An Inflation / Deflation Model for Price Stabilization in Networks

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Abstract: We consider a simple network model for economic agents where each can buy goods in the neighborhood. Their prices may be initially distinct in any node. However, by assuming some rules on new prices, we show that the distinct prices will reach an equilibrium price by iterating buy and sell operations. First, we present a protocol model in which each agent always bids at some rate in the difference between his own price and the lowest price in the neighborhood. Next, we show that the equilibrium price can be derived from the total funds and the total goods for any network. This confirms that the inflation / deflation occurs due to the increment / decrement of funds as long as the quantity of goods is constant. Finally, we consider how injected funds spread in a path network because sufficient funds of each agent drive him to buy goods. This is a monetary policy for deflation. A set of recurrences lead to the price of goods at each node at any time. Then, we compare two injections with half funds and single injection. It turns out the former is better than the latter from a fund-spreading point of view, and thus it has an application to a monetary policy and a strategic management based on the information of each agent.

1 INTRODUCTION

Motivation. Conventionally, the topics of price determination have been discussed in the context of microeconomics approach (J. E. Stiglitz, 2006). In supply and demand curves, if the price is higher (resp. lower) than an equilibrium, there is excess supply (resp. excess demand) and thus the price moves to the equilibrium. At the equilibrium price, the quantity of goods sought by consumers is equal to the quantity of goods supplied by producers. Neither consumers nor producers have an incentive to alter the price or quantity at the equilibrium. Since such a conventional approach cannot capture each person's behavior, it is difficult to reflect actual economic phenomena. So we considered a multiagent network model (J. Kiniwa and K. Kikuta, a; J. Kiniwa and K. Kikuta, b), in which each agent makes auctions and the price of goods is eventually determined. Our network model consists of nodes and edges as cities and their links to neighbors, respectively. Each node contains an agent which represents people in the city. Agents who want to buy goods make bids to the lowest-priced neighboring node, if any. Then, agents who want to sell the goods accept the highest bid. The process of price stabilization can be shown by using the idea of self-stabilization in distributed systems (S. Dolev, 2000). From any initial state, self-stabilizing algorithms eventually lead to a legitimate state without any aid of external actions. We notice that the properties of self-stabilization resemble those of price determination in convergence to a equilibrium without external operations.

Problem. The problem in our previous studies (J. Kiniwa and K. Kikuta, a; J. Kiniwa and K. Kikuta, b) is an ambiguous relation between the price and the amount of funds / goods. The most unsuccessful reason is that no other variables than "price" were used. There was no way to determine the next stage of the price other than using the prices of buyers and sellers. So we failed to explain why such an equilibrium price is determined. To estimate the equilibrium price, we need auxiliary variables which explain the next stage of the price under stabilization. In addition, our model failed to reflect the change of price due to various factors, called inflation or deflation. To explain the inflation / deflation, we need auxiliary variables which show the flow of money and goods under the process of such phenomena.

Solution. In this paper, we develop a new model containing a relation between the price and the amount

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of funds / goods. We assume that the price is proportional to the amount of funds and inversely proportional to the amount of goods at each node. Furthermore, the volume of trade is assumed to depend on the price difference between cities. As a result, the flow of money and goods is determined by the market principles, and thus the equilibrium price can be explained reasonably. Furthermore, it confirms that the inflation / deflation depends on the amount of funds as long as the amount of goods is constant.

Related Work. The classical theory of price determination in microeconomics is introduced, e.g., in (J. E. Stiglitz, 2006; N. G. Mankiw, 2012), and a survey is in (T. A .Weber, 2012). We review the theory from multiagent points of view. Though several economic network models have been already known (L. E. Blume, 2009; E. Even-Dar and S. Suri, 2007; S. M. Kakade and S. Suri, 2004), such models contain a bipartite structure (E. Even-Dar and S. Suri, 2007; S. M. Kakade and S. Suri, 2004) or traders who play intermediary roles (L. E. Blume, 2009). Agent-based stabilization has been discussed in (J. Beauquier and E. Schiller, 2001; S. Dolev and J. L. Welch, 2006; S. Ghosh, 2000; T. Herman and T. Masuzawa, 2001). Unlike our agents, their ideas are to use mobile agents for the purpose of stabilization. It is useful in designing protocols by what price we should make a bid. Several kinds of game theoretic flavors have appeared in self-stabilization, e.g., time complexity analysis (S. Dolev and S. Moran, 1995), strategies with optimal complexity (S. Dolev and P. Tsigas, 2008), relationships between Nash equilibria and stabilization (A. Dasgupta and S. Tixeuil, 2006; M. G. Gouda and H. B. Acharya, 2009). Our protocol in Section 3 can be considered as a kind of consensus algorithm. The consensus algorithm in decentralized systems is described in (N. A. Lynch, 1996), and its self-stabilizing version is described in (S. Dolev, 2000; S. Dolev and E. M. Schiller, 2010).

