Intuitionistic Fuzzy Sets with Shannon Relative Entropy

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Abstract: Bio-signal or bio-medical pattern recognition includes uncertainty. Intuitionistic fuzzy sets (IFSs) are effective representation of the uncertainty factor. We present a pattern recognition method based on the weighted distance of intuitionistic fuzzy sets (IFSs) in dealing with the fuzzy recognition problem. The proposed method has a particular focus on handling the problem of choosing feature weights and feature selection in the framework of IFSs. Depending on the idea of information-theoretic entropy and relative entropy, a method is presented in dealing with the said two key problems, i.e., choosing feature weights and feature selection. The proposed pattern recognition method in the framework of IFSs can not only represent the dissimilarity between pair of features based on choosing feature weights but also reduce the computational complexity depending on feature selection. Finally, a numerical example is utilized to validate the proposed pattern recognition method.

1 INTRODUCTION

Zadeh (1965) and Yager (2000) emphasized the importance of fuzzy set and its extended fuzzy sets in the field of recognition technology. In many domains such as finance, medicine, bio-medical, defence, politics and marketing, a central problem is object recognition under uncertainty (Larsen and Yager, 2000). In the bio-medical diagnosis, many problems include imprecise and imperfect facts. How to model these problems with uncertainty and hesitancy is still a challenge (Shinoj,2013) (Szmidt and Kacprzyk, 2001) (Chung-ming, 2009). Presently, fuzzy set and its extended fuzzy sets (such as interval-valued fuzzy sets, intuitionistic fuzzy set,L*-fuzzy set, intuitionistic[0,1]-fuzzy set, vague set, grey set) have been tobe effective techniques in dealing with above-mentioned classification problems with uncertainty (Deschrijver and Kerre, 2007). Among them, intuitionistic fuzzy sets (IFSs), proposed by Atanassov (1986; 1993), provide a flexible mathematical way to cope with the hesitancy originating from imperfect or imprecise information. In an IFS, the membership degree and non-membership degree are more or less independent, and the only constraint is that the sum of the two degrees must not exceed 1.

Various aspects of IFSs have been utilized for decision making, pattern classification, and fuzzy reasoning, where imperfect facts coexist with uncertain knowledge (Li, 2010) (Hung and Yang, 2008) (Ciftcibasi and Altunay, 1998) (Cornelis, Deschrijver and Kerre, 2004). In the context of pattern recognition and classification, distance measures, similarity measures, and correlation measures of IFSs have been utilized aiming at the pattern recognition problems under fuzzy environment successfully (Hung and Yang, 2008) (Liang and Shi, 2003) (Xu, 2007) (Wang and Xin, 2005) (Park et al., 2009). In the category of IFSs, the weighted distance measure, proposed by Xu (2007), takes into account the every element's weight. However, it is difficult to choose appropriate weight of each element aiming at certain pattern recognition problem under fuzzy environment. In this paper, we shall present a framework for recognition technology based on the weighted distance in the category of IFSs, especially emphasized on the choosing the feature weights and feature selection depending on information entropy and its relative theory(Hung and Yang, 2008)(Szmidt and Kacprzyk, 2001).

The remainder of this paper is organized as follows: In Section 2 we introduce some preliminary concepts. A distance measure of IFSs is introduced

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for pattern recognition problem using intuitionistic fuzzy information particularly emphasized on the choosing the weight of each feature and feature selection in Section 3. In Section 4, we utilize some pattern classification examples to validate the pattern recognition model. Finally the article concludes with a brief summary in Section 5.

