consider firstly the knowledge base K such that $K = \{a, \neg a, a \vee b, \neg b, b\}$. K is inconsistent. For example, the inconsistency measure $MIV_\#$ (resp. MIV_D) assigns the value 1 (resp. 1) to the formulae a, $a \vee b$ and b. However, we would like that $a \vee b$ has a better value, unlike to a and b, since the formulae a, $a \vee b$ and b have the same properties except that $a \vee b$ belongs to a MUS more connected than a and b. Consequently, our goal is to use the interactions between MUSes as a main element in order to evaluate the conflict brought by each formula of the knowledge base.

One of the most known structure able to capture the interactions between MUSes is undoubtedly the *argumentation tree*. Argumentation tree is a well known concept widely explored in the context of the classical logic argumentation. This tree offers a faithful image of the interactions of MUSes and allows then to reason more finely about inconsistency, since the attack relation definition ordinarily relates attacks to inconsistency. Note that this is unlike the oriented links between MUSes explored in (Benferhat and Garcia, 1998) which says that the resolution of one MUS allows automatically for the resolution of the other. So, by looking inside MUSes and taking into account the correlations between them, the proposed analysis of MUSes structure lead to several interesting measures different from the existing ones.

3 ARGUMENTATION-BASED INCONSISTENCY MEASUREMENT

In this section, we discuss how to use logical argumentation to address the problem of measuring the degree of inconsistency in knowledge bases. Our approach offers considerable advantage as it actually supports the use of diverse logics, not just propositional logic. In other words, our framework can be naturally extended to other logics where arguments are defined like first order logic (Besnard and Hunter, 2005), conditional logic (Besnard et al., 2013), modal logic (Raddaoui, 2013), description logic (Black et al., 2009), resource logic (Besnard et al., 2012), etc.

Before formalizing our inconsistency measurement framework, we need further notations that will be useful in the following section. Let $\mathcal{T}$ be a complete argumentation tree, $nodes(\mathcal{T})$ denotes the set of nodes of $\mathcal{T}$. $|\mathcal{T}|$ is the number of nodes (i.e., arguments) of $\mathcal{T}$. Let $n \in nodes(\mathcal{T})$, we denote by $\mathcal{H}(n)$ the height of n, i.e. the number of nodes from the root to n. We will also use $\mathcal{H}(\mathcal{T})$ to denote the height of $\mathcal{T}$. Given an argument $\langle\Phi,\alpha\rangle$, $\mathcal{U}ndercuts(\Phi)$ is the set of children of $\langle\Phi,\alpha\rangle$ in $\mathcal{T}$. For instance, the set of undercuts of an atomic argument $\langle\alpha,\beta\rangle$ is denoted as $Undercuts(\alpha)$.

We can now show that the inconsistency of the knowledge base is rooted by the presence of conflictual arguments.

Proposition 3. *Let K be a knowledge base. If K is inconsistent, then there exists at least one complete argumentation tree $\mathcal{T}$ such that $|\mathcal{T}| > 1$.*

The following result states the relationship between minimal inconsistent subsets and attacks between arguments in the sense that if an argument attacks another, then it must be that the support of the former is inconsistent with the support of the latter.

Proposition 4. *Let $\mathcal{T}$ be a complete argumentation tree in K s.t. $|\mathcal{T}| \geq 2$. For each argument $\langle\Phi,\alpha\rangle \in \mathcal{T}$, there exists a MUS $M \subseteq K$ such that $M \subseteq \Phi \cup \Psi$ where $\langle\Psi,\beta\rangle \in \mathcal{U}ndercuts(\Phi)$.*

As shown by Proposition 4, the complete argumentation tree can gather many minimal inconsistent subsets of a given knowledge base in the same structure, and thus it takes the dependencies between these MUSes into account.

Now, we characterize the notion of a free formula in the light of the complete argumentation tree as follows.

