

# Robust Flight' Scheduling in a Colombian Domestic Airline

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Abstract: Air traffic has been grown rapidly, increasing the airlines' competition, generating complex planning problems for airlines and major customers' demands. Airlines' profitability is highly influenced by its planners ability to face these challenges and build efficient schedules. In this paper, we developed a bi-objective optimization model for the timetabling problem of a Colombian domestic airline. Preliminary results show an increase of 12% respect to the current profitability of the airline.

## 1 INTRODUCTION

According to statistics from Colombian Civil Aviation Authority (Aeronautica Civil, 2013), the Colombian market for domestic air passengers increased by 21.7%, corresponding to 1.49 million of passengers, over the previous year in the first five months of 2013. In turn, the load factor of the market increased from 75.3% to 77.0% in the same period. Besides, international passengers exhibit a similar trend. By May of 2013, the number of international passengers increased in about 403,000 passengers compared to the same period of 2012. This traffic increase, the strong competition among airlines and passenger demand for better services, have created complex planning problems for airlines, which require new models and solution methods (Dorndorf et al., 2007).

All this has led to the airlines to spend considerable time in a complex decision process called airline planning (Cadarsoa and Marín, 2011). This process seeks to produce an operational program and it is composed of the following five stages: fleet planning, flights' network planning, revenue management, crew scheduling and planning of airport resources (Lohatepanont, 2002).

This paper focuses on a problem that arises in the flights' network planning stage. It begins about 12 months and lasts about 9 months before the deployment of the program (Lohatepanont, 2002). This stage comprises several subproblems since it has been deemed untreatable because of its computational complexity. Therefore, several subproblems are optimized sequentially and the

output of one is taken as the input of the next one (Papadakos, 2009). These subproblems are named: schedule design, fleet assignment, maintenance programming of aircrafts and sometimes also include crew scheduling (Barnhart et al., 2003).

Within the flights' network planning, the schedule design subproblem addresses the most important decisions for an airline. These decisions determine the profit of the airline because they define which markets operate, including cities, routes, frequencies and hours to be offered in the day (Weide, 2009). Usually, the schedule design problem is divided into two steps: the frequency planning and the development of the timetables (Cadarsoa and Marín, 2011). This paper focuses on the latter step, i.e., the development of the timetables.

In frequency planning, planners determine the appropriate number of frequencies for a market (Lohatepanont, 2002). Increases in the frequency of departures on a route, commonly improve convenience for customers and in turn the airline can benefit from increased traffic and associated revenues, provided that this increase is accompanied by a market study to ensure that the operation is profitable (Belobaba et al., 2009).

After, the frequency planning process, the next step is the development of the routes (also known as timetable development), where the planners decide the day and hour in which each flight will be offered. The result of the timetable development is a list of flights, with dates and departure and arrival times, called basic programming (or itinerary) (Rabetanety et al., 2006).

Airlines commonly design their schedules under the premise that all flights arrive and depart on the planned hours. This scenario is rarely met, leading the airline to incur in additional costs (Lan, 2003). An efficient flight scheduling can contribute to increase the level of service and customer satisfaction. Under these ideas, the quality of a scheduling is measured by its level of robustness (Bian et al., 2005). The robustness of a schedule can be defined as the ability to start all the flights scheduled on time despite of the delays in their predecessors. To achieve this goal, a schedule has to include some firewalls or time windows without programmed flights such that they can absorb the flights delays through the day and the following flights can depart on time.

This paper addresses the timetabling problem in a Colombian domestic airline scheduling design. Currently, the airline constructs its schedules based on its planners' expertise and lacks clearly defined robustness measures. In this paper, we propose a bi-objective optimization model and a solution method designed to achieve optimal schedules that increase profitability and take into account service measures such as robustness.

The remainder of this paper is organized as follows: Section 2 presents a brief literature review. Section 3 introduces the proposed bi-objective optimization problem. Section 4 summarizes the results of preliminary computational results. Finally, Section 5 gives conclusions and outlines future research possibilities.

## 2 LITERATURE REVIEW

Since the late 50's, operations research has played a fundamental role on helping airline industry to sustain high rates of growth. Thus, over 100 airlines and air transport associations created the Air Group of Operations Research Societies (AGIFORS) in 1961 (Barnhart et al., 2003). Within the list of air transport issues in which operations research has contributed through optimization and stochastic models are: airline fleet planning, maintenance planning, decision support tools for managing air operations, classical problems of flights scheduling and crews assigning, revenue management, flights performance management, among others (Barnhart et al., 2003).

