

# Low-cost EM-Simulation-based Multi-objective Design Optimization of Miniaturized Microwave Structures

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**Abstract:** In this work, a simple yet reliable technique for fast multi-objective design optimization of miniaturized microwave structures is discussed. The proposed methodology is based on point-by-point identification of a Pareto-optimal set of designs representing the best possible trade-offs between conflicting objectives such as electrical performance parameters as well as the size of the structure of interest. For the sake of computational efficiency, most operations are performed on suitably corrected equivalent circuit model of the structure under design. Model correction is implemented using a space mapping technique involving, among others, frequency scaling. Our approach is demonstrated using a compact rat-race coupler. For this specific example, a set of ten designs representing a Pareto set for two objectives (electrical performance and the layout area) is identified at the cost corresponding to less than thirty high-fidelity EM simulations of the structure.

## 1 INTRODUCTION

Design of miniaturized microwave structures for contemporary wireless communication systems is a challenging task. It involves, among others, adjustment of designable (usually geometry) parameters of the structure to satisfy multiple, often conflicting objectives such as size, bandwidth, phase response, etc. (Yeung and Man, 2011). Important characteristics of compact structures, e.g., folded or fractal-shaped couplers (Tseng and Chen, 2008; Ghali and Moselhy, 2008; Liao, *et al.*, 2005), are densely packed layouts. Due to considerable electromagnetic couplings between various parts of such circuits, high-fidelity electromagnetic (EM) analysis is the only way of accurate evaluation of their electrical performance parameters.

Unfortunately, high-fidelity EM simulation is computationally expensive, which turns out to be a fundamental issue in simulated-driven design of compact components. Conventional design strategies such as repetitive parameter sweeps guided by engineering experience or direct EM-driven optimization—using, e.g., gradient-based or derivative free methods (Nocedal and Wright, 2006; Rios and Sahinidis, 2013)—require large number of

EM analyses, the total cost of which may be unacceptable from practical point of view or even prohibitive. On the other hand, alternative techniques for performance evaluation (e.g., exploiting transmission line theory) are grossly inaccurate. This is particularly true for highly miniaturized circuits with coupled building blocks (e.g. Bekasiewicz and Kurgan, 2014; Wincza and Gruszczynski, 2013; Kurgan and Bekasiewicz, 2014; Tsai, 2013; Koziel, *et al.*, 2014).

These difficulties can be alleviated, to some extent, by means of surrogate-based optimization (SBO) techniques such as space mapping (SM), which have proven their computational superiority over traditional optimization algorithms applied to the design of conventional microwave circuits. SBO schemes benefit from low-cost surrogates that are aligned with high-fidelity EM models through adaptive corrections (Bandler *et al.*, 2004b; Koziel *et al.*, 2006; Koziel *et al.*, 2008). Because most of operations are carried out on the corrected low-fidelity model, and the high-fidelity EM simulation is only launched occasionally (to verify the current design and update the surrogate model), the overall cost of the SBO process can be kept low.

As opposed to conventional designs, compact

structures are typically developed based on novel topologies and the influence of the structure size on its performance capabilities cannot be foreseen beforehand (Kurgan *et al.*, 2012; Bekasiewicz *et al.*, 2012). To eliminate the risk of design failure in the case of excessively stringent specifications that cannot be met by a prototype circuit, multi-objective optimization becomes a necessity. The goal here, rather than a single optimum design, is to find the entire set of designs (a so-called Pareto set) representing the best possible trade-offs between non-commensurable objectives. The most popular solution approach is population-based metaheuristics (Afshinmanesh *et al.*, 2008; Deb, 2001; Jin and Rahmat-Samii, 2010; Koulouridis *et al.*, 2007; Koziel and Ogurtsov, 2013; Yeung and Man, 2011). While methods such as genetic algorithms or particle swarm optimizers are capable of identifying the entire Pareto set in one algorithm run, these methods are of limited use for compact circuit design due to a large number (from hundreds to tens of thousands) of objective function evaluations involved (Koziel *et al.*, 2014; Koziel and Ogurtsov, 2013).