2 PRELIMINARY

2.1 Review of IFSs

Since fuzzy set only gives a membership degree to each element of the universe (Zadeh,1965), Atanassov introduces the concept of IFS characterized by a membership function and a non-membership function, where non-membership is less thanor equals to one minus the membership degree (Atanassov, 1986). The concept of IFS is as follows:

Let Xbe a set. An IFS A in X is defined with the form

$$A=\{(x,\mu_A(x),\upsilon_A(x)|x\in X)\} \tag{1}$$

where

$$\mu_A: X\rightarrow[0,1], \upsilon_A: X\rightarrow[0,1] \tag{2}$$

are two maps satisfying

$$0\leq\mu_A(x)+\upsilon_A(x)\leq 1, \text{for all } x\in X. \tag{3}$$

The numbers $\mu_A(x)$ and $\upsilon_A(x)$ denote the membership degree and nonmembershipdegree of x to A, respectively. For each IFS A in X, we call

$$\pi_A(x)=1-\mu_A(x)-\upsilon_A(x) \tag{4}$$

The intuitionistic index of x in A. If $\pi_A(x)$ = 0, the IFS A reduces to a fuzzy set (Atanassov, 1986).

2.2 Relative Entropy

Relative entropy represents the amount of discrimination between two probability distributions (Shannon and Weaver, 1949).Let X be a discrete random variable, and $p(x)$ and $q(x)$ be two probability distributions for X. Kullback defined the relative entropy between $p(x)$ and $q(x)$ as

$$\mathrm{D}(p,q)=\sum_{x\in X}p(x)\log\frac{p(x)}{q(x)}, \tag{5}$$

where $0\log\frac{0}{q(x)}=0$ and $p(x)\log\frac{p(x)}{0}=\infty(p(x)\neq 0)$.

Lin(1991), Hung and Yang(2008) pointed out that p must be absolutely continuous with respect to q, that is $q(x)$= 0 whenever $p(x)$ =0. To overcome this restriction, a modified cross-entropy measure was introduces as (Hung and Yang, 2008) (Lin, 1991) (Vlachos and Sergiadis, 2007):

$$I(p,q)=\sum_{x\in X}p(x)\log\frac{p(x)}{\frac{1}{2}p(x)+\frac{1}{2}q(x)}. \tag{6}$$

Since $I(p,q)$is not a symmetric measure, Hung et al. introduced a symmetricmeasure H as follows:

$$H(p,q)=\frac{1}{2}(I(p,q)+I(q,p)). \tag{7}$$

According to the above-mentioned analysis, $H(p,q)$ is a symmetrical functionand provides a measure to represent the divergence between p and q.

3 PATTERN RECOGNITION UNDER INTUITIONISTIC FUZZY ENVIRONMENT

Distance measure is a term that represents the difference between pair of IFSs. As an important concept in the category of fuzzy sets, distance measures of IFSs have also gained much attention due to their extensive applications, such as decision making, pattern recognition, clustering and market prediction. So far, various calculation methods of distance measures between IFSs have been proposed in the latest decades. In the following part, we introduce several classical distance measures. Let $A=\{(x,\mu_A(x),\upsilon_A(x)|x\in X)\}$ and $B=\{(x,\mu_B(x),\upsilon_B(x)|x\in X)\}$ be two IFSs in $X=\{x_1,x_2,\dots,x_n\}$. Bustince and Burillo(1995) proposed the following two distance measures between A and B. The Hamming distance and the Euclidean distance are defined by Eq. (8) and (9). Szmidt and Kacprzyk(2000) extended the work of Bustince and

$$d_1(A,B)=\frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i)+\mu_B(x_i)|+|\mu_A(x_i)-\mu_B(x_i)|) \tag{8}$$

$$d_2(A,B)=\frac{1}{2n}\sum_{i=1}^{n}((\mu_A(x_i)+\mu_B(x_i))^2+(\mu_A(x_i)-\mu_B(x_i))^2) \tag{9}$$

$$d_3(A,B)=\frac{1}{2n}\sum_{i=1}^{n}(|(\mu_A(x_i)-\mu_B(x_i)|+|\upsilon_A(x_i)-\upsilon_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|) \tag{10}$$

$$d_4(A,B)=\frac{1}{2n}\sum_{i=1}^{n}(((\mu_A(x_i)-\mu_B(x_i))^2+(\upsilon_A(x_i)-\upsilon_B(x_i))^2+(\pi_A(x_i)-\pi_B(x_i))^2)^{\frac{1}{2}} \tag{11}$$

$$d_5(A,B)=(\frac{1}{2}\sum_{i=1}^{n}\omega_i(|(\mu_A(x_i)-\mu_B(x_i)|^{\lambda}+|\upsilon_A(x_i)-\upsilon_B(x_i)|^{\lambda}+|\pi_A(x_i)-\pi_B(x_i)|^{\lambda}))^{\frac{1}{\lambda}} \tag{12}$$