Regarding the frequency of scheduling, the literature distinguishes between daily, weekly and dated - problems. The first one assumes that the schedule repeats every day with the same flights in

each of them. The second one assumes that scheduling repeats weekly and the flight may vary on some days of the week. And the third one considers that there are no restrictions about the replication of flights for different days (Weide, 2009). In this work we address a daily scheduling problem.

Table 1: Solution Techniques for Flights Scheduling Models.

Solution techniques	Works
Lagrangian relaxation	(Chen et al. 2010)(Yan & Tseng 2002)(Sherali et al. 2009)
Network techniques	(Stojkovi & Solomon 2002)(Tang et al. 2008)
Heuristics	(Tang et al. 2008)(Kim & Kim 2011)(Babic et al. 2011)(Yan et al. 2008)(Clarke 1998)(Weide 2009)
Metaheuristics	(Jungai & Hongjun 2012)(Kim & Barnhart 2007)(Lee et al. 2007)(Burke et al. 2010)
Columns Generation	(Barnhart et al. 1998)(Papadakos 2009)
Benders decomposition	(Mercier et al. 2005)(Papadakos 2009)

Commonly, flights scheduling models are large scale in terms of the number of variables requiring solution methodologies that decompose and reduce the size of the problem. Usually, these problems have been solved by column generation, branch-and-cut or branch-and-price algorithms, lagrangian relaxation and Benders decomposition (Weide et al., 2008). However, as Table 1 illustrates, approximate techniques such as heuristics and metaheuristics have also been used.

The aforementioned techniques have been applied in several real world problems. For instance, (Kim and Kim, 2011) considered the planning of operations in a military aviation unit. They deal with the problem of assigning flight missions to aircraft and schedule those tasks. The authors developed heuristic algorithms to reduce the time required to complete all missions, they conclude that the heuristics reach near optimal solutions in reasonable computational times. Similarly, (Cadarsoa and Marín, 2011) presented a robust approach that integrates frequency planning and timetable development in a simple model in order to build economic solutions. Their model was implemented at IBERIA airline.

Several robustness measures have been studied, the most common are: the probability that a flight can connect to any next flight (Sohoni et al., 2011), the probability of having misconnected passengers (Sohoni et al., 2011), the deviations of optimal departing hours, the minimization of the costs of deviations from the optimal scheduling, the minimization of flight delays, and the capacity to recover the operation after a delay (Lan, 2003).

The reviewed literature reveals that the simultaneous consideration of schedule robustness and usual airlines targets is seldom studied. However, (Sohoni et al., 2011) proposed an integer programming formulation that perturbs optimally a given schedule in order to maximize expected profits while maintaining service levels.

In the Colombian context, flights schedules are generally built based only on the planners or managers expertise. Additionally, in the airline under study there is a lack of a solution method that targets the schedule robustness and take into account, simultaneously, the usual airlines goal of maximizing its profitability. This paper aims to address these two issues.

This paper aims to fill this gap through the construction of a bi-objective optimization model and solution method, which maximize airline profitability, considering the robustness of the flight schedule.

### 3 PROBLEM DEFINITION

Through this section, the term timetable or itinerary is defined as the final configuration of the schedule, which provides the time at which each route will be offered.

Given a set  $C = \{1, 2, \dots, c\}$  of cities to be connected the frequency  $f_{ij}$  represents the number of required flights connecting the origin city  $i \in C$  and the destination city  $j \in C$ . The set  $H = \{1, 2, \dots, h\}$  corresponds to the time slots (i.e., hours available to operate the flights), and  $BH_{ij} (i, j \in C)$  represents the duration of each flight leg.

#### 3.1 Problem Representation and Notation

The flight-scheduling problem is represented through a space-time graph, as it is shown in Figure 1. A vertical move in this graph represents a travel between cities, while horizontal moves represent a temporal movements between time slots Using this

representation, a feasible flight is represented by the arc that joins the origin city  $i \in C$  and the destination city  $j \in C$ , taking off at hour  $m \in H$  and landing at hour  $n \in H$ . The profitability of an arc  $r_a$  is defined as the revenue generated by operating a flight at a given departure hour. Figure 1 shows the arc representing a flight from city C to city E, starting at time 1 and ending at 5. Note that only feasible arcs are included in this representation (i.e., arcs where  $BH_{ij} = n - m$ )

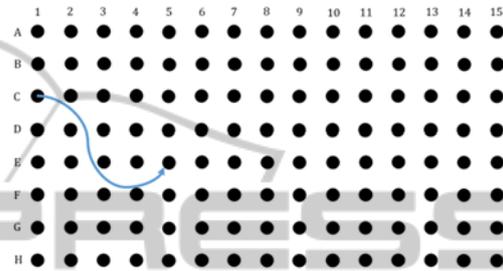


Figure 1: Space-time graph.