Contributions. We consider an inflation / deflation network model, where the price is proportional to the amount of funds, and is inversely proportional to the amount of goods at each node. First, we present a protocol in which each agent always offers a fixed price without considering other bidders' strategies. Then, we show that an equilibrium price is determined by the total amount of funds and goods, and confirm that inflation / deflation is determined by the amount of funds. Next we focus on path networks and reveal the price of each node and the amount of funds of each node at each time. Finally, we show that the injection of funds from two points is more effective than that from a single point.

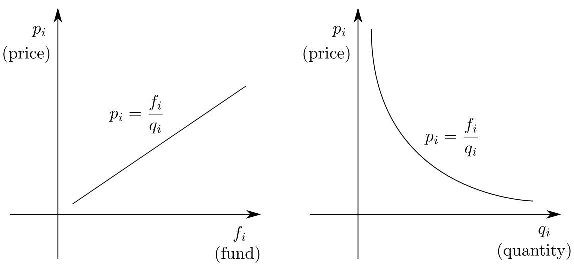
The rest of this paper is organized as follows. Section 2 states our model. Section 3 shows that our protocol can stabilize distinct goods prices. Section 4 analyzes the behavior of our protocol in detail. Section 4.1 investigates an equilibrium price in an arbitrary network. Then, Section 4.2 estimates the amount of funds at any node at any time for path networks. Furthermore, it suggests an effective fund-injection method for a central bank. Finally, Section 5 concludes the paper.

2 MODEL

Our system can be represented by a connected network $G=(V,E)$, consisting of a set of nodes V and edges E. In the network G, an arbitrary pair of nodes $i \in V$ and $j \in V$ represent cities and an edge $(i,j) \in E$ between them, called neighbors, represents direct transportation. Let N_i be a set of neighboring nodes of $i \in V$, and let $N_i^+ = N_i \cup \{i\}$. We assume that each node $i \in V$ has goods and their initial price may be distinct. Let $p_i(t)$, or denoted by p_i, be the price of goods at node i for the time step $t \in T = (0,1,2,\ldots)$. Each node $i \in V$ has exactly one representative agent a_i who always stays at i and can buy goods in the neighborhood N_i. Each agent a_i has funds f_i, that is, the total amount of money at i, and the quantity q_i of goods at i. The price p_i is determined by the relationship between the quantity of goods and the purchasing power, or called *supply-demand* balance. So we simply assume that the price is proportional to the amount of funds for constant goods (Figure 1(a)), and is inversely proportional to the amount of goods for constant funds (Figure 1(b)) at each node, that is,

$$p_i = \frac{f_i}{q_i}. \tag{1}$$

The *buy operation* is executed as follows. Each agent a_i assigns a *value* $v_i^j(t)$, or denoted by v_i^j, to the goods of any neighboring node $j \in N_i$, where the value means the maximum amount an agent is willing



(a) Constant goods (q_i)　(b) Constant funds (f_i)

Figure 1: Price determined by funds and goods at each node.

to pay. Agent a_i compares its own goods price p_i with the neighboring price p_j. If the cheapest price in N_i is p_j and is less than p_i, the agent a_i wants to buy it and submits a bid $b_i^j(t)$, or denoted by b_i^j, to node j. We consider $v_i^j(t) = p_i(t)$ for any $j \in N_i$ because he can buy it at price $p_i(t)$ in his node.

The *sell operation* is executed as follows. After accepting bids from N_j, agent a_j *contracts* with $a_i \in N_j$, an arbitrary one of agents who submitted the highest bid b_i^j. Then, a_j passes the goods to (receives money from) the contracted agent a_i until the price $p_j(t+1)$ becomes b_i^j derived from the supply-demand balance. We do not take the carrying cost of goods into consideration but focus on the change of prices. In this way, at every time, any price is updated if necessary. The state Σ_i of each node $i \in V$ is represented by the price, the quantity of goods and the amount of funds $(p_i(t), q_i(t), f_i(t))$.

