A Cooperative Game Approach to a Production Planning Problem

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Abstract: This paper deals with a production planning problem formulated as a Mixed Integer Linear Programming (MILP) model that has a competition component, given that the manufacturers are willing to produce as much products as they can in order to fulfil the market's needs. This corresponds to a typical game theoretic problem applied to the productive sector, where a global optimization problem involves production planning in order to maximize the utilities for the different firms that manufacture the same type of products and compete in the market. This problem has been approached as a cooperative game, which involves a possible cooperation scheme among the manufacturers. The general problem was approached by Owen (1995) as the "production game" and the core was considered. This paper identifies the cooperative game theoretic model for the production planning MILP optimization problem and Shapley Value was chosen as the solution approach. The results obtained indicate the importance of cooperating among competitors. Moreover, this leads to economic strategies for small manufacturing companies that wish to survive in a competitive environment.

1 INTRODUCTION

The high competition in the market has led many companies to adopt supply chain management in order to obtain better results and competitive advantages to achieve a good positioning in them.

For this reason, businesses today search for an optimal performance of their overall operations in important areas like Production and Logistics (Gimenez and Ventura, 2005). In order to do this, many authors have provided contributions in this field: Optimizing Inventory Operations (Hartman and Dror, 2003); Optimal operations planning (Li et al., 2003); Optimal price and return policy (Mukhopadhyay and Setaputro, 2004); Optimal operations of transportation fleet (Kang et al., 2008); Optimal multi-stage logistic and inventory policies (Hsiao, Lin & Huang, 2010); Optimal production planning (Shi et al., 2011); Optimal deteriorating items production inventory models (Widyadana and Wee, 2011); Optimal production management (Cadenillas et al., 2013); Optimal production planning (Gong and Zhou, 2013); Optimal transportation and business cycles (Das et al., 2014); and Optimal dynamic policies for integrated production (Chen, 2014).

Optimal production is directly related to increased capacity and thus, a business is able to offer more to their clients. Yet, the overall performance of a business is not guaranteed by this, given that there are many other factors (financial, marketing, commercial) that affect the business' performance and could be even more important than production itself. Production planning optimization problems have been approached to obtain the best solution that maximizes or minimizes an objective aimed by the business or group of businesses. This solution, in many cases, seems an unrealistic solution given that the businesses are observing a static market. Getting a view of the competitors' movements, on the other hand, makes the decision even a more competitive one. Not only this, but if integrating the competitor's decisions in the market to the production planning problem, could result in a more plausible solution. When tackling this type of problems, with a competition component involved, a game theoretic solution approach should be considered.

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Production planning has been widely studied in many of its components and applications in the industry (Khaledi and Reisi-Nafchi, 2013), where mathematical models have been the most representative of this type of problems, on both static and dynamic models (Missbauer and Uzsoy, 2011). Moreover, the competitive component of the production planning has been approached by a few authors and most of them have offered game theoretic solutions to these problems. In (Zhou, Xiao and Huang, 2010), the authors proposed a game theoretic mathematical solution to generate the optimal process plan for multiple jobs; (Manupati et al., 2012) presented a scheme for generating optimal process plans for multiple jobs in a networked based manufacturing system by applying non-cooperative games; (Aydinliyim and Vairaktarakis, 2013) considered a competitive scheduling setting using a cooperative game theoretic approach to achieve the maximum savings possible.

Generally, production planning problems are formulated using mixed integer linear programming (MILP) models, issue that has had a great development in the literature. Lütke Entrup et al., (2005) developed a MILP model that integrates shelf-life issues into production planning and scheduling. In (Ertugrul and Isik, 2009), the authors presented a MILP model to wine production planning. In (Doulabi et al., 2012), the formulation of an open shop scheduling problem was developed as two different MILP models. Jolayemi (2012) developed a MILP model for scheduling projects under penalty and reward arrangements, while in (L'Heureux et al., 2013), the authors presented a MILP model to solve a short term planning problem. In (Mattik et al., 2014), an MILP optimization model based on the block planning principle was developed to obtain optimal production scheduling.

On the other hand, the application of game theory to solve the production planning problem has shown great impact during the last years. In (Li et al., 2012), the authors developed an application of game theory in planning and scheduling integration, using Nash equilibrium to deal with multiple objectives; In (Zamarripa et al., 2012), a multi-objective MILP model was developed, to optimize the planning of supply chain with a game theoretic approach; In (Yin et al., 2013) a game theoretic model to coordinate single manufacturer and multiple suppliers with asymmetric quality information was proposed.