Burillo, the improved Hamming distance and Euclidean distance are formulated by Eq. (10) and (11), respectively. Basing on the above-mentioned work, Xu (2007) introduced the distance measure by Eq.(12) where $\lambda \geq 1$, and ω_i (i=1, 2,…,n) denotes the weight of x_i (i=1, 2,…,n),which satisfies $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$. According to the distance proposedby Xu, (9) and (10) can be obtained from (11). More distance measures ofIFSs were proposed in recent years from different angles (see (Hung and Yang, 2008)(Xu, 2007) (Wang and Xin, 2005)(Szmidt and Kacprzyk, 2000)(Guha and Chakraborty, 2010)(Zeng and Guo, 2008).

In general, the different features have different importance in pattern recognition problem. Actually a feature having great dissimilarity compared with other features should be endowed with great weight value (Seoung and Panaya, 2011). Therefore, it is necessary to have a feature selection or choose appropriate weights of features. Since the weighted distance measures takes into account the weights' divergence, it is helpful to describe the importance of each feature. However, how to choose the weights of features under intuitionistic fuzzy environment belongs to a difficult problem which is the research focus of this paper.

In the following part, we establish a pattern recognition method based on the weighted distance measure of IFSs, and particularly have an emphasis on choosing the weight vector. Assume $\{A_1, A_2, \dots, A_m\}$ and B be m+1 IFSs in the set X=$\{x_1, x_2, \dots, x_n\}$, where A_j $(j = 1,2,\dots,m)$, B, and X denote the prototype, the unknown type and the feature set (or attribute set), respectively.

According to the analysis in 2.2, $H(p,q)$ is symmetrical function andcan be utilized to specify the dissimilarity between the probability distributions$p$ and q. For an IFS A in X, for all $\in X$, we have $\pi_A(x)+\mu_A(x)+\upsilon_A(x) = 1$, $0 \leq \mu_A(x), \upsilon_A(x), \pi_A(x) \leq 1$ .This implies that $(\mu_A(x), \upsilon_A(x), \pi_A(x))$ may be regarded as a probability distribution. Therefore, we can utilize the function H to consider the dissimilarity between IFSs. Meanwhile, to represent the importance degree of different attributes for the pattern recognition, the weight vector is introduced and defined.

The weight ω_j ($j = 1,2,\dots,n$)is defined as follows:

$$\omega_j = \frac{\exp(\eta \cdot \rho(x_j))}{\sum_{l=1}^{n} \exp(\eta \cdot \rho(x_l))}, j = 1,2,\dots,n \qquad (13)$$

where $\eta \geq 0$and

$$\rho(x_j) = \frac{1}{2}\sum_{k=1}^{m}\sum_{s=1}^{m} H\left(A_k(x_j), A_s(x_j)\right) \qquad (14)$$

Obviously, $\rho(x_j) \geq 0$. Since $\rho(x_j)$ is the sum of symmetric measure, it can indicate the dissimilarity degree of an attribute x_j. So $\rho(x_j)$ is suitable to represent the weight of each attribute. Meanwhile exponent expression ensures the weight always more than zero. So the weight vector ω is an alterable vector depending on choosing the differentvalues of η. We have the following proposition as follows:

Proposition 3.1.$\omega_i = \frac{1}{n}(i = 1,2,\dots,n)$, if $\eta = 0$ or $\rho(x_i) = \rho(x_j)$ for all $i,j \in \{1,2,\dots,n\}$.(2)$\omega_{i^*} = 1$, if $\rho(x_{i^*}) = \max\{\rho(x_1), \rho(x_2), \dots, \rho(x_n)\}$ $(i^* \in \{1,2,\dots,n\})$, $\rho(x_{i^*}) > \rho(x_j)$ for all $j \neq i^*(j \in \{1,2,\dots,n\})$, and $\lim \eta = +\infty$.