We assume a *synchronous model*, that is, every agent periodically exchanges messages and knows the states of neighboring agents. The global state of all nodes is called a *configuration*. The set of all configurations is denoted by $\Gamma = \Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_{|V|}$. An *atomic step* consists of reading the states of neighboring agents, a buy / sell operation, and updating its own state. Then, a configuration is changed from $c_j \in \Gamma$ into $c_{j+1} \in \Gamma$ (or c_{j+1} is reached from c_j) by the atomic step. An *execution E* is a sequence of configurations $E = c_0, c_1, \ldots, c_j, c_{j+1}, \ldots$ such that $c_{j+1} \in \Gamma$ is reached from $c_j \in \Gamma$.

3 PROTOCOL DESIGN

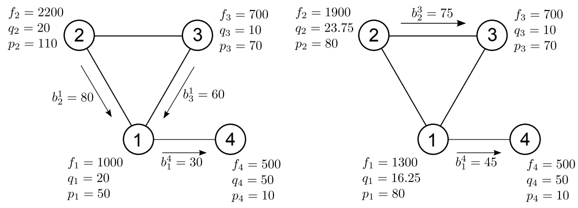
In this section, we consider a protocol model, called **FundBidding**, in which each agent a_i always makes a bid b_i^j $(p_j(t) \leq b_i^j \leq p_i(t))$ to an agent $a_j \in N_i^+$ with the lowest price in the neighborhood. For simplicity, let k be a constant rate so that b_i^j lies between $p_j(t)$ and $p_i(t)$, where the price may not be an integer.

FundBidding

- Each agent a_i makes a bid

$$b_i^j(t) = p_j(t) + \left(\frac{p_i(t) - p_j(t)}{k} \right), \qquad (2)$$

  where $k \geq 1$, to node $j \in N_i^+$ which has the lowest-priced goods in N_i^+.
- The agent a_j contracts with the neighboring a_i who has submitted the highest bid $\max_{i \in N_j} b_i^j(t)$. If a_j has submitted his bid to neighboring node at the same time, it is postponed until the next time step. The goods of a_j and the money of a_i are exchanged, that is, the goods are moved from q_j to q_i and the money is moved from f_i to f_j as long as $p_i > p_j$. The prices $p_i(t+1)$ and $p_j(t+1)$ are determined by the funds and the amount of goods.
- If several agents make bids to node j with the same highest price, agent a_j makes deals with one of them at random.



(a) Bids before trading (b) Bids after trading between 1 and 2

Figure 2: An illustration of protocol **FundBidding**.

Example 1. *Figure 2 shows an example of our network system consisting of 4 nodes* $V = \{1,2,3,4\}$. *For the bidding price (2), let* $k = 2$. *At time t, the prices of goods are* $(p_1(t), p_2(t), p_3(t), p_4(t)) = (50, 110, 70, 10)$ *as shown in Figure 2(a). Each agent* a_i *wants to buy the lowest-priced goods at node* $j \in N_i$ *if its price is lower than* p_i*, that is,* $p_i > \min_{j \in N_i} p_j$. *Thus, agent* a_1 *makes a bid to node 4 with price* $b_1^4 = 30$. *Likewise, agents* a_2 *and* a_3 *make bids to node 1, respectively. Then, only* a_2*'s bid is successful, and* a_2 *makes a contract with* a_1.

At time $t+1$, *the prices become* $(p_1(t+1), p_2(t+1), p_3(t+1), p_4(t+1)) = (80, 80, 70, 10)$ *as shown in Figure 2(b). Since price* p_1 *has been changed, agent* a_1*'s bid* b_1^4 *is resubmitted as* $(80+10)/2 = 45$. *Since the bids* b_2^3 *and* b_1^4 *are independent, they are executed in parallel at time* $t+1$. □

We are concerned with whether or not the prices of goods eventually reach an equilibrium price even if they are initially distinct. So we define the legitimacy of a configuration as follows.

Definition 1 (legitimate configuration)**.** *A configuration is legitimate if the goods in every node have the same price.* □

Let $C_t \subseteq V$ be the set of nodes that have updated their prices from time t to $t+1$. The following lemma proves that the protocol **FundBidding** is free from deadlocks.

Lemma 1. *The protocol* **FundBidding** *is deadlock-free. That is, there exist some nodes in* C_t *as long as the configuration is illegitimate.*

Proof. First notice that no cycle is generated by the chain of bidding requests, as depicted in Figure 2, because every bidding request occurs from a higher priced node to a lower priced node.

