Computationally Efficient Multi-Objective Optimization of and Experimental Validation of Yagi-Uda Antenna

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Abstract: In this paper, computationally efficient multi-objective optimization of antenna structures is discussed. As a design case, we consider a multi-parameter planar Yagi-Uda antenna structure, featuring a driven element, three directors, and a feeding structure. Direct optimization of the high-fidelity electromagnetic (EM) antenna model is prohibitive in computational terms. Instead, our design methodology exploits response surface approximation (RSA) models constructed from sampled coarse-discretization EM simulation data. The RSA model is utilized to determine the Pareto optimal set of the best possible trade-offs between conflicting objectives. In order to alleviate the difficulties related to a large number of designable parameters, the RSA model is constructed in the initially reduced design space, where the lower/upper parameter bounds are estimated by solving appropriate single-objective problems resulting in identifying the extreme point of the Pareto set. The main optimization engine is multi-objective evolutionary algorithm (MOEA). Selected designs are subsequently refined using space mapping technique to obtain the final representation of the Pareto front at the high-fidelity EM antenna model level. The total design cost corresponds to less than two hundred of EM antenna simulations.

1 INTRODUCTION

Contemporary antenna structures have to be designed to satisfy very strict performance requirements concerning various characteristics such as reflection, gain (Sharaqa and Dib, 2013; Koulouridis et al., 2007), cross polarization (Afshinmanesh et al., 2008; Chamaani et al., 2011) or side-lobe level (Kuwahara, 2005; Jin and Rahmat-Samii, 2007). At the same time, antenna topologies become more and more complex and their electromagnetic (EM) models have to account for various interactions with the environment (e.g., housing, connectors, installation fixtures, etc.). Standard design procedures based on repetitive parameter sweeps guided by engineering experience are normally laborious and typically fail to find a truly optimum results. On the other hand, automated design using numerical optimization procedures (e.g., Nocedal and Wright, 2006; Conn et al., 2009) may be associated with prohibitively high computational costs when accurate, high-fidelity EM

simulations are used for antenna performance evaluation.

In general, antenna design is a multi-objective process where various (often conflicting) objectives have to be simultaneously accounted for (Kolundzija and Olcan, 2006; Yang et al., 2008). The goal of multi-objective optimization is normally to identify a set of designs (also referred to as a Pareto-optimal set) representing the best possible trade-offs between non-commensurable objectives (Deb, 2001). Compared to conventional single-objective optimization, multi-objective design posed additional challenges, both conceptual (due to possible non-commensurability of vector-valued objective function) and computational (related to the necessity of identifying multiple solutions).

Conventional direct-search methods (including gradient based algorithms) are not suitable for solving multi-objective design tasks (Deb, 2001). Population-based metaheuristic algorithms, such as genetic algorithms (Ding and Wang, 2013; Junwei *et al.*, 2009; Koulouridis *et al.*, 2007;) and particle swarm optimizers (Chamaani *et al.*, 2011; Jin and

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Rahmat-Samii, 2007; Jin and Rahmat-Samii, 2010), are more attractive because of their ability of finding the entire representation of the Pareto set in a single algorithm run (Deb, 2001). Unfortunately, disadvantage of population-based search techniques is their high computational cost which is a result of processing large sets of candidate solutions - typical number of objective function evaluations are a few thousands to tens of thousands for a single algorithm run (Afshinmanesh et al., 2008; Kuwahara, 2005). This is a serious drawback when handling EManalyzed antenna models: simulation time for realistic setups that take into account not only the antenna structure itself but also its environment (feeding structure, connectors, installation fixtures) may take as long as a few hours per design.

Utilization of surrogate-based optimization (SBO) techniques (Bandler et al., 2004; Koziel et al., 2013) can alleviate the difficulities related to high cost of metaheuristic optimization. In SBO scheme, the high-fidelity antenna model is replaced by a computationally cheap, yet less accurate surrogate (a so-called low-fidelity model), which can be 10 to 50 times faster than its high-fidelity counterpart. In case of antennas, a surrogate is usually a coarsely-meshed version of the highfidelity model, evaluated in the same electromagnetic (EM) solver (Koziel et al., 2014; Bekasiewicz et al., 2014a). The optimization burden in SBO is shifted to a surrogate, which is iteratively refined in a prediction-correction loop. The numerical expenses related to massive evaluations of the low-fidelity model during metaheuristic optimization may be further reduced bv incorporation of response surface approximation (RSA) techniques (Koziel and Bandler, 2012; Koziel and Ogurtsov, 2013). However, the cost of RSA model preparation grows exponentially with the number of designable parameters, which reduces potential applications of this approach to lowdimensional antenna design cases only.