Others have used cooperative game theory for the formation of alliances in other contexts other than production. For example, in (Okada, 2010) the author proposed a cooperative game that describes an economic situation in which n individuals can communicate and form coalitions with each other under the concept that such a strategic alliance would increase individual income per participant.

The purpose of this paper is to illustrate an approach to solving problems of production planning with a *competitive component* through the application of Game Theory.

2 PROBLEM FORMULATION

2.1 Mathematical Model for the Production Planning Problem

We consider a production planning problem as a MILP model in order to obtain the maximum income for each of the m manufacturers involved in a specific market, which considers the production of n different products. The following model is represented for each manufacturer.

Notation:

$i = 1, \ldots, n$ Product (good) type

$j = 1, \ldots, cp$ Production facilities

$l = 1, \ldots, m$ Manufacturing firms

$w = 1, \ldots, raw$ Type of raw materials

$k = 1, \ldots, cl$ Client types

Parameters:

Cap_{ijl} = Production capacities of product type i at production facility j of the manufacturing firm l.

$CapMP_{wjl}$ = Raw material type w available at production facility j of manufacturing firm$l$.

Dem_{ik} = Quantity demanded of product type i at client k.

p_{ikl} = Price of product type i offered to client k by manufacturing firml.

c_{ijl} = Cost of manufacturing product type i at production facility j by manufacturing firm$l$.

a_{iw} = Quantity of raw material w required producing product type i

Variables:

X_{ijlk} = Quantity of product type i produced at production facility j by manufacturing firm l sold to client k.

$$q_{ilk} = \begin{cases} 1, if\ prod\ i\ is\ sold\ by\ manuf\ firm\ l\ to\ cliente\ k \\ 0, \quad on\ the\ contrary \end{cases}$$

Objective Function:

Maximize

$$f(x) = \sum_i \sum_k p_{ikl} \sum_j q_{ilk} X_{ijlk} - \sum_k X_{ijlk} c_{ijl} \tag{1}$$

Subject to

$$\sum_k X_{ijlk} \leq \sum_k q_{ilk} Cap_{ijl} \quad \forall i, j, l \tag{2}$$

$$\sum_{i,k} a_{iw} X_{ijlk} \leq CapMP_{wjl} \quad \forall w, j, l \tag{3}$$

$$\sum_{j,l} q_{ilk} X_{ijlk} \geq Dem_{ik} \quad \forall i, k \tag{4}$$

$$X_{ijlk} \in \mathbb{Z}^+ \qquad \forall i, j, l, k \tag{5}$$

Equation (1) establishes the objective function of the production problem, which aims to maximize the total utilities of the manufacturers. Equation (2), (3) and (4) establish capacity and demand restrictions.

For a single manufacturing firm this model is simple (the decision variable q_{ilk} should not be included) and an optimal solution is guaranteed, which makes the capacity restriction the main concern to obtaining greater income for each manufacturer.

Given that there are multiple manufacturers integrated in the same optimization problem, when competing in the same market, the solution is not that simple. Moreover, if some of the manufacturing companies are small and, as an overall, the industry is affected by external competitors that are threatening to take away a part of their own market, a strategy besides working at optimal conditions, has to be implemented by the manufacturers.

2.2 Cooperative Game Model

The "Production Game" (Owen, 1995) is defined as a set of players $l = \{1, 2, \ldots, m\}$, each player has a batch of w kinds of raw material. Player 1 has b_{11} units of raw material 1, b_{12} units of raw material 2, and $b_{1,raw}$ units of raw material w; Player 2 has units b_{21} raw material 1, b_{22} units of raw material 2 and $b_{2,raw}$ units of raw material w; player 3 has b_{31} units of raw material 1, b_{32} units of raw material 2, and $b_{3,raw}$ units of raw material w,…, player m has b_{m1} units of raw material 1, b_{m2} units of raw material 2,…, $b_{m,raw}$ units of raw material w. The products do not have value for themselves, except that they are used to produce goods x_1, x_2,…, x_n which can be sold at prices set in the market. A linear production process is assumed, in which one unit of the product 1 requires a_{11} raw material 1, a_{12} units of the raw material 2 and $a_{1,raw}$ units of the raw material w; a unit of the product 2 requires a_{21} units of raw material 1, a_{22} units of raw material units 2 and $a_{2,raw}$ units of the raw material w, one unit of the product n requires a_{n1} units of raw material 1, a_{n2} units raw material 2 and $a_{n,raw}$ units of the raw material w. Products x_1,x_2,…, x_n can be sold at p_1, p_2,…, p_z dollars respectively.