For revealing the variety of the weights resulting from the different values of η, in the following we introduce an example.

Example 3.1. The goal of this example is to reveal the relationship between feature weightω_j and η in equation 13, so we can assume that the value of $\rho(x_i)$ ($i = 1,2,3,4,5,6$) is known, then we will analyze the variety of ω_i ($i = 1,2,3,4,5,6$) with the different values of η. Let$\rho(x_i)$ $(i = 1,2,3,4,5,6)$ be the following values: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0.

Now we set η to be 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0, respectively and compute the corresponding ω_j. Basing on (13), the weights of X with different values of η are listed in Table 1 and shown in Figure 1. It implies that on the one hand we can adjust the weights of features by choosing different η, on the other hand it is an effective way to have a feature selection. In the following, we construct the feature selection model and pattern recognition model under intuitionistic fuzzy environment.

Algorithm 1: Feature Selection

Let $\delta (0 \leqslant \delta < 1)$ be a constant as the accepted threshold of weights. The feature selection rule is defined as follows:

a) Accept the feature $x_i (i = 1,2,\dots,n)$, if $\omega_i \geq \delta$

b) Reject the feature x_i $(i = 1,2,\dots,n)$, if $\omega_i < \delta$.

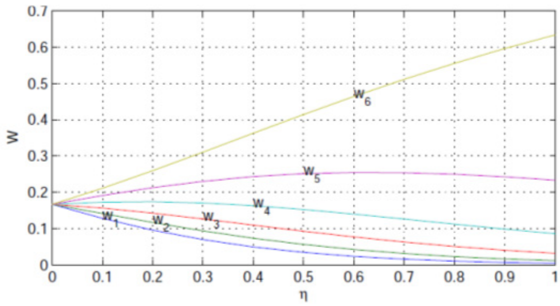

Figure 1: The weights of X with different values of.η.

Remark: since δis the threshold of feature selection, if$\omega_i < \delta$, it means attribute $x_i$is considered to be useless for classification and will be ignored. Thus δ should be a positive real number near zero, in this paper we let δ=0.03. Also, there are some other methods to decide the value of δ, for example, $\delta = \mathrm{mean}(\omega_i)/\mathrm{n}$, where n is the number of attributes.

Basing on the rule of feature selection rule, we construct the pattern recognition method under intuitionistic fuzzy environment as below:

Algorithm 2: Pattern Recognition Based on IFS with Shannon Relative Entropy

Step 1. Initialize λ and δ. λ is usually set to be 1 or 2.

Step 2. Compute $\omega_i(i = 1,2,\ldots,n)$ according to (13) and (14), and havea feature selection.

Step 3. Compute the weights of the selective features based on (13) and(14).

Step 4. Compute $\min\{d(A,A_j)\}$ ($j \in \{1,2,\ldots,m\}$) according to equation (12).

Step 5. If $d(A,A_r) = \min\{ d(A,A_j)\}$ ($r \in \{1, 2,\ldots, m\}$), then A belongs toA_r, $A_r$is a known pattern.

Remark: How to choose the parameter η is a difficult problem. Since η is an adjusted parameter, we suggest the parameter η satisfying $\frac{\max(\omega_1,\omega_2,\ldots,\omega_n)}{min(\omega_1,\omega_2,\ldots,\omega_n)} \leq 10$. This strategy ensures a majority of attributes to remain their contributions to the classification and a minority of attributes to be ignored.

4 NUMERICAL EXAMPLE AND ANALYSIS

In this Section, we utilize two numerical examples in the scenarios of the classification of building material recognition and medical diagnosis to validate the said pattern recognition method in the framework of IFSs. Meanwhile we compare the results obtained by the Hamming distance and the Euclidean distance defined by Szmidt and Kacprzyk (2001).