In this paper, we propose a computationally efficient procedure for multi-objective simulation-driven design of compact microwave passives. Our methodology exploits surrogate-based optimization, an equivalent circuit representation of the structure, and space mapping correction techniques to perform a point-by-point Pareto set identification. Our approach is illustrated using a compact rat-race coupler design.

## 2 CASE STUDY: COMPACT RAT-RACE COUPLER

In this section, we provide a description of a specific miniaturized microwave circuit to be used for explaining and demonstrating the proposed multi-objective design optimization methodology. We also describe the design objectives that will be of interest.

### 2.1 Compact Rat-Race Coupler

Consider a novel structure of an equal-split miniaturized rat-race coupler (RRC) shown in Fig. 1.

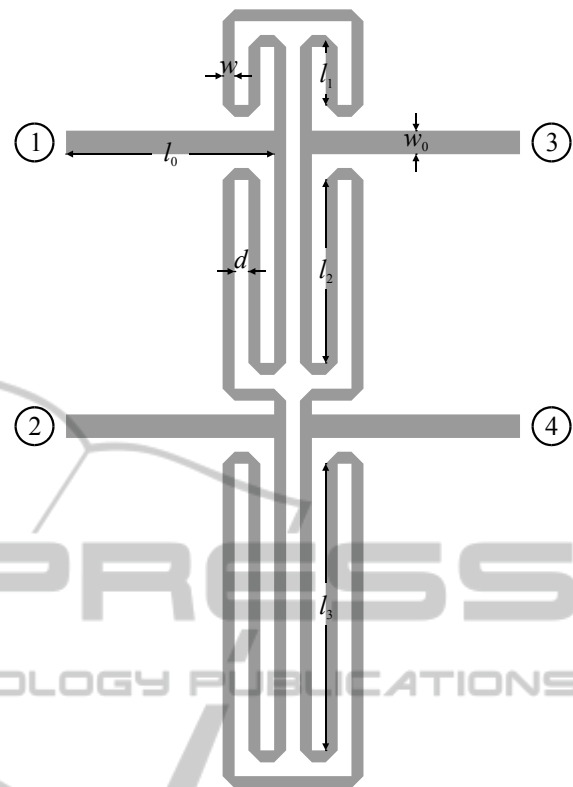


Figure 1: Geometry of a considered compact rat-race coupler.

The structure miniaturization is achieved by folding each  $70.7 \Omega$  section. The considered RRC is designed on Taconic RF-35 substrate ( $\epsilon_r = 3.5$ ,  $h = 0.762$  mm,  $\tan\delta = 0.018$ ). The input impedance is  $50 \Omega$ . The vector of coupler dimensions is:  $\mathbf{x} = [l_1 \ l_2 \ l_3 \ d \ w]^T$ , whereas  $w_0 = 1.7$ ,  $l_0 = 15$  remain fixed (all dimensions in mm). The low- and high-fidelity models of the structure are prepared with Agilent ADS (Agilent ADS, 2011) and CST Microwave Studio (CST, 2013) ( $\sim 220,000$  mesh cells and simulation time  $\sim 15$  minutes per design), respectively. Lower/upper bounds  $\mathbf{l}/\mathbf{u}$  of the solution space are represented by the following vectors:  $\mathbf{l} = [2 \ 10 \ 17 \ 0.2 \ 0.5]^T$  and  $\mathbf{u} = [8 \ 16 \ 25 \ 1.2 \ 1.5]^T$ . The initial design is:  $\mathbf{x} = [5 \ 14 \ 21 \ 0.7 \ 0.9]^T$ .

### 2.2 Design Objectives

There are two objectives considered in the coupler design:  $F_1$  – maximization of bandwidth (defined as intersection of  $|S_{11}|$  and  $|S_{41}|$  below  $-20$  dB) centred around the operating frequency of 1 GHz, and  $F_2$  – minimization of the RRC footprint (layout area).

These two objectives are generally conflicting, which means that reducing the layout area results in reduction of the  $-20$  dB bandwidth. The purpose of multi-objective design in this case is to find out possible trade-offs between the objectives. Knowledge about these trade-offs is of fundamental importance for the designer, especially when selecting a structure for a particular application.