Next suppose that the configuration is illegitimate at time t. Then, there is a pair of neighboring nodes $i, j \in V$ such that $p_i(t) = \max_{h \in N_j} p_h(t)$ and $p_j(t) = \min_{h \in N_i} p_h(t)$, where $p_i(t) - p_j(t)$ is the maximum price difference in the neighborhood. In this case, agent a_i makes a bid to node j and agent a_j accepts the price. Since $p_j(t)$ is increased at time $t+1$, $j \in C_t$ holds. □

In (J. Kiniwa and K. Kikuta, b), we investigated a condition such that any protocol satisfying the framework of **FundBidding** achieves price stabilization. Suppose that agents a_i and a_j make bids to node h. We say that *bids have the same order as values* if $v_i^h \leq v_j^h$ implies $b_i^h \leq b_j^h$ for the goods of node h. Next lemma shows that the bids having the same order as values is necessary for price stabilization.

Lemma 2. *(J. Kiniwa and K. Kikuta, b) If bids do not always have the same order as values, price stabilization is not guaranteed.* □

The following theorem further shows that an additional condition leads to the price stabilization.

Theorem 1. *(J. Kiniwa and K. Kikuta, b) Suppose that bids have the same order as values. If any contract price lies between buyer's price and seller's price, price stabilization occurs.* □

Since we assume that $v_i^j(t) = p_i(t)$ for any neighboring node $j \in N_i$ and a_i makes a bid by (2), **FundBidding** satisfies the condition above.

4 ANALYSIS

In this section, we investigate several aspects of our **FundBidding** for arbitrary networks and path networks.

4.1 Arbitrary Network

The following theorem claims that the equilibrium price is determined by the total amounts of funds and the goods regardless of the network topology.

Theorem 2. *Let F be the total amount of funds, and Q the total amount of goods. The equilibrium price, denoted by P_e, is*

$$P_e = \frac{F}{Q}$$

regardless of the network topology.

Proof. By definition, the price of goods at node i is $p_i = f_i/q_i$. Suppose that the equilibrium prices are different for each stabilization process. Then, $p_i(t) \neq p_i(t')$ for time t and t' ($t \neq t'$) holds. Since $f_i = p_i(t)q_i$ and $f_i' = p_i(t')q_i'$ hold for any node i, where $F = \sum_i f_i = \sum_i f_i'$, we have

$$p_i(t) \cdot \sum_i q_i = p_i(t') \cdot \sum_i q_i'.$$

Since the total amount of goods Q is identical, we have

$$Q = \sum_i q_i = \sum_i q_i'.$$

Thus we obtain $p_i(t) = p_i(t')$, a contradiction. Therefore, the equilibrium price P_e is identical for each stabilization process.

Next, since $f_i = P_e \cdot q_i$ holds for every node i, the total funds sum up to

$$F = P_e \cdot Q.$$

Thus we obtain $P_e = F/Q$. □

The theorem above is known as the *Fisher's quantity equation* (N. G. Mankiw, 2012) $FV = P_e Q$ if the velocity of money V equals to 1. This means the correctness of our assumption (1) at each node. Thus, in our inflation / deflation model, the inflation (resp. deflation) occurs if the total amount of funds increase (resp. decrease) as long as the total amount of goods is constant.

4.2 Path Network

In what follows, we restrict our concern to path networks. The path networks probably represent the distance feature in arbitrary networks. Then, we consider how injected funds spread in the path network because sufficient funds of each agent drives him to buy goods. This is a monetary policy for deflation. Section 4.2.1 considers the situation that incremental funds are injected from a single point. Section 4.2.2 considers the situation that the half of incremental funds are injected from two points.

4.2.1 Single Injection

We investigate the amount of funds at each node of a path $P = (1, 2, \ldots, n)$ at any time. For simplicity, let $k = 2$ and let $b_i^j(t) = (p_i(t) + p_j(t))/2$ in (2). Suppose that we inject funds m into node 1, called an *injection point*. Let $p_i^c(t)$ be the temporary, intermediate price of node i reached by trading exhaustively for a contract between t and $t+1$.

Lemma 3. *Let q_i be the quantity of goods, and f_i the funds of agent a_i before the trade at node i. Then, the price after the trade will be*

$$p_i^c(t) = \frac{f_{i-1} + f_i}{q_{i-1} + q_i}.$$

Proof. Suppose that (q_{i-1}, f_{i-1}) and (q_i, f_i) change into (q'_{i-1}, f'_{i-1}) and (q'_i, f'_i) after the trade, respectively. Let $F_{i-1,i}$ and $Q_{i-1,i}$ be a sum of funds and a sum of quantities of goods at nodes $i-1$ and i, respectively. Since no other funds and goods do not come into these values, we have

$$\begin{aligned} f_{i-1} + f_i &= f'_{i-1} + f'_i = F_{i-1,i} \\ q_{i-1} + q_i &= q'_{i-1} + q'_i = Q_{i-1,i}. \end{aligned}$$