Example 4.1. There are four material prototypes and an unknown type denoted by IFSs in X=$\{x_1, x_2, \ldots, x_{12}\}$in this pattern recognition problem (Wang and Xing, 2005). The four prototypes and the unknown type are represented as Table 2, where A_i ($i = 1,2,3,4$) and B denote the prototype and the unknown type respectively.

Table 1: The weights of X with different values of.□.

η	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
0	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667
0.1	0.1279	0.1414	0.1562	0.1727	0.1908	0.2109
0.2	0.0954	0.1166	0.1424	0.1739	0.2124	0.2594
0.3	0.0693	0.0935	0.1262	0.1704	0.2300	0.3105
0.4	0.0491	0.0732	0.1092	0.1629	0.2430	0.3626
0.5	0.0340	0.0560	0.0924	0.1523	0.2512	0.4141
0.6	0.0231	0.0421	0.0767	0.1397	0.2546	0.4639
0.7	0.0154	0.0311	0.0626	0.1260	0.2538	0.5111
0.8	0.0102	0.0226	0.0504	0.1121	0.2495	0.5552
0.9	0.0066	0.0163	0.0401	0.0985	0.2424	0.5961
1.0	0.0043	0.0116	0.0315	0.0858	0.2331	0.6337

Table 2: Data of the prototypes and the unknown type.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
$\mu_{A_1}(x)$	0.173	0.102	0.530	0.965	0.420	0.008	0.331	1.000	0.215	0.432	0.750	0.432
$\nu_{A_1}(x)$	0.524	0.818	0.326	0.008	0.351	0.956	0.512	0.000	0.625	0.534	0.126	0.432
$\mu_2(x)$	0.510	0.627	1.000	0.125	0.026	0.732	0.556	0.650	1.000	0.145	0.047	0.760
$\nu_{A_2}(x)$	0.365	0.125	0.000	0.648	0.823	0.153	0.303	0.267	0.000	0.762	0.923	0.231
$\mu_{A_3}(x)$	0.495	0.603	0.987	0.073	0.037	0.690	0.147	0.213	0.501	1.000	0.324	0.045
$\nu_3(x)$	0.387	0.298	0.006	0.849	0.923	0.268	0.812	0.653	0.284	0.000	0.483	0.912
$\mu_{A_4}(x)$	1.000	1.000	0.857	0.734	0.021	0.076	0.152	0.113	0.489	1.000	0.386	0.028
$\nu_{A_4}(x)$	0.000	0.000	0.123	0.158	0.896	0.912	0.712	0.756	0.389	0.000	0.485	0.912
$\mu_B(x)$	0.978	0.980	0.798	0.693	0.051	0.123	0.152	0.113	0.494	0.987	0.376	0.012
$\nu_B(x)$	0.003	0.012	0.132	0.213	0.876	0.756	0.721	0.732	0.368	0.000	0.423	0.897

Table 3: The weights with different.η.

η	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}	ω_{11}	ω_{12}
0	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
0.2	0.0829	0.0878	0.0753	0.092	0.0759	0.0884	0.0728	0.0885	0.082	0.0916	0.0798	0.083
0.4	0.0819	0.092	0.0677	0.101	0.0688	0.0932	0.0633	0.0935	0.0803	0.1001	0.0759	0.0822
0.6	0.0806	0.0959	0.0605	0.1103	0.0621	0.0978	0.0547	0.0982	0.0782	0.1089	0.0719	0.0809
0.8	0.0789	0.0995	0.0539	0.1198	0.0557	0.1021	0.047	0.1026	0.0758	0.1178	0.0677	0.0793
1.0	0.0768	0.1027	0.0477	0.1295	0.0497	0.106	0.0402	0.1067	0.0731	0.1268	0.0635	0.0773
1.2	0.0744	0.1055	0.042	0.1394	0.0441	0.1096	0.0343	0.1105	0.0702	0.1359	0.0592	0.075
1.4	0.074	0.1111	0.0379	0.1538	0.0402	0.1162	0	0.1173	0.069	0.1493	0.0566	0.0746
1.6	0.0708	0.1126	0.033	0.1633	0.0352	0.1185	0	0.1198	0.0654	0.1578	0.0522	0.0715
1.8	0.0694	0.1171	0	0.1779	0.0317	0.1241	0	0.1255	0.0635	0.1712	0.0493	0.0702
2.0	0.0676	0.1208	0	0.1922	0	0.1288	0	0.1305	0.0612	0.1842	0.0462	0.0685

Table 4: The weights of X with different values of ηafter feature selection.

