

# Ontology Hierarchy Self Generation using Algebraic Multi-Grid (AMG)

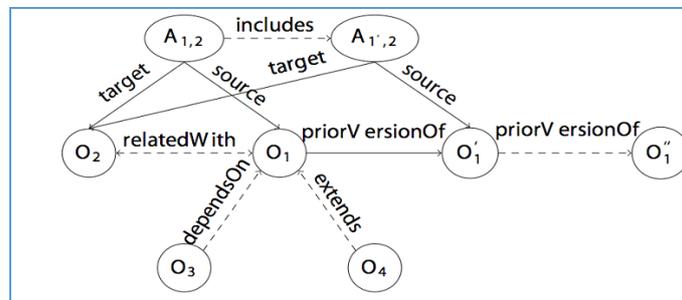
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**Abstract.** In Computer Science an ontology is a standardized representation of knowledge as a set of concepts within a domain, and the relationships between those concepts. This can be described as a graph or a network. In this paper we claim that the topology itself of the ontology contains significant semantics. Then we discuss the possibility of revealing a hierarchy of element-class from the network itself of an abstract ontology. We will use the Algebraic-Multi-Grid (AMG) concepts and tools for the Laplace equation defined on this network.

## 1 Introduction

An ontology is a body of formally represented knowledge based on a conceptualization: the objects, concepts, and other entities that are assumed to exist in some area of interest and the relationships that hold among them [1]. An example of a small fraction of an ontology is shown in Figure 1.



**Fig. 1.** An example of an ontology for software components.

It turns out that we can gradually eliminate information about the relationships among concepts – e.g. relationship labels and/or relationship directionality – and still preserve significant amounts of semantics in abstract ontologies containing only concept labels.

The central idea of this work is to look at abstract ontologies as networks amenable to algebraic treatment, viz. by AMG methods, ultimately leading back to semantic gains.

Algebraic Multi-Grid (AMG) [4] is a family of numerical methods that was

developed in order to accelerate computations for Linear and Non-Linear sets of equations with many (hundreds of thousands and more) of variables. The Algebraic MultiGrid method was developed over the past 40 years and proved useful in many cases. The general idea behind the method is based upon constructing a hierarchy of variables such that a coarser level has less variables than its finer level. The connections between these coarse level variables is extracted from the finer level variables that they represent. Eventually the coarsest level contains only few variables and are easily solved. The solution is then down-propagated to the finer levels.

In this paper we explore Ontology hierarchy self-generation, starting from abstracted ontologies, using AMG inspired methods.

## 1.1 Related Work

Here, a very concise review of related work is provided.

This work is within the field of automatic ontology summarization. Peroni, Motta, and d'Aquin [2] describe such an algorithm based on number of criteria, drawn from cognitive science, network topology, and lexical statistics.

Tho, Quan Thanh, et al [3] describe how to build an ontology from text with ideas that are similar to some of the ideas in this paper however without the algebraic infrastructure.

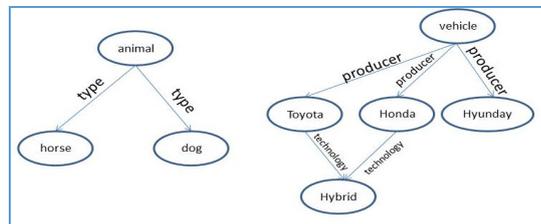
Clustering based on eigenvalues of the some similarity matrix is found in [4,5]. The AMG approach is similar to the algebraic approaches since it is also related to eigenvectors with low eigenvalues. However the spectral methods completely ignore any information that is coded in the “geometrical” nature of the network whereas the AMG is trying to encompass it in a hierarchal fashion.

## 2 From Ontology to Abstract Networks

When one deals with an ontology one perceives the property that some "concepts" (nodes) are related to each other such that they form a “group” – or as we would like to formalize it a new ontology element emerges such it will be a “class” where the elements in the group would now be elements in the "class". The process may be repeated several times. Several levels of element-class relation would emerge.

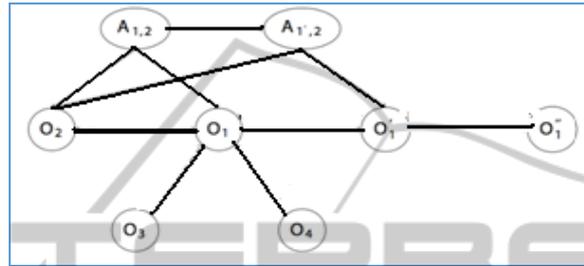
**Claim** : The topology itself of the ontology contains significant semantics.

In order to illustrate the claim let's look at figure 2.



**Fig. 2.** Moving objects ontology: animals in the l.h.s. and vehicles in the r.h.s.

Even when we ignore the labels on the links it is clear that a "horse" is not related to "Honda" and that "Hybrid" is remotely related to "Hyundai". Hence in order to simplify (and actually to enable) the following discussion we will ignore the relationship labels and will not discern between several types of relationships but rather will discuss a general form of "relation". Moreover, we claim that a significant part of the information is preserved when we ignore the directionality of the relations (this constraint is not an absolute must, but it simplifies the discussion). Hence we are dealing now with an undirected graph. Figure 3 is the undirected graph of Figure 1.



**Fig. 3.** Ontology abstract network. This is the same ontology as in Fig. 1 with removed relationship labels and directionality.

In a real semantic ontology network several hierarchies may co-exist over a single network, each class would represent a different set of "relations". E.g. "dog" may be an element in the class of "canine" and also a member of the class "three letters word".