In (Koziel and Ogurtsov, 2013), a surrogatebased multi-objective optimization scheme for seeking the representation of a Pareto optimal-set using population-based metaheuristics has been proposed. The approach partially addressed the problem of RSA model construction for antennas with up to 8 geometrical parameters by means of structure decomposition into a radiator and its feeding network; however, this is not possible for majority of modern antenna designs. Also, decomposition is not practical when the number of parameters in the decomposed parts is still too large.

In this work, we discuss a simple yet robust methodology for design space reduction aimed at generation of an accurate RSA model to extend the applicability of the technique described in (Koziel and Ogurtsov, 2013). Our approach is based on identification of the extreme points of the Pareto set by performing separate single-objective optimizations with respect to each design goal, one at a time. The reduced design space is a hypercube determined by these extreme points and it is orders of magnitude (volume-wise) smaller than the initial one, which allows for a construction of an accurate RSA model even for large number of design variables. For the sake of demonstration, we consider a 12-variable planar Yagi-Uda antenna optimized with respect to minimum voltage standing wave ratio (VSWR) and maximum average gain within the frequency band of interest. A set of designs selected from the Pareto set obtained through optimization have been fabricated and measured to experimentally validate the design methodology. PUBLIC ATIONS

2 MULTI-OBJECTIVE DESIGN OPTIMIZATION

In this section, we recall the formulation of the multi-objective optimization problem. We also discuss the optimization algorithm as well as the design space reduction scheme. In Sections 3 and 4, our design methodology is demonstrated using a planar Yagi-Uda antenna. Experimental validation is provided in Section 5.

2.1 Formulation of Multi-Objective Antenna Design Problem

We will denote by $\mathbf{R}_{j}(\mathbf{x})$ as an accurate (or highfidelity) model of the antenna under optimization. The model is obtained using EM simulation at fine discretization. Antenna response (gain, side-lobe level or VSWR) is denoted by \mathbf{R}_{j} , whereas \mathbf{x} is a vector of designable (normally, geometry) parameters.

Let $F_k(\mathbf{R}_j(\mathbf{x}))$, where $k = 1, ..., N_{obj}$, be a *k*th objective. A typical design objective could be related to minimization of an antenna side-lobe level, reduction of the occupied area or maximization of gain. In multi-objective scheme we seek for a representation of a so-called Pareto optimal-set X_P , which is composed of non-dominated designs such that for any $\mathbf{x} \in X_P$, there is

no other design y for which the relation $y \prec x$ is satisfied $(y \prec x, i.e., y$ dominates over x, if $F_k(\mathbf{R}_f(y)) \leq F_k(\mathbf{R}_f(x))$ for all $k = 1, ..., N_{obj}$, and $F_k(\mathbf{R}_f(y)) < F_k(\mathbf{R}_f(x))$ for at least one k) (Deb, 2001).

For the sake of optimization we also consider a coarse-discretization version \mathbf{R}_{cd} of \mathbf{R}_{f} , referred to as the low-fidelity model. \mathbf{R}_{cd} is evaluated using the same solver as \mathbf{R}_{f} and it is typically at least one order of magnitude faster than the high-fidelity model.

2.2 Multi-Objective Optimization Methodology

Both the high-fidelity model R_f and its low-fidelity (coarse-discretization) counterpart R_{cd} are too expensive to be directly optimized in multi-objective sense. For that reason, a kriging interpolation model R_s is prepared (Simpson *et al.*, 2001) using a set of training samples acquired by simulating R_{cd} at the predetermined training locations (Koziel *et al.*, 2013). Here, the samples are distributed using Latin Hypercube Sampling (LHS) algorithm (Beachkofski and Grandhi, 2002) within the previously reduced design space. The methodology for a solution space reduction is briefly described in Section 2.3.