At an equilibrium, since

$$\frac{f'_{i-1}}{q'_{i-1}} = \frac{f'_i}{q'_i} = P_e,$$

$q'_{i-1} = f'_{i-1}/P_e$ and $q'_i = f'_i/P_e$. Then,

$$q'_{i-1} + q'_i = (f'_{i-1} + f'_i)/P_e,$$

that is, $Q_{i-1,i} = F_{i-1,i}/P_e$ holds. Thus, we have

$$P_e = \frac{F_{i-1,i}}{Q_{i-1,i}} = \frac{f_{i-1} + f_i}{q_{i-1} + q_i}.$$

This means we can find the equilibrium price before the trade. □

The following Figure and Example present a behavior of price diffusion in a path.

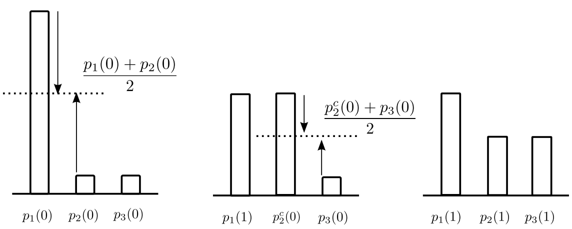


Figure 3: Price diffusion in a path.

Example 2. *Figure 3 illustrates price diffusion in a path $(1,2,3)$, where the price at node 1 is initially higher than others because funds have been injected. The intermediate state of p_i between $t = 0$ and $t = 1$ is denoted by $p_i^c(0)$ for convenience. Let $k = 2$ for the expression (2). First,*

$$p_1(1) = \frac{p_1(0) + p_2(0)}{2} = p_2^c(0),$$

and then

$$p_2(1) = \frac{p_2^c(0) + p_3(0)}{2} = \frac{p_1(1) + p_3(1)}{2} = p_3(1)$$

holds. □

Thus, in general, the price $p_j(t)$ at node $j \in P$ $(2 \leq j \leq n-1)$ can be represented as follows.

$$p_1(t) = \frac{1}{2} \cdot p_1(t-1) + \frac{1}{2} \cdot p_2(t-1) \quad (3)$$

$$p_j(t) = \frac{1}{2} \cdot p_{j-1}(t) + \frac{1}{2} \cdot p_{j+1}(t-1) \quad (4)$$

$$p_n(t) = \frac{1}{2} \cdot p_{n-2}(t) + \frac{1}{2} \cdot p_n(t-1) \quad (5)$$

From (4), we have

$$\sum_{t\geq 1} p_j(t)x^t = \frac{1}{2} \cdot \sum_{t\geq 1} p_{j-1}(t)x^t + \frac{1}{2} \cdot \sum_{t\geq 1} p_{j+1}(t-1)x^t.$$

Using $R_j(x) = \sum_{t\geq 0} p_j(t)x^t$, we obtain

$$\begin{aligned} R_j(x) - p_j(0) &= \frac{1}{2}(R_{j-1}(x) - p_{j-1}(0)) + \frac{x}{2} R_{j+1}(x) \\ 2R_j(x) &= R_{j-1}(x) + xR_{j+1}(x) + (2p_j(0) - p_{j-1}(0)) \end{aligned}$$

For simplicity, we assume $2p_j(0) - p_{j-1}(0) = 0$ and replace j by $j-1$. Then,

$$xR_j - 2R_{j-1} + R_{j-2} = 0.$$

So we have

$$R_j = A_1 \left(\frac{1 + \sqrt{1-x}}{x} \right)^j + A_2 \left(\frac{1 - \sqrt{1-x}}{x} \right)^j.$$

Using our initial conditions $R_0(x) = \sum_{t\geq 0} p_0(t)x^t = 0$ and $R_1(x) = \sum_{t\geq 0} p_1(t)x^t \approx p_1(0)$,

$$\begin{cases} A_1 + A_2 = 0 \\ A_1(\frac{1+\sqrt{1-x}}{x}) + A_2(\frac{1-\sqrt{1-x}}{x}) = p_1(0) \end{cases}$$

lead to

$$R_j = p_1(0) \cdot \frac{x}{2\sqrt{1-x}} \left\{ \left(\frac{1+\sqrt{1-x}}{x} \right)^j - \left(\frac{1-\sqrt{1-x}}{x} \right)^j \right\}.$$