η	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}	ω_{11}	ω_{12}
0	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
0.2	0.0829	0.0878	0.0753	0.0920	0.0759	0.0884	0.0728	0.0885	0.0820	0.0916	0.0798	0.0830
0.4	0.0819	0.0920	0.0677	0.1010	0.0688	0.0932	0.0633	0.0935	0.0803	0.1001	0.0759	0.0822
0.6	0.0806	0.0959	0.0605	0.1103	0.0621	0.0978	0.0547	0.0982	0.0782	0.1089	0.0719	0.0809
0.8	0.0789	0.0995	0.0539	0.1198	0.0557	0.1021	0.0470	0.1026	0.0758	0.1178	0.0677	0.0793
1.0	0.0768	0.1027	0.0477	0.1295	0.0497	0.1060	0.0402	0.1067	0.0731	0.1268	0.0635	0.0733
1.2	0.0744	0.1055	0.0420	0.1394	0.0441	0.1096	0.0343	0.1105	0.0702	0.1359	0.0592	0.0750
1.4	0.0718	0.1078	0.0368	0.1493	0.0390	0.1128	0.0291	0.1138	0.0670	0.1449	0.0550	0.0725
1.6	0.0690	0.1098	0.0322	0.1593	0.0344	0.1156	0.0245	0.1168	0.0638	0.1539	0.0509	0.0697
1.8	0.0661	0.1114	0.0280	0.1692	0.0302	0.1180	0.0206	0.1194	0.0605	0.1629	0.0469	0.0668
2.0	0.0630	0.1126	0.0243	0.1791	0.0264	0.1201	0.0173	0.1216	0.0571	0.1717	0.0430	0.0638

Aiming at this pattern recognition problem, we adopt two cases as follows:

4.1 Case 1

Assume that $\lambda = 1{,}2$, and η be with following values: 0, 0.2, 0.4, 0.6, 0.8, 1.0,1.2, 1.4, 1.6, 1.8 and 2.0.

Step 1. According to the different values of η, the weights of X are shownin Table 3.

Step 2. Let $\lambda = 1{,}2$, compute the distance $d(A_i, B)$ (i=1, 2, 3, 4) as Table 4.

Step 3. Since $d(A_4, B)$=min $d(A_1, B)$,$d(A_2, B)$, $d(A_3, B)$,$d(A_4, B)$ (η=0, 0.2,…, 1.8, 2.0; λ=1, 2) for every η, B belongs to the prototype A_4.

Table 4 shows that $d(A_4, B)$ is a strictly monotone decreasing function with the strictly monotone increasing value of η . The result of distance measureswith different η shows that the proposed pattern recognition method can represent the dissimilarity between pair of features. Especially, when η=0 and λ=1, 2, the weighted distance measures reduce to the improved Hamming distance measure and the improved Euclidean distance, respectively (Szmidt and Kacprzyk, 2001).

4.2 Case 2

Assume that δ = 0.03, λ=1, 2, and η be with following values: 0, 0.2, 0.4,0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0. The pattern recognition process is asfollows:

Step 1. The weights of X={$x_1, x_2, \dots, x_{12}$} based on the said values of $\eta$are shown in Table 3.

Step 2. Let $\delta = 0.03$, the weights of X= {$x_1, x_2, \dots, x_{12}$} are shown in Table 5 after a feature selection process.

Step 3. Compute the distance $d(A_i, B)$ (i=1, 2, 3, 4) based on the obtain weights in Step 2 with λ= 1, 2, and the results are shown in Table 6.

Step 4. Since $d(A_4, B)$=min{$d(A_1, B)$,$d(A_2, B)$, $d(A_3, B)$,$d(A_4, B)$}(η=0, 0.2,…, 1.8, 2.0; λ=1, 2) for assumed parameters, B belongs to the prototypeA_4.