Proposition 5. *Let K be a knowledge base. Let α be a formula in K. α is a free formula of K iff for each complete argumentation tree $\mathcal{T}$ such that $\langle\alpha,\beta\rangle$ is the root of $\mathcal{T}$, $|\mathcal{T}| = 1$.*

Now we can explore the advantages of considering argumentation to quantify the degree of conflict in knowledge bases. In the following, we introduce the degree of inconsistency measure of each formula belonging to a given knowledge base K. More precisely, we give in the following a family of degree of inconsistency measure, denoted as I_{ARG}, in terms of complete argumentation tree. These measures aim specially to take into account not only the attacks between arguments and their number but also the quality of each attack.

In logical argumentation, arguments may attack each other, which is captured by logical conflict. This requirement reflects a fundamental assumption in logical argumentation, namely that the conflict among arguments is related to attack between them. So, the amount of the contradiction in the arguments can then be viewed as the number of their attackers. More formally, the following result holds.

Proposition 6. *Let K be a knowledge base and $\langle\Phi,\alpha\rangle$ be an atomic argument. Then, $|\mathcal{U}ndercuts(\Phi)| = |\{M \mid M \in MUSes(K),\ \Phi \cap M \neq \emptyset\}|$.*

Proof. We see from Proposition 4, for each undercut $\langle\Psi,\beta\rangle$ for $\langle\Phi,\alpha\rangle$, there exists a MUS M s.t. $M \subseteq \Psi \cup \Phi$. Then, it easy to see that the number of undercuts of $\langle\Phi,\alpha\rangle$ is equal to the number of MUSes containing Φ. □

Proposition 6 suggests that the existence of undercuts of a given argument depends on the set of MUSes involving its support. So, the evaluation of the contradiction of a formula (i.e. support of the argument) is linked to the set of counter-arguments of the argument containing this formula. According to this observation, one can notice that the $MIV_{\#}$ measure is simply the number of canonical undercuts that defeat the argument $\langle\alpha,\beta\rangle$, i.e., $MIV_{\#}(\alpha) = |\mathcal{U}ndercuts(\alpha)|$. Then, we can see that each time a counter-argument exists, we increase the degree of inconsistency by 1. However, according to the argumentation tree, the initial argument can be challenged, as well as counter-arguments to the initial argument can themselves be challenged, and so on, recursively. This means that the amount of conflict is splitting among the whole argumentation tree, telling that the intuition behind that the conflict lie in the counter-arguments, counter counter-arguments, etc.

Our goal is then to claim that the degree of inconsistency of the formula supported the initial argument must decrease when the counter-arguments of this argument are themselves be attacked, and so on. This is the direct formulation of the idea that the more arguments needed to produce the argumentation tree, the less inconsistency there is in the root. To this end, it should be natural to take all attacks among arguments in order to evaluate more finely the inconsistency of the formulae. Starting from this, it is obvious to see that $MIV_{\#}$ measure provides a local evaluation of the inconsistency since it considers only the neighborhood of α (e.g., only MUSes containing α), and consequently only the counter-arguments of the initial argument are taken into account. For instance, the $MIV_{\#}$ measure assigns the same blame to each formula belonging to the same number of interconnected MUSes. This result shows that the $MIV_{\#}$ measure is not sufficiently discriminating for our purposes since just one level of inconsistency is considered. Note that the same reasoning can be obtained if we consider the MIV_D or the MIV_C measures.

In the sequel, we will present different measures able to consider such aspects by tacking the whole structure of the argumentation tree into account to evaluate the responsibility/contribution of each formula in the inconsistency of the knowledge base.

To address this need, let us now introduce a uniform definition of an inconsistency degree under the complete argumentation tree in order to make a distinction among the formulae of the knowledge base according to their participation in the inconsistency.