Using $\frac{x}{2\sqrt{1-x}} = z$,

$$\begin{aligned} R_j &= \frac{p_1(0)}{2z} \{ (\frac{1}{x} + z)^j - (\frac{1}{x} - z)^j \} \\ &= \frac{p_1(0)}{2z} \{ 2 \binom{j}{1} \left(\frac{1}{x} \right)^{j-1} z + 2 \binom{j}{3} \left(\frac{1}{x} \right)^{j-3} z^3 + \cdots \} \\ &= p_1(0) \sum_{r\geq 1} \binom{j}{2r-1} \left(\frac{1}{x} \right)^{j-(2r-1)} z^{2r-2} \\ &= p_1(0) \sum_{r\geq 1} \binom{j}{2r-1} \left(\frac{1}{x} \right)^{j-(2r-1)} \left(\frac{1-x}{x^2} \right)^{r-1}. \end{aligned}$$

Thus,

$$\begin{aligned} R_j &= p_1(0) \sum_{r\geq 0} \binom{j}{2r+1} x^r \cdot \frac{1}{(1-x)^{j-1}} \\ &= p_1(0) \sum_{r\geq 0} \binom{j}{2r+1} x^r \sum_{s\geq 0} \binom{j+s-2}{s} x^s \\ &= p_1(0) \sum_{r\geq 0} \left(\sum_{0\leq s\leq r} \binom{j}{2(r-s)+1} \binom{j+s-2}{s} \right) x^r. \end{aligned}$$

Therefore, we have

$$p_j(t) = p_1(0) \sum_{0 \le s \le \lfloor (j-1)/2 \rfloor} \binom{j}{2(r-s)+1} \binom{j+s-2}{s}.$$

Then, $p_1(t)$ and $p_n(t)$ can be described as follows:

$$p_1(t) = \frac{p_1(0)}{2^t} + \sum_{0 \le k \le t-1} \frac{p_2(k)}{2^{t-k}},$$

and

$$p_n(t) = \frac{p_n(0)}{2^t} + \sum_{1 \le k \le t} \frac{p_{n-2}(k)}{2^{t-k+1}}.$$

Let $b^j_{j-1}(t) = (p_{j-1}(t) + p_j(t))/2$ be the bidding price of node $j-1$ to node j. Let $p^c_j(t)$ (or simply p^c_j) denote the temporary, intermediate price at node j between time t and $t+1$. Then, the amount of goods $q_j(t+1)$ can be determined as follows.

Lemma 4. *The amount of goods at time $t+1$ is*

$$q_j(t+1) = \frac{(b^j_{j-1} + p_j)(b^{j+1}_j + p^c_j)}{(p^c_j + b^j_{j-1})(p_j + b^{j+1}_j)} \cdot q_j(t)$$

Proof. Let x (resp. y) be the amount of goods moved from node j to node $j-1$ (resp. node $j+1$ to node j). First, we consider the trade between node $j-1$ and node j. Notice that the funds of agent j reach $f_j(t) + x \cdot b^j_{j-1}(t)$ and the amount of goods at node j becomes $q_j(t) - x$. By Lemma 3, when the price $p^c_j(t)$ reaches $p^c_j(t) = (f_{j-1} + f_j)/(q_{j-1} + q_j)$,

$$\begin{aligned} \frac{f_j + x \cdot b^j_{j-1}}{q_j - x} &= p^c_j \\ x &= \frac{p^c_j q_j - f_j}{p^c_j + b^j_{j-1}}. \end{aligned}$$

Since $f_j = p_j q_j$, we have

$$q^c_j = q_j - x = \frac{q_j(b^j_{j-1} + p_j)}{p^c_j + b^j_{j-1}}.$$

Likewise, for the trade between node j and node $j+1$,

$$\begin{aligned} \frac{p^c_j q^c_j - y \cdot b^{j+1}_j}{q^c_j + y} &= p_j \\ y &= \frac{p^c_j q^c_j - p_j q^c_j}{p_j + b^{j+1}_j}. \end{aligned}$$

Thus,

$$\begin{aligned} q_j(t+1) &= q^c_j + y \\ &= \frac{q_j(b^j_{j-1} + p_j)}{p^c_j + b^j_{j-1}} + \frac{p^c_j q^c_j - p_j q^c_j}{p_j + b^{j+1}_j} \\ &= \frac{(b^j_{j-1} + p_j)(b^{j+1}_j + p^c_j)}{(p^c_j + b^j_{j-1})(p_j + b^{j+1}_j)} \cdot q_j(t). \end{aligned}$$