### 3 MULTI-OBJECTIVE DESIGN METHODOLOGY

In this section, we formulate the multi-objective design problem, discuss surrogate modelling using space mapping, as well as describe the proposed multi-objective optimization approach. The numerical results obtained for the example structure of Section 2 are presented in Section 4.

#### 3.1 Multi-Objective Design Problem Formulation

Let  $F_k(\mathbf{R}_f(\mathbf{x}))$ , where  $k = 1, \dots, N_{obj}$ , be a  $k$ th design objective. Typical objectives include electrical performance parameters as well as the component size (in particular the area occupied by the circuit layout), the latter being critical for the design of compact structures. In multi-objective scheme we seek for a representation of a so-called Pareto-optimal set  $X_p$ , which is composed of non-dominated designs such that for any  $\mathbf{x} \in X_p$ , there is no other design  $\mathbf{y}$  for which the relation  $\mathbf{y} \prec \mathbf{x}$  is satisfied ( $\mathbf{y} \prec \mathbf{x}$ , i.e.,  $\mathbf{y}$  dominates over  $\mathbf{x}$ , if  $F_k(\mathbf{R}_f(\mathbf{y})) \leq F_k(\mathbf{R}_f(\mathbf{x}))$  for all  $k = 1, \dots, N_{obj}$ , and  $F_k(\mathbf{R}_f(\mathbf{y})) < F_k(\mathbf{R}_f(\mathbf{x}))$  for at least one  $k$ ) (Deb, 2001).

#### 3.2 Low-Fidelity Model. Surrogate Modelling using Space Mapping

The most popular solution approaches for multi-objective problems are undoubtedly population-based metaheuristics (Venkatarayalu *et al.*, 2005; Guimaraes *et al.*, 2006; Yang *et al.*, 2008), including genetic algorithms (Kuwahara, 2005) or particle swarm optimizers (Jin and Rahmat-Samii, 2007). The most important advantage of these techniques is their ability to identify the entire Pareto set in a single algorithm run. Unfortunately, the computational cost of metaheuristic algorithms is normally very high – typically, thousands or tens of thousands of objective function evaluations (Afshinmanesh *et al.*, 2008; Chamaani *et al.*, 2011;

Kuwahara, 2005). Consequently, metaheuristics are only suitable for handling problems where computational cost of objective evaluation is not of a major concern. Here, EM-simulated high-fidelity model is too expensive to be directly handled in a multi-objective optimization setting. Therefore, we use an auxiliary equivalent circuit (low-fidelity) model  $\mathbf{R}_c$ , evaluated by means of a circuit simulator (Bandler *et al.*, 2001; Koziel *et al.*, 2008; Bandler *et al.*, 2002), here, Agilent ADS (Agilent ADS, 2011). Figure 2 shows the circuit model for the coupler structure of Fig. 1.

The low-fidelity model is very fast, however, it is not an accurate representation of  $\mathbf{R}_f$ . Its corrected version, a surrogate model  $\mathbf{R}_s$ , will be utilized in the optimization process (Bandler *et al.*, 2004a; Cheng *et al.*, 2004). Based on initial inspection of the type of misalignment between the low- and high-fidelity models, implicit and frequency space mapping (SM) seem to be the most suitable correction techniques. More specifically, the surrogate model is defined as

$$\mathbf{R}_s(\mathbf{x}) = \mathbf{R}_{c,F}(\mathbf{x}; \mathbf{f}, \mathbf{p}) \quad (1)$$

where  $\mathbf{R}_{c,F}$  is a frequency-scaled low-fidelity model, whereas  $\mathbf{f}$  and  $\mathbf{p}$  are frequency SM and implicit SM parameters, respectively.