When a coalition S is formed, members will contribute to each of their products in order to maximize profits from the sale of products on the market. Therefore, the characteristic function is given by the following linear equation:

$$\mathrm{v(S)} = \sum_i p_i x_i = p_1 x_1 + p_2 x_2 + p_z x_n \tag{6}$$

Subject to:

$$\sum_i a_{iw} x_i \leq b_w(S), \qquad \forall\, w \tag{7}$$

Where:

$$b_w(S) = \sum_{l \in S} b_{lw} \tag{8}$$

3 SOLUTION APPROACH

3.1 Application of the MILP to the Cooperative Game Model

The model described in section 2.1 is integrated to the Cooperative Game Model described in section 2.2. For the implementation of the game, the following cooperation strategies were considered:

- When cooperating, each player is allowed to share its capacity with the others that form the coalition.
- Utilities are transferable among players that form the same coalition.

3.1.1 Definition of the Cooperative Game

Consider the m manufacturers, players of the game. Each player has a manufacturing facility with available raw materials for production and clients requiring each type of product. Each one yields for the maximum payoff, according to the MILP formulated in section 2.1. When cooperating, the production is set on two strategies: (i) more capacity is available and (ii) prices are stabilized according to the market's needs.

3.1.2 Characteristic Function

Given the optimal income function $f(x)$ presented previously and $v(S)$ for the general problem, the resulting characteristic function evaluated is:

$$\text{v(S)} = \sum_{i}\sum_{j}\sum_{k}\sum_{l\in S} X_{ijlk}\left(p_{ik}^{*} - c_{ijl}\right) \quad (9)$$

where, p_{ik}^{*} is the average price of product type i offered to client k for all players belonging to the coalition S, That is, every player belonging to the coalition S, offers to each client k, a product type i with a p^{*} price. On the other hand, the cost involved corresponds to the facility that is actually managing the production of the type of product sold. The facilities chosen to manufacture a product are subject to the capacity restriction that was previously stated in the MILP formulation, and adapted to the cooperative model as follows in eq. 10.

$$\sum_{k} X_{ijlk} \leq \sum_{k} Cap_{ijl} \quad \forall i,j,l \quad (10)$$

3.2 Shapley Value

Shapley Value is a solution approach to cooperative games and is given by the following equation:

$$\varphi_i(v) = \sum_{S\subset N} \frac{(s-1)!\,(n-1)!}{n!}[v(S) - v(S-i)] \quad (11)$$

Where N is any finite v company, with $|N| = n$. This formulation expresses the Shapley value for each player i in a game v as a weighted sum of terms of the form $[v(S) - v(S-i)]$, which is the contribution of player i to coalition S (Roth, 1988).

In this way, the contribution of each player can be calculated by using an algorithm that evaluates the Shapley Value, which is explained in the following sections.

3.2.1 Solution Algorithm

Calculating the Shapley Value has been a research topic of interest. Its computational complexity is combinatorial given that it requires knowing all possible combinations among the n different players, that is, $2^n - 1$. The model proposed in this paper presents an efficient algorithm that can be applied to many players, given that it integrates the probabilistic aspect of the Shapley Value formula and the possible margin of contribution that any player is able to make in a coalition. Similar to the expected value a decision making model under uncertainty restrictions, the Shapley Value is the expected value of each player under the different coalition scenarios. The table 1 explains the calculations executed in this algorithm with an example of four players.

Table 1: Shapley Value calculation for 4 players.

ith pl.	1	2	3	4
1	v(1)	v(1,2) – v (2) + v (1,3) – v(3) + v (1,4) - v (4)	v(1,2,3) - v (2.3) + v(1,2,4) - v(2.4) + v(1,3,4) - v(3,4)	v (1,2,3,4) - v(2,3,4)
2	v(2)	v (1,2) - v (1) + v (2,3) - v (3) + v (2,4) - v (4)	v (1,2,3) - v (1,3) + v (1,2,4) - v (1,4) + v (2,3,4) - v (3,4)	v (1,2,3,4) -v (1,3,4)
3	v(3)	v (1,3) - v (1) + v (2,3) - v (2) + v (3,4) - v (4)	v (1,2,3), v (1,2) + v (2,3,4), v (2,4) + v (1,3,4), v (1,4)	v (1,2,3,4) -v (1,2,4)
4	v(4)	v (1,4) - v (1) + v (2,4) - v (2) + v (3,4), v (3)	v (1,2,4), v (1,2) + v (1,3,4), v (1,3) + v (2,3,4), v (2,3)	v (1,2,3,4) -v (1,2,3)

3.2.2 Pseudo Code

The resulting program code for the solution algorithm generated is showed in the Appendix section.