The main tool for identification of Pareto optimal solutions is a multi-objective evolutionary algorithm (MOEA) with fitness sharing, mating restrictions and Pareto dominance tournament selection (Talbi, 2009). The solutions obtained by MOEA define the initial approximation of the Pareto optimal-set of interest. In the next stage, we select K designs from the initial solution set: $\mathbf{x}_{s}^{(k)}, k = 1, ..., K$. The chosen designs are subsequently refined using SBO to find their corresponding high-fidelity model R₊ responses. The description of the SBO scheme given below assumes two design objectives: F_1 and F_2 ; however, the procedure can be easily generalized to any number of objectives. For each $x_s^{(k)}$, the corresponding high-fidelity model solution $x_f^{(k)}$ is found using the output space mapping (OSM) algorithm of the form (Koziel et al., 2008):

$$\begin{aligned} \mathbf{x}_{f}^{(k,i+1)} &= \\ &= \arg\min_{\mathbf{x}, F_{2}(\mathbf{x}) \le F_{2}(\mathbf{x}_{s}^{(k,i)})} F_{1} \Big(\mathbf{R}_{s}(\mathbf{x}) + [\mathbf{R}_{f}(\mathbf{x}_{s}^{(k,i)}) - \mathbf{R}_{s}(\mathbf{x}_{s}^{(k,i)})] \Big) \end{aligned}$$
(1)

where $\mathbf{x}_{f}^{(k,i)}$ is the *i*th approximation of $\mathbf{x}_{f}^{(k)}$ (the process (1) is iterated until convergence).

The objective of design refinement is to minimize F_1 for each $x_f^{(k)}$ without degrading F_2 . The utilization of OSM ensures perfect alignment of the surrogate model R_s with the high-fidelity model at

the beginning of each iteration of (1). Usually, 2 to 3 iterations are required to find the desired high-fidelity model solutions $\mathbf{x}_{f}^{(k)}$. The OSM-driven refinement procedure is repeated for all *K* chosen samples. One should emphasize that the evaluation of the high-fidelity model \mathbf{R}_{f} is performed only during the refinement step. In this work, the construction of kriging interpolation model is performed using a DACE toolbox (Lophaven *et al.*, 2002). The block diagram of the optimization procedure is shown in Fig. 1. More detailed explanation of the optimization algorithm can be found in (Koziel and Ogurtsov, 2013).

2.3 Design Space Reduction Algorithm

Response surface approximation model R_s , once set up, is very fast and easy to optimize; however the cost of gathering training data for its construction increases exponentially with the number of design variables, which makes utilization of such a model questionable if the number of antenna geometry parameters is larger than 5 or 6 (Bekasiewicz *et al.*, 2014b; Koziel and Ogurtsov, 2013). Consequently, the reduction of the design space is a crucial step to make the RSA model setup feasible.

The Pareto optimal-set is usually located in a small region of the initially defined design space: normally, the frontiers for each geometry parameter of an antenna are defined rather wide to ensure that the desired solutions are located within these prescribed limits.



Figure 1: Design flow of the proposed multi-objective optimization procedure.

Figure 2 shows a typical example of such a situation, here, for a UWB monopole antenna (Bekasiewicz *et al.*, 2014c). Nonetheless, setting up the RSA model in such a large solution spaces is virtually impractical. In the proposed approach, frontiers of the solution space are reduced using single-objective optimizations with respect to each design goal. Consider l and u as initial lower/upper bounds for the design variables. Let

$$\mathbf{x}_{cd}^{*(k)} = \arg\min_{l \le \mathbf{x} \le \mathbf{u}} F_k \left(\mathbf{R}_{cd}(\mathbf{x}) \right)$$
(2)

where $k = 1, ..., N_{obj}$, is an optimal design of the low fidelity-model R_{cd} with respect to the *k*th objective.

These extreme (or corner) points of the Pareto optimal-set are denoted by $x_{cd}^{*(k)}$. Frontiers of the reduced design space can be then defined as follows:



Figure 2: (a) Visualization of the Pareto optimal set (\circ) in 3dimensional solution space (data for UWB monopole antenna, Bekasiewicz et al., 2014). The portion of the design space that contains the part of the Pareto set we are interested in (red cuboid, where $F_1 \leq -10$) is only a small fraction of the initial space. (b) the Pareto set of interest (\Box) versus the entire design space mapped to the feature space (×).

$$l^{*} = min\{x_{cd}^{*(1)}, \dots, x_{cd}^{*(Nobj)}\}$$
(3)

$$\boldsymbol{u}^{*} = \min\{\boldsymbol{x}_{cd}^{*(1)}, \dots, \boldsymbol{x}_{cd}^{*(Nobj)}\}$$
(4)

The reduced solution space is usually orders of magnitude (volume-wise) smaller than the initial one, which makes the generation of an accurate RSA model possible at a low computational expense. One should note that utilization of the proposed method cannot ensure the existence of all Pareto optimal solutions within refined design space; however, the majority of them are usually accounted for.