Considering the feasible set of arcs the notation used to formulate the problem is as follows:

#### Sets

- $C$  Set of cities that makes up the origin and destination of a leg
- $A$  Set of available aircraft
- $H$  Set of day's hours
- $ARC$  Set of feasible arcs

#### Parameters

- $f_{ij}$  Frequency for leg  $(i, j) i, j \in C$
- $r_a$  Profitability of arc  $a \in ARC$
- $o_a$  Origin of leg  $a \in ARC$
- $d_a$  Destination of leg  $a \in ARC$
- $hi_a$  Start time of leg  $a \in ARC$
- $hf_a$  Ending time of leg  $a \in ARC$

- $MP_i$  Maximum allowed arrival time of an aircraft to its base  $i \in C$
- $AV_i$  Number of aircraft at base  $i \in C$
- $\rho$  A non-negative real number  $\rho \in [0, 1]$
- $pmax$  Maximum profitability
- $rmax$  Maximum robustness

Some assumptions were made for the construction of the following optimization model: (i) The rotation time required to prepare an aircraft for the next flight is included into the length of time for each flight arc. (ii) Two flights with the same destination cannot take off at the same time from the same city of origin. (iii) It is possible to have more than one

plane parked at the same time in any city. (iv) An aircraft returns to its base at the end of the day. (v) The fleet is homogenous, i.e., all airline aircrafts are of the same type.

### 3.2 Mathematical Model

The flight-scheduling problem was formulated as a bi-objective optimization model where the two objectives are profitability and robustness. These objectives were combined in a single objective using a weighting method after scaling their magnitudes to make them comparable.

$$\text{Max } \rho * \sum_{a \in \text{ARC}} \frac{r_a * x_a}{p_{\text{max}}} \quad (1)$$

$$+ (1 - \rho) * \sum_{a \in \text{ARC} | o_a = d_a} \frac{x_a}{r_{\text{max}}} \quad (1)$$

$$\sum_{\substack{a \in \text{ARC} \\ |o_a = i \text{ and } d_a = j}} x_a = f_{ij} \quad \forall i \in C, j \in C \quad (2)$$

$$\sum_{\substack{a \in \text{ARC} \\ |o_a = i \text{ and } h_{i_a} = 1}} x_a = AV_i \quad \forall i \in C \quad (3)$$

$$\sum_{\substack{a \in \text{ARC} \\ |d_a = i \text{ and } h_{f_a} = MP_i}} x_a = AV_i \quad \forall i \in C \quad (4)$$

$$\sum_{\substack{a \in \text{ARC} \\ |d_a = q \text{ and } h_{f_a} = p}} x_a = \sum_{\substack{a \in \text{ARC} \\ |o_a = q \text{ and } h_{i_a} = p}} x_a \quad (5)$$

$$\forall p \in h; q \in C | p \neq 1, p \neq MP_q$$

$$x_a \in \{0,1\} \quad \forall a | o_a \neq d_a \in \text{ARC} \quad (6)$$

$$x_a \in \mathbb{Z} \quad \forall a | o_a = d_a \in \text{ARC} \quad (7)$$

The first term of the objective function (1) aims at maximizing the profitability of the timetable, while the second part seeks to maximize the scheduling robustness by creating time windows with aircraft parked at some cities to absorb the delay of previous flights. The constraint set (2) ensures that the number of frequencies defined for each leg is met. Constraints (3) guarantee that at the beginning of the day the number of planes that leave each city is equal to the number of planes located in each base. Constraints (4) requires that to each base city arrive the total number of aircrafts corresponding to them at the maximum arrival time  $MP_i$ . Constraints (5) maintain the balance between the number of planes

that goes in and that comes out at each node in the graph. Finally, constraints (6) and (7) define the nature of the decision variables.

Notice that decision variables take binary values for the cases in which the origin and destination cities are different. This is due to the assumption that two flights with the same destination cannot take off at the same time in a given origin city. On the other hand, decision variables can take integer values for the cases in which these cities are the same. This was necessary in order to model the fact that one or more aircrafts can be parked in the same city during a given period of time. In that case, the arc corresponding to the same origin and destination would take the value of the number of aircrafts parked in that city.

## 4 COMPUTATIONAL EXPERIMENTS

The model (1)-(7) was implemented using Xpress 7.5 and Gurobi 5.6.2 and all the computational experiments were run in a computer with an Intel Corei3-2350M processor running under Windows 8 at 2.30GHz with 6GB of RAM.