Let  $\mathbf{R}_c(\mathbf{x}) = [R_c(\mathbf{x}, \omega_1) R_c(\mathbf{x}, \omega_2) \dots R_c(\mathbf{x}, \omega_m)]^T$ , where  $R_c(\mathbf{x}, \omega_j)$  is evaluation of the circuit model at a frequency  $\omega_j$ . Then,  $\mathbf{R}_{c,F}(\mathbf{x}; \mathbf{f}, \mathbf{p}) = [R_c(\mathbf{x}, f_0 + \omega_1 f_1 \mathbf{p}) \dots R_c(\mathbf{x}, f_0 + \omega_m f_1 \mathbf{p})]^T$ , with  $f_0$  and  $f_1$  being frequency scaling parameters. Here, implicit SM parameters  $\mathbf{p}$  are dielectric permittivity as well as thickness of the microstrip components of the circuit corresponding to selected groups of components as indicated in Fig. 2. SM parameters are extracted to minimize misalignment between  $\mathbf{R}_s$  and  $\mathbf{R}_f$  as follows:

$$[\mathbf{f}^*, \mathbf{p}^*] = \arg \min_{\mathbf{f}, \mathbf{p}} \|\mathbf{R}_f(\mathbf{x}) - \mathbf{R}_{c,F}(\mathbf{x}; \mathbf{f}, \mathbf{p})\| \quad (2)$$

Figure 3 shows the responses of the high- and low-fidelity model at certain design  $\mathbf{x}$ , as well as the response of the surrogate model  $\mathbf{R}_s$  at the same design. It can be observed that the model alignment is greatly improved, however, generalization capability of the surrogate is limited (cf. Fig. 3(b)). In particular, it is not possible to find a single set of SM parameters that would ensure surrogate model accuracy across the entire design space. As a consequence, in order to lead towards a satisfactory design, the surrogate has to be iteratively refined during the optimization process.

### 3.3 Optimization Algorithm

Due to the limited generalization capability of the SM surrogate model mentioned in Section 3.2, as well as required reduction of the computational cost of the multi-objective optimization process, our design approach is based on point-by-point identification of the Pareto set. In the first step, the coupler structure is optimized taking into account the first objective only (here, electrical parameters). The obtained value  $F_1(\mathbf{R}_f(\mathbf{x}_p^{(1)}))$  at the optimum design  $\mathbf{x}_p^{(1)}$  determines, together with the corresponding value of the second objective (here, layout area),  $F_2^{(1)} = F_2(\mathbf{R}_f(\mathbf{x}_p^{(1)}))$ , the extreme points of the Pareto set.

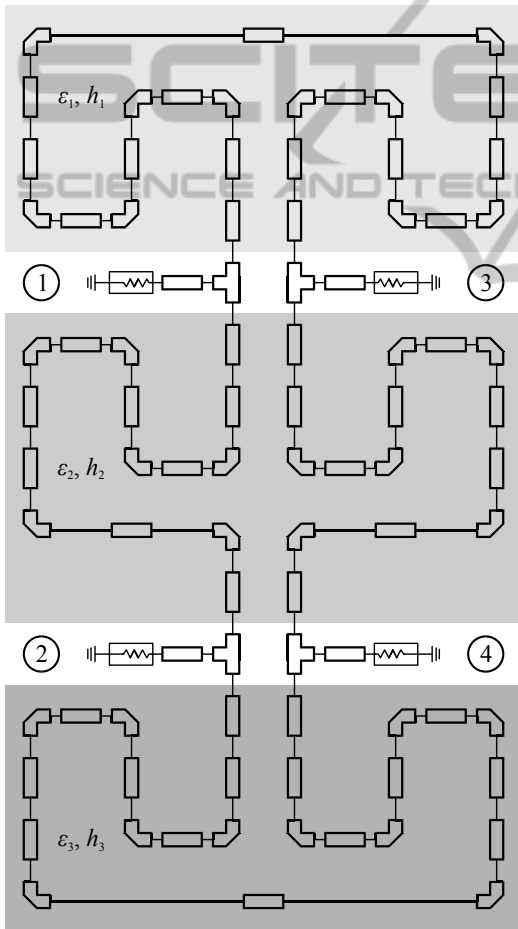


Figure 2: Equivalent circuit model of the coupler structure of Fig. 1 with highlighted regions with different implicit SM parameters  $\mathbf{p}$  (model implemented in Agilent ADS).