3 CASE STUDY: PLANAR YAGI-UDA ANTENNA

In order to demonstrate the presented design concepts, let us consider a planar Yagi-Uda antenna shown in Fig. 3. The antenna is designed to work on Taconic RF-35 substrate ($\varepsilon_r = 3.5$, $tan\delta = 0.0018$, h = 0.762 mm). The structure is an extended version of antenna discussed in (Qian *et al.*, 1998) and it comprises a driven element feed by a microstrip-to-coplanar strip transition, three directors and a microstrip balun. The input impedance is 50 Ω . In the design process, the following two objectives are considered:

- F_1 minimization of VSWR (with the maximum allowed VSWR equal to 2 for the entire frequency range of interest, here, 5.2 GHz to 5.8 GHz), and
- F_2 maximization of average gain in 5.2 GHz to 5.8 GHz frequency range.

The antenna geometry is described by 12 parameters: $\mathbf{x} = [s_1 \ s_2 \ s_3 \ s_4 \ v_1 \ v_2 \ v_3 \ v_4 \ u_1 \ u_2 \ u_3 \ u_4]^T$. Parameters $w_1 = 1.7, w_2 = 3, w_3 = 0.85$ and $w_4 = 0.85$ remain fixed (all dimensions in mm). The highfidelity model \mathbf{R}_f of the antenna (~600,000 mesh cells, average evaluation time of 25 minutes) and its low-fidelity counterpart \mathbf{R}_{cd} (~110,000 mesh cells, evaluation time of 150 seconds) are both implemented in CST Microwave Studio (CST, 2013) and evaluated using its transient solver. The initial lower/upper bounds are $\mathbf{l} = [2 \ 2 \ 2 \ 2 \ 18 \ 7 \ 7 \ 3 \ 7 \ 2 \ 1]^T$, and $\mathbf{u} = [10 \ 10 \ 10 \ 30 \ 15 \ 15 \ 15 \ 12 \ 16 \ 6 \ 3]^T$.

4 OPTIMIZATION RESULTS

In the first stage of the design process, the technique described in Section 2.3 has been applied to determine the reduced search space boundaries:

 $l^* = [4.05 \ 3.75 \ 2.93 \ 2 \ 22.89 \ 13 \ 14.6 \ 8 \ 4.93 \ 12.34 \ 4.2 \ 1.96]^T$, and $u^* = [7.39 \ 9.75 \ 8.93 \ 10 \ 24.22 \ 15 \ 14.6 \ 15 \ 8.93 \ 13.01 \ 4.2 \ 2.62]^T$. Compared to the initial bounds, a five orders of magnitude reduction (volume-wise) has been obtained.

In the next stage, the response surface approximation model was constructed using 1300 R_{cd} training samples allocated by means of Latin Hypercube Sampling (Beachkofski and Grandhi, 2002). The generalization error of the model estimated using cross-validation (Queipo et al., 2005) is only 1% for VSVR and 0.1% for gain. It should be reiterated that it is not possible to construct such accurate RSA models in the original design space unless significantly larger number of training samples are utilized.

The initial Pareto optimal set has been identified using multi-objective evolutionary algorithm applied to the surrogate model R_s . In the last stage, a set of ten designs has been selected from the initial Pareto set and refined using the procedure described in Section 2.2. The results are shown in Table 1 (detailed antenna dimensions for the selected designs) and Fig. 4 (initial and refined Pareto sets).



Figure 3: Layout of 12-variable planar Yagi-Uda antenna.



Figure 4: Pareto set of optimized RSA model \mathbf{R}_s (×) obtained by the proposed multi-objective optimization procedure and 11 refined high-fidelity designs \mathbf{R}_f (\Box) obtained by (1).

It can be observed that the minimum antenna VSVR is 1.177 (with the corresponding average gain of 6.47 dB). The maximum average gain possible for this antenna is 8 dB while still maintaining the VSVR level of 2 within the entire frequency band of interest. Figure 5 shows the frequency responses of the antenna for a few designs selected along the Pareto set.

It is interesting to analyse the cost of multiobjective antenna design using the proposed technique. In the first stage (design space reduction), 334 R_{cd} evaluations were used to execute singleobjective evaluations (220 R_{cd} and 114 R_{cd} evaluations for minimization of F_1 and maximization of F_2 , respectively). The response surface approximation model was constructed using additional 1300 R_{cd} samples. Multi-objective optimization of the RSA model is a very fast process, the cost of which corresponds to less than one high-fidelity model evaluation. Finally, the refinement step requires 30 R_f evaluations. The total aggregated cost of Yagi-Uda antenna optimization is about 194 R_f simulations (~81 hours of CPU time), which is very low compared to direct multi-objective optimization using population-based metaheuristic, the latter requiring at least a few thousands of highfidelity model evaluations (estimated using the number of evaluations of R_s model during MOEA optimization).