### 3 Laplace Equation

The first step in our algebraic treatment is to define a set of linear equations based on the network. One of the natural candidates is the Laplace equation. This will be briefly explained in the following section. The important aspect is that there is a variable for each node. An equation for each node is also built from the variable in the node and the variables of the nodes adjacent to this node. An elaborated version can be found in [7]. The non-expert can skip section 3.1.

#### 3.1 Laplace Equation – Definition

Let us define a variable  $x_i$  for each node (concept/object) in the network. Let  $A_{i,j}$  be the adjacency matrix. That is  $A_{i,j}=1$  if and only if the  $i$ 's object is related to the  $j$ 's object, otherwise  $A_{i,j}=0$ . Now let us define  $D$  to be a diagonal matrix such that

That is  $D$  contains on the diagonal the number of neighbours of the node  $i$ .

$$D_{i,i} = \sum_j A_{i,j}. \quad (1)$$

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The un-normalized Laplace operator over the network  $I$  is now defined as  $L=D-A$ . and the set of equations is

$$Lx = f \quad (2)$$

When  $f=0$ , eq (1) has a simple interpretation. It states that the value at the node  $i$  ( $x_i$ ) equals the average of its neighbors (the elements  $j$  such that  $A_{ij}=0$ ).

The normalized versions of the laplacian operator are

$$L_{rw} = D^{-1}Lx = f \quad (3)$$

and

$$L_{sym} = D^{-1/2}LD^{-1/2}x = f \quad (4)$$

In the following we will refer to definition (2). The rest of the discussion can be adapted to (3) and (4) and verify that at least qualitatively they bring similar results.

#### 4 Algebraic Multi-Grid in a Nutshell

Algebraic Multi Grid (AMG) [6] is a variant of the Multi-Grid methods. The Multi-Grid (MG) methods were formed in order to solve a set of differential equations usually related to some physical problem. The approach is based on representing the problem on small scales and on large scales (see figure 4). The small scale provides the required accuracy where the large scale would provide the faster convergence. In the geometrical MG the small and large scales are extracted using the given geometry of the problem. In Algebraic MG the small and large scales are constructed during the solution process using the equations themselves.

Thus the AMG methods contain an additional step – define the coarse degrees of freedom based on the equations themselves without external reference to the geometry (if it exists). The AMG methods contain a step of recursively defining coarse degrees of freedom using the knowledge that is embedded in the equations defining the problem at hand.



**Fig. 4.** MultiGrid scheme. The fine level ( $h$ ) is transferred to the coarser level ( $2h$ ), and then recursively until the coarsest level is reached. Then backwards the solution is transferred to the finer level etc. until the finest level is reached again.

#### 5 AMG and Ontology Networks

In this work we suggest to use the Algebraic Multi-Grid [6] both as a set of concepts and as a practical tool for defining an hierarchy of element-class. Each level will be

formed in two steps:

1. "Class objects" –will be defined based on the structure of the network of relations of the current level, such that the elements will be members of these classes.
2. *Class relations* – The relations between the classes will be automatically generated based on the relations of the elements that form the class and their relations.

The first step for each level is achieved by decomposing the elements of this level into two disjoint subsets: Coarse elements and Fine elements. Technically it can be thought of a coloring process were each element is colored wither by "C" or by "F". The coloring is required to fulfil two requirements:

R1: each "F" element should have at least one "C" element that is strongly connected to it;

R2: "C" elements should not be strongly connected to each other.

There are a few heuristics that strive to optimize this step.

In the second step a new set of equations is built between the coarse points. This set of equations is a coarse representation of the fine equations such that solving this set is a coarse solution to the fine equations.

It is important to understand that after repeating the process once or more, then the set of equations that are generated in the higher level are not anymore a "simple" Laplacian. The strength of the elements  $L(i,j)$  in higher levels are not anymore restricted to be 0 or 1. They now represent in a finer degree the strength of connection between these "higher-level" elements.

The coarse elements of the current level will now be considered elements of the next level and the process of forming "super classes" can now be continued until only a single class is remained.

Note: The AMG process depends only on the equations and not of the required outcome. That is for equation (3) the process depends only on  $L$  and not on  $f$ .

## 5.1 Overall AMG Procedure

The overall AMG Procedure in pseudo-code is:

- Start from a network of concepts  $N$  as explained in section 2.
- Using the network  $N$  define a set of equations (the Laplace equation) as explained in section 3.
- On this set of equations we apply the AMG method. A central component of the AMG method is the construction of a multi-level structure of coarse and fine variables. The coarse variables at each level represent a set of variables in the lower level (the fine points).

In the ontology language, we will claim that this process creates a hierarchy such that the coarse "concept" is representing the concepts in the hierarchy below it. At this point it is important to state that we do not impose from the outside how many levels are needed nor how many fine elements are represented by any coarse element (even though these numbers can be controlled using some parameters of the heuristics).

## 6 Discussion

In a formal sense we could measure to which degree does this "coarsening" do justice or is this particular choice of coarse/fine (or class/element) relations faithful to the intended semantics. This metric could measure whether solving the coarse equation on the coarse level contributes to the solution of the fine level after a lower (interpolated) level. Again formally, we should measure if the AMG process is more efficient than say a simple Gauss-Seidel process for the same topology. If it is - we will claim that the coarsening procedure represents (at least in some sense) the finer level.

In the spirit of the paper published in SKY2010 [8], we could also claim that we have extracted information about the system since we are now able to solve it faster.

### 6.1 Open Issues and Future Work

There is a series of interesting open issues to be dealt with in future work:

How do we "test" whether the scheme succeeded to capture the multi-level structure of the ontologies from?

Is the obtained hierarchy semantically meaningful?

When the AMG scheme is expected to break down and when it is expected to succeed?

Finally, to implement the method in a tool, to enable performance of actual tests, initially with small, abstract case studies, which will be gradually increased to actual practical problems.

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