This solution approach was first applied to other applications related to supply chain, resulting in interesting results. In the electric energy industry, where a two-level game was proposed, in which the first one looks for a Stackelberg Equilibrium solution where the leader is a generator, in particular, then the second-level obtains the coordination among a group of marketers following a cooperative game, where Shapley Value is calculated for each player as a result of their coordination (Guzmán et al., 2008). Also, in the furniture industry, with respect to the competitive value of both supplier and manufacturing companies (Puello-Pereira and Ramírez-Ríos, 2014).

4 RESULTS

4.1 Numerical Example

For the numerical example, a 4-player game is considered, where each player represents a manufacturing company that competes for a single client with four different products. The information below includes the market price and consumption of raw material per type of product.

Table 2: Market price:

Product type	Price P_z
1	40
2	50
3	45
4	35

It is assumed that the fabrication of product requires four different materials in the proportions showed in Table 4.

Table 3: Amount of raw material.

Raw material	Player 1	Player 2	Player 3	Player 4
1	200	150	130	180
2	100	210	190	140
3	50	155	230	160
4	300	135	180	90

Table 4: Raw material requirement.

Row material	x_1	x_2	x_3	x_4
1	5	6	6	5
2	6	2	1	5
3	1	2	5	1
4	3	5	1	6

4.1.1 Optimal Solution for the Competitive Model

This problem was solved initially as global optimization model that didn't consider possible cooperation among the agents.

By using an optimization engine (GAMS), an optimal solution was generated, with a total utility of \$5.155, where the optimal value, corresponding to each player, is presented in table 5.

Table 5: Solution generated.

P. type	Player 1	Player 2	Player 3	Player 4
Prod 1	10	0	2	0
Prod 2	20	25	20	15
Prod 3	0	0	0	15
Prod 4	0	0	0	0
Utilities	1400	1250	1080	1425

4.1.2 Cooperative Game Solution to the Problem

For this numerical example, the possible coalitions are the following: $v(1)$, $v(2)$, $v(3)$, $v(4)$, $v(1,2)$, $v(1,3)$, $v(1,4)$, $v(2,3)$, $v(2,4)$, $v(3,4)$, $v(1,2,3)$, $v(1,2,4)$, $v(1,3,4)$, $v(2,3,4)$ y, $v(1,2,3,4)$.

According to the solution approach implemented, after weighing the coalitions, an optimization engine is integrated to generate the optimal value for each scenario, resulting in each contribution to the coalition, as was presented in table 1.

For each scenario generated, the FO value for each player is considered as the contribution of each one to the coalition. In the first case, when considering individual coalitions, that is, $\{1\}$, $\{2\}$, $\{3\}$ and $\{4\}$, the optimal solution would be the ones considered in the optimization model previously solved if solved individually. Thus for Player 1, it turns to be optimal to manufacture 10 units of product 1 and 20 units for product 2. Nevertheless, when it comes to sharing demanded quantity, the solutions change for the other players.
After solving for all scenarios, optimal values for each coalition are given in the following table.

Table 6: Optimal value.

Coalition	Optimal value	Coalition	Optimal value
v (1)	1400	*v (2,3)*	2333.3
v (2)	1250	*v (2,4)*	2687.5
v (3)	1083.33	*v (3,4)*	2583.3
v (4)	1425	*v (1,2,3)*	4000
v (1,2)	2916.66	*v (1,2,4)*	4416.66
v (1,3)	2750	*v (1,3,4)*	4250
v (1,4)	3166.66	*v (2,3,4)*	3833.33
v (1,2,3,4)		5500	

4.1.3 Shapley Solution

In the previous subsection coalitions were formed and also optimal values for each coalition were calculated, the next step is find optimal coalitions

$$\varphi_i(V) = \sum_{\{S\epsilon N: i\epsilon S\}} \frac{(s-1)!\,(n-s)!}{n!} \tag{12}$$