□

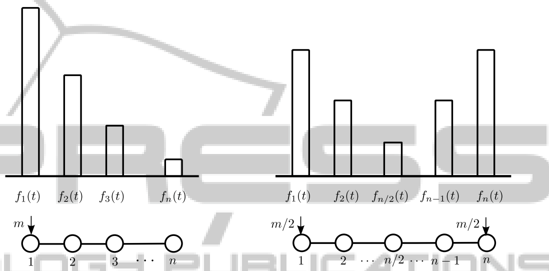
Theorem 3. *The amount of agent a_j's funds at time t is*

$$f_j(t) = p_j(t) \prod_{i=1}^{t-1} \frac{(b^j_{j-1} + p_j)(b^{j+1}_j + p^c_j)}{(p^c_j + b^j_{j-1})(p_j + b^{j+1}_j)} \cdot q_j(0).$$

□

4.2.2 Double Injections of Half Funds

This section considers the half of incremental funds are injected from two points. Figure 4 illustrates (a) single injection and (b) double injections of half funds.



(a) Single injection of funds m at 1 (b) Double injections of funds $m/2$ at 1 and n

Figure 4: Injection of funds.

First, we focus on the asymptotic behavior of the terminal agent a_n. Notice that agent a_n's funds only increases and the amount of goods only decreases under stabilization. Next, we show that the method of double injections of half funds is better than that of single injection from the fund-spreading point of view. The investigation is motivated by exploring a good monetary policy.

Lemma 5. *Let $p^c_{n-1}(0)$ be the price at node $n-1$ immediately before bidding for node n. Then,*

$$p^c_{n-1}(0) = \frac{p_1(0)}{2^{n-1}} + p_n(0) \left(1 - \frac{1}{2^{n-1}}\right)$$

if we assume $p_2(0) = \cdots = p_n(0)$.

Proof. First, agent a_1 makes a bid to node 2 with $b^2_1(0) = (p_1(0) + p_2(0))/2$. Then, agent a_2 makes a bid to node 3 with $b^3_2(0) = (b^2_1(0) + p_3(0))/2$, and so on. The bidding reaches node n with $b^n_{n-1}(0) = (b^{n-1}_{n-2}(0) + p_n(0))/2$. Thus, we have

$$p^c_{n-1}(0) = \frac{p_1(0) + p_2(0)}{2^{n-1}} + \frac{p_3(0)}{2^{n-2}} + \cdots + \frac{p_n(0)}{2}.$$

If we assume $p_2(0) = \cdots = p_n(0)$,

$$\begin{aligned} p^c_{n-1}(0) &= \frac{p_1(0)}{2^{n-1}} + p_n(0) \left(\frac{1}{2} + \cdots + \frac{1}{2^{n-1}}\right) \\ &= \frac{p_1(0)}{2^{n-1}} + p_n(0) \left(1 - \frac{1}{2^{n-1}}\right). \end{aligned}$$

□

After the trade at time t, suppose that agent a_n's funds become $f_n + x \cdot b^n_{n-1}$ and the amount of goods becomes $q_n - x$. Since the price reaches b^n_{n-1},

$$\begin{aligned} \frac{f_n + x \cdot b^n_{n-1}}{q_n - x} &= b^n_{n-1} \\ x &= \frac{1}{2}q_n - \frac{f_n}{2b^n_{n-1}} \end{aligned}$$

holds. Thus,

$$q_n(t+1) = q_n(t) - x = \frac{1}{2}q_n + \frac{f_n}{2b^n_{n-1}}.$$

Let us denote $q_t = \frac{1}{2}q_{t-1} + \frac{f_{t-1}}{2b_{t-1}}$ for simplicity. Then,

$$\begin{aligned} q_n(t) &= \frac{1}{2}\left(\frac{1}{2}q_n(t-2) + \frac{f_n(t-2)}{2b^n_{n-1}(t-2)}\right) + \frac{f_n(t-1)}{2b^n_{n-1}(t-1)} \\ &= \frac{1}{2^2}q_n(t-2) + \frac{1}{2}\frac{f_n(t-2)}{2b^n_{n-1}(t-2)} + \frac{f_n(t-1)}{2b^n_{n-1}(t-1)} \\ &\vdots \\ &= \frac{1}{2^t}q_0 + \sum_{0\le k\le t-1}\frac{1}{2^k}\left(\frac{f_n(t-1-k)}{2b^n_{n-1}(t-1-k)}\right). \end{aligned}$$