Table 1: Multi-objective optimization results for planar Yagi-Uda antenna.

		Selected designs				
		$x_{f}^{(1)}$	$x_{f}^{(3)}$	$x_{f}^{(5)}$	$x_{f}^{(8)}$	$x_{f}^{(10)}$
VSVR Average gain [dB]		1.177	1.240	1.573	1.801	2.003
		6.472	7.425	7.771	7.957	8.041
Design variables [mm]	S_1	7.38	6.19	6.03	4.87	4.30
	<i>s</i> ₂	3.79	3.82	7.57	9.46	9.75
	<i>S</i> ₃	3.08	7.24	6.94	8.10	8.44
	S_4	9.41	9.61	9.75	9.98	9.92
	v_1	24.03	23.96	23.00	23.03	22.96
	v_2	14.65	14.95	14.97	14.92	14.99
	v_3	14.60	14.60	14.60	14.60	14.60
	\mathcal{V}_4	11.52	14.81	14.78	15.00	15.00
	u_1	8.08	7.36	5.49	5.44	5.11
	u_2	12.36	12.39	12.40	12.35	12.34
	u_3	4.20	4.20	4.20	4.20	4.20
	u_4	2.24	2.59	1.98	2.01	2.35



Figure 5: Frequency characteristics of the designs from Table I: $\mathbf{x}_{f}^{(1)} - (\cdots), \mathbf{x}_{f}^{(3)} - (-\cdot), \mathbf{x}_{f}^{(5)} - (--), \mathbf{x}_{f}^{(8)} - (--), \mathbf{x}_{f}^{(10)} - (\cdots), \mathbf{$

It should be emphasized that multi-objective optimization is essential to obtain comprehensive information about the structure under design, here, the considered Yagi-Uda antenna. The knowledge about possible trade-offs between conflicting objectives is fundamental for making design decisions, in particular selecting the antenna structure for a particular application. The proposed technique allows us to gather this information at a low computational cost and it is doable on a singleprocessor machine in hours rather than weeks (the latter typical for metaheuristic-based optimization, see, e.g., Chamaani et al., 2011).

5 EXPERIMENTAL VALIDATION

Selected antenna designs $(x_f^{(1)}, x_f^{(5)}, x_f^{(10)})$ – see Table 1 for dimensions) have been fabricated in order to

carry out experimental verification of the proposed multi-objective design and optimization technique. Both, reflection and gain have been measured using vector network analyser. The latter has been determined using three antenna method based on Friis transmission equation (Balanis, 2005). A comparison of simulation and measurement results is shown in Fig. 6, whereas a photograph of the fabricated circuits is shown in Fig. 7.



Figure 6: Simulation (—) and measurement (…) results of optimized Yagi-Uda antennas in terms of voltage standing wave ratio and gain: (a) $\mathbf{x}_{f}^{(1)}$; (b) $\mathbf{x}_{f}^{(5)}$; (c) $\mathbf{x}_{f}^{(10)}$.



Figure 7: Photograph of a fabricated antennas, from the top: $\mathbf{x}_{f}^{(1)}, \mathbf{x}_{f}^{(5)}, \mathbf{x}_{f}^{(10)}$.

A slight difference in VSWR factor can be observed for $\mathbf{x}_{f}^{(5)}$ and $\mathbf{x}_{f}^{(10)}$ antenna realizations, which is due to the lack of connector in the EM antenna model. Slight differences between gain responses can also be observed. They are introduced by impedance and polarization matching errors during measurement procedure.

6 CONCLUSIONS

In this work, a simple yet robust and computationally efficient technique for multiobjective optimization of multi-parameter Yagi-Uda antenna has been presented. Our approach exploits variable-fidelity EM simulations and a response surface approximation (RSA) model (here, realized as kriging interpolation). An important step of the process, i.e., initial reduction of the design space, allows for constructing the RSA surrogate using a limited number of training samples, even though the number of designable parameters is relatively large. The Pareto optimal-set is obtained at a cost of less than 200 high-fidelity model evaluations, which is a considerable speedup in comparison to direct multiobjective optimization using population-based metaheuristics. The selected designs have been fabricated and measured for the sake of an additional validation of the design procedure.

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