When η = 1.4, 1.6, 1.8, 2.0 and δ = 0.03, the pattern recognition method firstly have a feature selection. The pattern recognition results are the same as the results without feature selection. From the above-mentioned theoretical analysis and the numerical results, it implies that the proposed pattern recognition method provide an effective way to have a feature selection and can reduce the computational complexity.

The pattern recognition results of both Case 1 and Case 2 are the same as the results of (Hung and Yang, 2008)(Xu, 2007) (Wang and Xin, 2005)(Vlachos and Sergiadis, 2007)(Szmidt and Kacprzyk, 2000)(Guha and Chakraborty, 2010).

Example 4.2. In this example we utilize the data set from literature (Szmidt and Kacprzyk, 2004) to verify the performance of our method in the bio-medical diagnosis application. Here there are four patients: P={Al, Bob, Joe, Ted}. Disease classification includes: D={viral Fever, Malaria, Typhoid, Stomach problem, Chest problem}

The symptoms is defined by set S={temperature, headache, stomach pain, cough, chest-pain}. The relationship between patients and their symptoms, symptoms and diseases are represented by IFSs as table 7 and 8, respectively.

Table 5: Distances between A_i(i=1, 2, 3, 4) and *B* with different parameters.

η	λ=1				λ=2			
	$\mathrm{d}(\mathrm{A}_1,\mathrm{B})$	$\mathrm{d}(\mathrm{A}_2,\mathrm{B})$	$\mathrm{d}(\mathrm{A}_3,\mathrm{B})$	$\mathrm{d}(\mathrm{A}_4,\mathrm{B})$	$\mathrm{d}(\mathrm{A}_1,\mathrm{B})$	$\mathrm{d}(\mathrm{A}_2,\mathrm{B})$	$\mathrm{d}(\mathrm{A}_3,\mathrm{B})$	$\mathrm{d}(\mathrm{A}_4,\mathrm{B})$
0	0.8744	0.8922	0.6434	0.4677	0.9192	0.9301	0.846	0.7901
0.2	0.8745	0.8935	0.6428	0.4603	0.9183	0.9278	0.8408	0.7826
0.4	0.874	0.8946	0.6421	0.4529	0.9171	0.9254	0.8357	0.7751
0.6	0.8731	0.8953	0.6415	0.4456	0.9156	0.9227	0.8306	0.7675
0.8	0.8715	0.8958	0.6407	0.4384	0.9137	0.9199	0.8257	0.7599
1	0.8695	0.8959	0.64	0.4313	0.9115	0.917	0.8207	0.7522
1.2	0.867	0.8957	0.6391	0.4244	0.9089	0.9138	0.8159	0.7445
1.4	0.8639	0.8952	0.6381	0.4175	0.906	0.9106	0.8111	0.7368
1.6	0.8605	0.8943	0.637	0.4108	0.9028	0.9072	0.8063	0.7291
1.8	0.8565	0.8932	0.6357	0.4042	0.8992	0.9036	0.8015	0.7214
2	0.8522	0.8918	0.6344	0.3978	0.8952	0.9	0.7968	0.7138

Table 6: Distances between A_i(i=1, 2, 3, 4) and *B* with different parameters.

η	λ=1				λ=2			
	$d(A_1,B)$	$d(A_2,B)$	$d(A_3,B)$	$d(A_4,B)$	$d(A_1,B)$	$d(A_2,B)$	$d(A_3,B)$	$d(A_4,B)$
0	0.8744	0.8922	0.6434	0.4677	0.9192	0.9301	0.846	0.7901
0.2	0.8745	0.8935	0.6428	0.4603	0.9183	0.9278	0.8408	0.7826
0.4	0.874	0.8946	0.6421	0.4529	0.9171	0.9254	0.8357	0.7751
0.6	0.8731	0.8953	0.6415	0.4456	0.9156	0.9227	0.8306	0.7675
0.8	0.8715	0.8958	0.6407	0.4384	0.9137	0.9199	0.8257	0.7599
1	0.8695	0.8959	0.64	0.4313	0.9115	0.917	0.8207	0.7522
1.2	0.867	0.8957	0.6391	0.4244	0.9089	0.9138	0.8159	0.7445
1.4	0.8629	0.8883	0.6329	0.4084	0.9025	0.9046	0.8015	0.7269
1.6	1.7223	1.7768	1.2655	0.8114	1.2743	1.2776	1.1311	1.0235
1.8	2.5935	2.6815	1.9067	1.2151	1.565	1.5691	1.3882	1.2534
2	3.4475	3.584	2.5377	1.5982	1.8017	1.8075	1.5935	1.4333