We replace s for each player:

$$\varphi_1 = \sum_{\{S\epsilon N: j\epsilon S\}} \frac{(1-1)!\,(4-1)!}{4!} = 0{,}25 \tag{13}$$

$$\varphi_2 = \sum_{\{S\epsilon N: j\epsilon S\}} \frac{(2-1)!\,(4-2)!}{4!} = 0{,}083 \tag{14}$$

$$\varphi_3 = \sum_{\{S\epsilon N: j\epsilon S\}} \frac{(3-1)!\,(4-3)!}{4!} = 0{,}083 \tag{15}$$

$$\varphi_4 = \sum_{\{S\epsilon N: j\epsilon S\}} \frac{(4-1)!\,(4-4)!}{4!} = 0,25 \quad (16)$$

The resulting solution that gives the Shapley Value is given in table 7, as shown in the last column, which is considered as the payoff that should be assigned to each player in the coalition $S = (1,2,3,4)$, also known as the grand coalition.

Table 7: Shapley values.

ith player	1	2	3	4	ϕ
1	1400	5075	5062.5	1667	1611.5
2	1250	4029	3750	1250	1273
3	1083	3592	3312.5	1083	1117
4	1425	4500	4500	1500	1498
p_i	0.25	0.0833	0.0833	0.25	5500

According to the Shapley value, the distribution of the profits associated with each player in the grand coalition are as follows:

For player one USD $ 1,611.45.
For player two USD $ 1,273.26.
For the player three USD $ 1,273.26.
For the player four USD $ 1,273.26.
Value of grand coalition USD $ 5,500.00.

The results, as compared to the individual payoffs observed in table 5, show the feasibility of the solution and the economic incentive for cooperating. Table 8 show the comparison of the results obtained.

Table 8: Results compared.

Player	Individual	SV	% Improvement
1	1400	1611,5	15%
2	1250	1273	2%
3	1083	1117	3%
4	1425	1498	5%

4.2 Analysis Results Generated

After solving the numerical example shown above, it can be observed that cooperation is possible among competitors, which assume the share of demanded quantities for each one of the products offered. The grand coalition sets an overall of $5.500, much greater than what the global model considered initially, $5.155. In the resulting cooperative model, player 1 is most strategically benefited as shown by the Shapley values generated. Yet, overall, all players are benefitted, obtaining greater benefits than operating individually.

5 CONCLUSIONS

The increase of market competitiveness generates a growing interest in companies to improve their processes and operations in order to obtain satisfactory results and become well positioned. This has encouraged many of them to integrate with their competitors where the implementation of strategies focused on collaboration between several companies with a common goal. Nevertheless, this is not always true due to the lack of incentives that businesses have to cooperate. For this reason, many companies decide to continue working independently. In this particular case, cooperative game theory offers solutions such as the Shapley Value that allows an efficient distribution of incentives among each player, thus, resulting in a contribution received by each player, according to its objective function.

In this paper, we considered a problem of production planning in manufacturing companies, with a cooperative game model that integrated with MILP models that made possible the determination of optimal coalitions and the amount of each type of product to be manufactured by each player. The results generated, indicate that involving competition to obtain optimal benefits is not as simple as solving for a MILP model. Involving competition requires generating previous decisions, which are considered in several scenarios that must be evaluated. Moreover, if cooperation is considered, the implications make it a more dynamic and complex model.

The Shapley value calculation determine an efficient way of distributing their income and a solution algorithm was implemented in order to calculate the value among many companies.

This solution approach demonstrated that cooperation is not only recommended at a strategic level, but also is considered an important strategy for companies that are struggling in a competitive market and are striving to succeed.

Future research directions are considered reducing the complexity of coalition formation when addressing Shapley Value. Also, there are multiple applications where cooperation is needed and more and more companies are searching for a way to cooperate without losing money.

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APPENDIX

Solution algorithm

```
Count = number of coalitions formed.
Count ≤ 2^m-1
begin
   g = m, w= 0
   For l = 1 to m
      S_lg = assign l to coalition S
      v(S_lg)=Max f(x)
   Next l
   w= w + m
   Do while w <= Count
      Do
      z=1
      h=g-1
    For j = 1 to h
            Do
              S_wh = j
              For i = h+1 to g
                 S_wi = j+z
                 v(S_wi)= Max f(x_j)∀ j∈S_wi
                 z = z +1
              Next i
              w=w+1
            while j+z = m
    Next j
      h= h-1
      while h > 0
   Loop
   For l=1 to m
      For r=1 to g
       Calculate marginal payoffs
       MV_r= sum_r[v(S_rl)-v(S_rl-l)]
     Calculate p_r
      Next r
   Next l
   Calculate shapley value SV_l
   SV_l=sum_l[p_r*MV_l]
end
```