Definition 10. *Let K be a knowledge base s.t. $\alpha \in K$. Let $\langle\alpha,\beta\rangle$ be an atomic argument. Let $\mathcal{T}$ be a complete argumentation tree s.t. $\langle\alpha,\beta\rangle$ is the root of $\mathcal{T}$. The inconsistency degree of α, denoted $I_{ARG}(\alpha,K)$, is defined as:*

$$I_{ARG}(\alpha,K) = \begin{cases} 0 & if\ |\mathcal{T}| = 1 \\ \frac{|\mathcal{U}ndercuts(\alpha)|}{|\mathcal{T}|-1} & otherwise \end{cases}$$

I_{ARG} measure assigns as an inconsistency degree the ratio between the number of counter-arguments of $\langle\alpha,\beta\rangle$ and the size of the argumentation tree, but 1 must be subtracted to not count the root of the tree. Hence, more no challenged counter-arguments for the initial argument means higher degree of inconsistency. This allows us to draw a more precise picture of the inconsistencies of the formulae in the knowledge base. Note that assigning the maximum value of 0 as a the degree of conflict of a free formula seems to be very natural, since a free formula has nothing to do with the conflicts of the knowledge base.

Next, we show that the inconsistency measure defined above satisfies the consistency, and free formula independence properties.

Proposition 7. *Let K be a knowledge base and $\alpha \in K$. The inconsistency measure I_{ARG} satisfies the following properties:*

- *$I_{ARG}(\alpha,K) = 0$ if K is consistent (consistency)*
- *$I_{ARG}(\alpha,K) = 0$ iff $\alpha \in free(K)$ (free formula independence)*

Note that according to the I_{ARG} measure, the inconsistency can decrease when new formulae are added in the knowledge base. This is explained by the fact that the number of MUSes can increase as well as the number of counter-arguments in the argumentation tree. Hence, the I_{ARG} measure is not monotonic.

Example 5. *Let us consider the knowledge base of Example 4. Then, the I_{ARG} value gives as result:*

$I_{ARG}(\{a \wedge b\},K) = \frac{3}{7}$, $I_{ARG}(\{\neg a \wedge \neg d\},K) = \frac{1}{3}$, $I_{ARG}(\{\neg b \vee \neg c\},K) = \frac{1}{5}$, $I_{ARG}(\{c\},K) = \frac{2}{7}$, $I_{ARG}(\{d\},K) = \frac{2}{7}$, $I_{ARG}(\{\neg c\},K) = \frac{2}{9}$, $I_{ARG}(\{e \rightarrow f\},K) = 0$.

The following result suggests that free arguments are not challenged by any other arguments. This means that these arguments have nothing to do with the conflicts of the knowledge base.

Proposition 8. *Let K be a knowledge base and $\alpha \in K$. If $\langle\alpha,\beta\rangle$ is a free argument, then $I_{ARG}(\alpha,K) = 0$.*

It is interesting to note that the argumentation tree associated to the formula α does not usually involve all MUSes of the knowledge base but only those interconnected with the MUSes containing α. What happens is that while such interconnected MUSes do not contain cycles (e.g. each argument is not duplicated in many branches of the tree), the number of nodes remains the same for each formula belonging to these interconnected MUSes. In this case, the inconsistency measure is only sensitive to the number of the canonical undercuts of the initial argument. As consequently, the size and the dependencies between the set of MUSes the formula α belongs to, can have an impact on the evaluation of the inconsistency. To illustrate this, let us consider the following example.

Example 6. *Let us consider the knowledge bases K_1 and K_2 such that $K_1 = \{a, \neg a \wedge b, \neg a \wedge \neg b\}$, and $K_2 = \{\neg a, a, \neg a \vee b, b, \neg b\}$. From this, we obtain the following complete argumentation trees for $a \vee b$. Then, we have $I_{ARG}(a,K_1) = \frac{1}{2}$ while $I_{ARG}(a,K_2) = \frac{2}{3}$.*

The evaluation of the inconsistency value of each formula given by I_{ARG} is still very rough. In particular, the I_{ARG} measure is not yet able to distinguish finely between the formulae of the knowledge base. Indeed, if we consider again the knowledge base K_2 of Example 6, the values of inconsistency of $\neg a$, $\neg a \vee b$

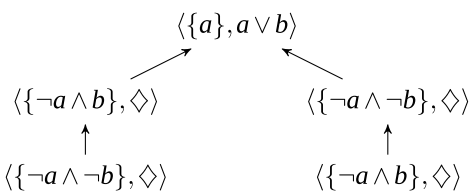

Figure 2: Complete argumentation tree for $a \vee b$.