Table 7: The IFSs of patients and their symptoms.

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
Ted	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Table 8: The IFSs of diseases and the symptoms.

R	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.4,0.0)	(0.7,0.0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
Headache	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0.0,0.8)
Stomach pain	(0.1,0.7)	(0.0,0.9)	(0.2,0.7)	(0.8,0.0)	(0.2,0.8)
Cough	(0.4,0.3)	(0.7,0.0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
Chest pain	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8,0.1)

Table 9: The weights with different.η.

η	ω_1	ω_2	ω_3	ω_4	ω_5
0	0.2	0.2	0.2	0.2	0.2
0.2	0.2173	0.1777	0.2188	0.1978	0.1882
0.4	0.2345	0.1570	0.2380	0.1944	0.1760
0.6	0.2516	0.1378	0.2571	0.1898	0.1635
0.8	0.2682	0.1202	0.2761	0.1843	0.1510
1	0.2842	0.1042	0.2948	0.1778	0.1387
1.2	0.2996	0.0899	0.3131	0.1706	0.1266
1.4	0.3141	0.07713	0.3307	0.1629	0.1150
1.6	0.3277	0.0658	0.3476	0.1547	0.1040
1.8	0.3404	0.0559	0.3636	0.1463	0.0936
2	0.3520	0.0474	0.3789	0.1378	0.0839

Table 10: Distances between the patients and diseases with.$\eta = 1$.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Al	0.3009	**0.2159**	0.3268	0.5857	0.5513
Bob	0.5034	0.6293	0.3835	**0.1379**	0.4493
Joe	0.3486	0.3934	**0.3312**	0.5660	0.5499
Ted	**0.2566**	0.2999	0.3339	0.4621	0.5025

Here we assume that $\lambda = 2$, and η be 1.0.

Step 1. According to the different values of η, the weights of X are shownin Table 9.

Step 2. Let $\lambda = 2$, compute the distance $d(P_i, D_j)$ (i=1, 2, 3, 4,5 and j =1,2,3,4,5) as Table 10.

Step 3. Since $d(P_1, D_2)$=min $d(P_1, D_1)$,$d(P_1, D_2)$, $d(P_1, D_3)$,$d(P_1, D_4)$, $d(P_1, D_5)$) (η=1.0; λ=2), so the patient P1, that is, Al suffers from malaria. Similarly, Bob suffers from stomach problems, Joe from typhoid and Ted from fever.

The diagnosis results are same with the method proposed in (Szmidt and Kacprzyk, 2004), which illustrates the effectiveness of our method.

Remark. The pattern recognition method based the weighted distance measure of IFSs under intuitionistic fuzzy environment not only provides a calculation method for choosing weights of features but also gives a method for feature selection.

5 CONCLUSIONS

In this paper, we construct the pattern recognition method based on the weighted distance measures of IFSs under fuzzy environment, especially emphasize on feature selection and choosing feature weights. This pattern recognition method provides a way to choose the feature weights and to have a feature selection depending on the information entropy theory. The proposed pattern recognition method not only provides a tool to represent the dissimilarity of different features but also can reduce the computational complexity through feature selection. To illustrate that the pattern recognition method is well suited in dealing with the fuzzy recognition problems, we borrowed the data set from (Wang and Xin, 2005). The results indicate that the proposed pattern recognition method is good in representing the feature weights and feature selection, and can give the accurate pattern recognition results.

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