The data to run the model was gathered from a Colombian domestic airline. Based on this data, we created a realistic instance corresponding to their flight-scheduling decision. From this instance, we created other 11 instances that correspond to interesting scenarios that the company may face in the near future. The convention used for naming the instances is such that, the first two digits indicate the number of aircrafts included, the following two digits represent the number of legs to be scheduled

Table 2: Results for instances.

Instance	Profitability	Robustness	O.F	Time (s)
043210	575	236	811	1.3
043411	604	194	798	2.2
043410	600	201	801	1.8
053812	760	347	1107	5.4
054213	964	215	1179	6.2
054613	993	132	1125	6.1
065014	1100	269	1369	3.8
065215	1134	222	1356	10.7
065415	1146	186	1332	35.5
076014	1273	307	1580	4.3
075813	1280	349	1629	1.1
044211	944	33	977	5.6

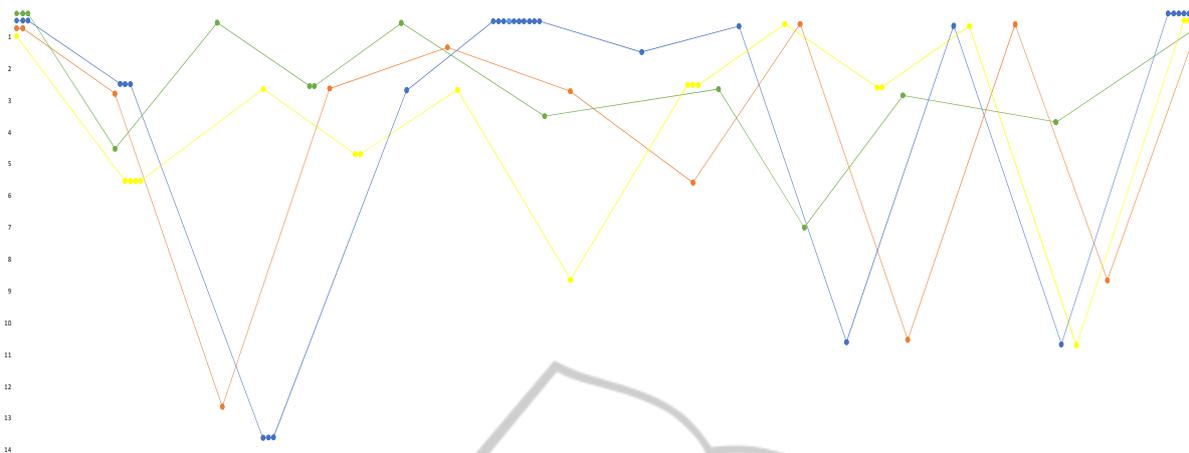


Figure 2: Solution obtained.

and the last two digits stand for the number of cities to be connected. These instances are available from the author upon request. Table 2 shows the results obtained for each of these instances.

For the realistic instance 044211, Figure 2 shows the structure of the solution obtained. Each line shows the path of each aircraft (flight legs flown for each aircraft).

To explore the trade-off between objectives we changed systematically  $\rho$  ( $\rho = \{0, 0.05, \dots, 0.95, 1.0\}$ ) to approximate the efficient frontier for the bi-objective flight-scheduling problem. The trade-off between the two objectives using different weights, for the more realistic instance, is shown in Figure 3. Moreover, in this instance, and where the two objectives have the same importance, the model obtains a good solution in terms of both profitability and robustness since this model found a solution that improves the daily average airline profitability by 12%. While in terms of robustness the solution presents 155 minutes of firewalls to recover the operation if case of some flight delays.

In terms of computing time, taking into account that this type of decisions is revised every year and

since the solution for each value of  $\rho$  takes less than 6 seconds, these running times seem reasonable.

### 5 CONCLUSIONS

The computational experiments shows that is possible to solve exactly the timetabling problem related to the flight scheduling of a Colombian domestic airline. Moreover, the efficient frontier obtained with a weighting method reveals that there exist a trade-off between profitability and robustness. However, the robustness measure used in this paper has some limitations, since it does not take into account aspects like the durations of the firewalls and their spread through the day that are important in terms of the quality of the delays absorption. However, we are already working on improving it.

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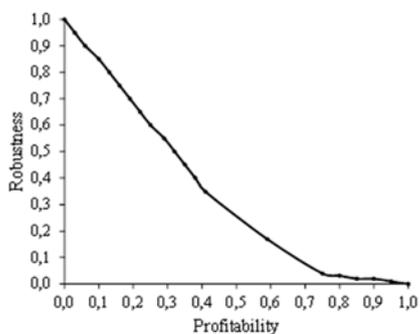


Figure 3: Trade-off between objectives.

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