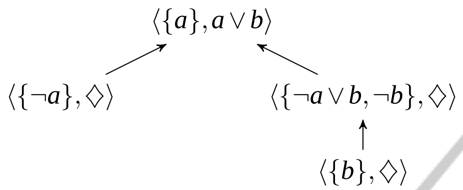

Figure 3: Complete argumentation tree for $a \vee b$.

and $\neg b$ are equal. So it could prove better not to simply take the number of counter-arguments of the root of an argument tree, but to take into account the height of the argumentation tree as well as the distance between nodes.

To address this need, we see in the following that by taking a more refined inconsistency measure on knowledge bases, we get a better assignment to formulae.

Definition 11. *Let K be a knowledge base such that $\alpha \in K$. Let $\langle \alpha, \beta \rangle$ be an atomic argument. Let $\mathcal{T}$ be a complete argumentation tree s.t. $\langle \alpha, \beta \rangle$ is the root of $\mathcal{T}$. The inconsistency degree of α is defined as:*

$$I^*_{ARG}(\alpha, K) = |\mathcal{U}ndercuts(\alpha)| \times f(\mathcal{T})$$

where f is a function that takes as input the complete argumentation tree $\mathcal{T}$.

The above definition is a general definition that allows for a range of possible measures to be proposed. Note that instances of I^*_{ARG} depend on the choice of function f.

Next we will introduce three types of functions as follows:

$$f^1(\alpha) = \frac{1}{\mathcal{H}(\mathcal{T})}$$

$$f^2(\alpha) = \frac{1}{\sum\limits_{n \in nodes(\mathcal{T})} \mathcal{H}(n)}$$

$$f^3(\alpha) = \sum_{n \in nodes(\mathcal{T})} \frac{1}{\mathcal{H}(n)}$$

The differently defined functions lead to different inconsistency measures. Let us explain the resulting measures as follows: Firstly, $I^1_{ARG}(\alpha, K) = |\mathcal{U}ndercuts(\alpha)| \times f^1(\alpha)$ takes into account the height of the complete argumentation tree associated to α. secondly, $I^2_{ARG}(\alpha, K) = |\mathcal{U}ndercuts(\alpha)| \times f^2(\alpha)$ considers the ratio between the counter-arguments of the initial argument and the sum of all other arguments of the complete argumentation tree where each one is represented by its distance to the root node of the tree. Finally $I^3_{ARG}(\alpha, K) = |\mathcal{U}ndercuts(\alpha)| \times f^3(\alpha)$ computes the weighted sum of each node of the tree, where the weight is the inverse of the hight of the corresponding node.

Although, the three above inconsistency measures are quite different, they allow to analyse more deeply the structure of the complete argumentation tree by taking into account the dependencies between MUSes in the knowledge base. This allows us to define a much more precise view of the inconsistency, as illustrated in the following example.

Example 7. *Let us consider the knowledge base $K = \{\neg a, a, \neg a \vee b, b, \neg b\}$. Then, we have:*

- $I^1_{ARG}(a) = 1, I^1_{ARG}(\neg a \vee b) = \frac{1}{2}, I^1_{ARG}(\neg a) = \frac{1}{3}$
- $I^2_{ARG}(a) = \frac{1}{2}, I^2_{ARG}(\neg a \vee b) = \frac{1}{5}, I^2_{ARG}(\neg a) = \frac{1}{6}$
- $I^3_{ARG}(a) = 5, I^3_{ARG}(\neg a \vee b) = 2, I^3_{ARG}(\neg a) = \frac{11}{6}$

Interestingly, we note that by using the family I^*_{ARG} of inconsistency measures we have the following relation: $I^*_{ARG}(\neg a) = I^*_{ARG}(b) < I^*_{ARG}(\neg a \vee b) < I^*_{ARG}(a) = I^*_{ARG}(\neg b)$. We can notice that now we can make a distinction between the formulae $\neg a$, $\neg a \vee b$, and $\neg b$.