Since $\frac{1}{2^t}q_0 \to 0$ for large t and $\frac{f_0}{2b_0} > \cdots > \frac{f_{t-1}}{2b_{t-1}}$,

$$q_t \le \frac{1}{2^t}q_0 + \sum_{0\le k\le t-1}\frac{1}{2^k}\left(\frac{f_n(0)}{2b^n_{n-1}(0)}\right)$$

Since $b^n_{n-1}(0) = (p^c_{n-1}(0) + p_n(0))/2$, we obtain

$$\frac{1}{2^t}q_0 + \frac{2f_n(0)}{p_1(0)/2^{n-1} + P_e(2-1/2^{n-1})} < q_t \quad \text{and}$$

$$q_t < \frac{1}{2^t}q_0 + \frac{2f_n(0)}{p_1(0)/2^{n-1} + p_n(0)(2-1/2^{n-1})} \tag{6}$$

by Lemma 5.

Consider which is better for money spreading, from one injection point or from two injection points with half funds. We compare two cases, (a) one injection point is node 1, and (b) two injection points are node 1 and node n. Clearly, the node with the minimum funds at equilibrium, called *min-funds node*, in case (a) is node n, and that in case (b) is node $\lceil n/2 \rceil$ (simply denoted by $n/2$). Thus, we have only to compare f_n in case (a) and $f_{n/2}$ in case (b). The following lemma shows that the quantity of goods at the min-funds node in case (b) is less than that in case (a).

Notice that $q^{[m\to h]}_i(t)$ means the quantity of goods at node i at time t on condition that incremental funds m are initially injected into node h.

Lemma 6. *For any* $t > 0$,

$$q^{[m/2\to 1, m/2\to n]}_{n/2}(t) < q^{[m\to 1]}_n(t)$$

holds.

Proof. From equation (6), we have only to compare $q^{upper}(n/2)$, the upper bound of $q_{n/2}(t) - \frac{1}{2^t}q_0$, and $q^{lower}(n)$, the lower bound of $q_n(t) - \frac{1}{2^t}q_0$. That is, they can be described as

$$q^{upper}(n/2) = \frac{2f_{n/2}(0)}{p_1(0)/2^{n/2} + p_{n/2}(0)(2-1/2^{n/2-1})}$$

and

$$q^{lower}(n) = \frac{2f_n(0)}{p_1(0)/2^{n-1} + P_e(2-1/2^{n-1})}.$$

Since $f_n(0) = f_{n/2}(0)$ and $p_n(0) = p_{n/2}(0)$ at time $t = 0$, we have

$$\begin{aligned} \frac{q^{lower}(n)}{q^{upper}(n/2)} &= \frac{p_1(0)/2^{n/2} + p_n(0)(2-1/2^{n/2-1})}{p_1(0)/2^{n-1} + P_e(2-1/2^{n-1})} \\ &= \frac{p_n(0)}{P_e}\cdot\frac{p_1(0)/p_n(0)2^{n/2} + (2-1/2^{n/2-1})}{p_1(0)/P_e 2^{n-1} + (2-1/2^{n-1})} \\ &\approx 2^{n/2-1} > 1. \end{aligned}$$

Thus, $q^{upper}(n/2) < q^{lower}(n)$ holds. So the lemma follows. □

From Lemma 6, we claim the following theorem because the equilibrium price is equal for each case. Notice that $f^{[m\to h]}_i$ means the amount of funds at node i on condition that incremental funds m are initially injected into node h.

Theorem 4. *At an equilibrium, we have*

$$f^{[m\to 1]}_n < f^{[m/2\to 1, m/2\to n]}_{n/2}.$$

□

The theorem above suggests that the multiple injection points is better than the single injection point for effective spreading of funds.

5 CONCLUSION

In this paper we considered a new network model for the price stabilization. First, we presented a system model in which the price of goods is proportional to the amount of funds and is inversely proportional to the amount of goods at each node. Then we provided a protocol which stabilizes price and moves money / goods. Next, we showed that the equilibrium price is determined by the total amount of funds and the total amount of goods. Then, we concentrated on path networks to reveal the behavior of the protocol more precisely. We considered the price under stabilization at each node. Finally, we investigated which injection method is better from the fund-spreading point of view, motivated by an application to monetary policy.

In summary, our network model reveals the following facts.

- The equilibrium price of goods can be estimated if the price is proportional to the amount of funds and is inversely proportional to the amount of goods at each node.
- The price under stabilization at each node in a path is investigated.
- The two injections with half funds is better than the single injection from fund-spreading point of view.

Our future work includes investigating an asynchronous system and developing other protocols.

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