3.1 Logical analysis

As seen earlier, the I^1_{ARG} measure combines the cardinality of the set of canonical undercuts, and the inverse of the height of the complete argumentation tree in order to quantify the participation of each formula in the inconsistencies. Note that adding new formulae to a knowledge base may grow the height of the argumentation tree (by adding new counter-arguments in the tree), and consequently the I^1_{ARG} value decreases. This means that I^1_{ARG} is not monotonic. To illustrate, let us consider the knowledge base $K = \{a, \neg a \wedge b, \neg b \wedge c, \neg c \wedge d, \neg d \wedge e\}$. Then, we have $I^1_{ARG}(a) = \frac{1}{4}$. Now if we add the new formula $\neg e$ to K, then the degree of inconsistency of a in $K \cup \{\neg e\}$ becomes $\frac{1}{5}$.

In contrast, I^2_{ARG} value counts the inverse of all distances from the root node to each argument in the tree. Moreover, adding new formulae cannot decrease the number of counter-arguments, and cannot increase the distance of existing *MUSes* from the root node, thus the inverse of the distances will be non-decreasing. Consequently, the I^2_{ARG} measure is monotonic. By the same reasoning, I^3_{ARG} is a monotonic measure.

3.2 Quantifying the Conflict of a Set of Formulae

In this section, we consider another inconsistency measure which aims to evaluate the amount of conflict of a set of formulae. To do this, the I_{ARG} measure can be naturally extended to a consistent subset of formulae, by just taking this subset as a support of the root argument in the complete argumentation tree.

Now, the inconsistency measure I_{ARG} can be defined as follows:

Definition 12. *Let K be a knowledge base and $\mathcal{S}$ a consistent subset of K. Let $\mathcal{T}$ be a complete argumentation tree s.t. $\langle \mathcal{S}, \alpha \rangle$ is the root of $\mathcal{T}$. The degree of inconsistency of S is defined as:*

$$I_{ARG}(\mathcal{S},K) = \begin{cases} 0 & if\ |\mathcal{T}| = 1 \\ \frac{|\mathcal{U}ndercuts(\mathcal{S})|}{|\mathcal{T}|-1} & otherwise \end{cases}$$

This definition allows us to define to what extent a subset of formulae inside a formula is concerned with the inconsistencies of the knowledge base.

Note that instances of I^*_{ARG} measure can be obviously extended to evaluate the inconsistency of a set of formulae by just considering a consistent subset of the knowledge base as a support of the root argument of the complete argumentation tree.

Example 8. *Let the knowledge base $K = \{a \wedge b, c, a \to \neg b \vee c, a \to \neg b, d, \neg d, \neg c, d \to a\}$. Then $I_{ARG}(\{a \wedge b, a \to \neg b \vee c\}, K) = \frac{1}{3}$.*

Now, we can show that for a particular case of interconnected MUSes, the following result holds.

Proposition 9. *Let K be a knowledge base and $S \subseteq K$ s.t. $S \nvdash \perp$. If there exists no chain of interconnected MUSes of a length greater then 2, then the maximum value reached by $I_{ARG}(\mathcal{S},K)$ is equal to 1 and $I^1_{ARG}(\mathcal{S},K)$ is equal to $|MUSes(K)| - |selfC(K)|$.*

4 CONCLUSION

In this paper, we have presented a new framework for defining inconsistency values that allow to associate each formula with its degree of contribution for the conflict of the whole base. Our approach is based on logical argumentation which is shown to be a useful approach to take the interaction between MUSes into account. We have also shown that such a framework can be extend to quantify the degree of conflict of a consistent subset of formulae. We also proposed some logical properties to characterize our inconsistency measures.

In future work, we plan to investigate the computational complexity of I_{ARG} family of inconsistency measures, and develop algorithms and implementations, possibly based on techniques of the computation of arguments (Besnard et al., 2010). Additionally, we will study how our inconsistency measures could be used to direct step-wise resolution of inconsistency. Finally, we plan to undertake case studies of applications of our framework of inconsistency degrees.

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