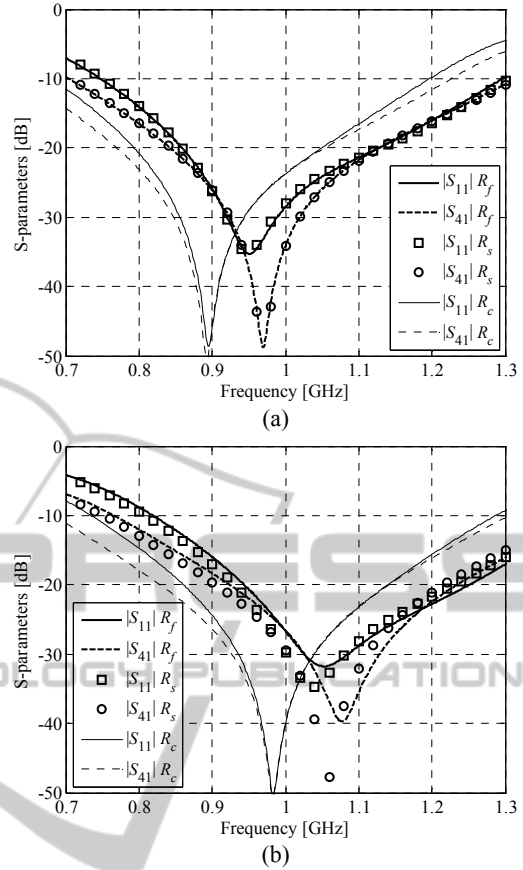


Figure 3: Responses of the high- and low-fidelity coupler models as well as the SM surrogate (a) at certain design  $\mathbf{x}$  (at which the surrogate is extracted), and (b) at some other design. Plot (b) indicates limited generalization capability of the surrogate.

In the subsequent steps, we set the threshold values for the second objective  $F_2^{(j)}$ , and optimize the structure with respect to the first objective so that the above threshold value is preserved:

$$\mathbf{x}_p^{(j)} = \arg \min_{\mathbf{x}, F_2(\mathbf{R}_f(\mathbf{x})) \leq F_2^{(j)}} F_1(\mathbf{R}_f(\mathbf{x})) \quad (3)$$

Here,  $\mathbf{x}_p^{(j)}$  is the  $j$ th element of the Pareto set. The process is continued until  $F_1(\mathbf{R}_f(\mathbf{x}_p^{(j)}))$  is still satisfactory from the point of view is given design specifications.

Problem (3) is solved using the SM surrogate model (cf. Section 2.2) and it is itself realized as an iterative process

$$\mathbf{x}_p^{(j,k)} = \arg \min_{\mathbf{x}, F_2(\mathbf{R}_f^{(j,k)}(\mathbf{x})) \leq F_2^{(j)}} F_1(\mathbf{R}_f^{(j,k)}(\mathbf{x})) \quad (4)$$

where

$$\mathbf{R}_s^{(j,k)}(\mathbf{x}) = \mathbf{R}_{c,F}(\mathbf{x}; \mathbf{f}^{(j,k)}, \mathbf{p}^{(j,k)}) \quad (5)$$

and

$$[\mathbf{f}^{(j,k)}, \mathbf{p}^{(j,k)}] = \arg \min_{\mathbf{f}, \mathbf{p}} \|\mathbf{R}_f(\mathbf{x}^{(j,k)}) - \mathbf{R}_{c,F}(\mathbf{x}^{(j,k)}; \mathbf{f}, \mathbf{p})\| \quad (6)$$

The starting point for the algorithm (4) is  $\mathbf{x}_p^{(j-1)}$  (the previously obtained Pareto set point). Normally, two iterations of (4) are sufficient to obtain  $\mathbf{x}_p^{(j)}$ , which is because the starting point is already a good approximation of the optimum. In practice, the thresholds  $F_2^{(j)}$  can be obtained as  $F_2^{(j)} = \alpha F_2^{(j-1)}$  with  $\alpha < 1$  (e.g.,  $\alpha = 0.95$ ), or  $F_2^{(j)} = F_2^{(j-1)} - \beta$  with  $\beta > 0$  (e.g.,  $\beta = 0.05 F_2^{(1)}$ ).

The computational cost of the entire multi-objective design process using the proposed methodology can be estimated (in terms of the number of EM simulations of the structure) as  $N \cdot K$ , where  $N$  is the number of point in the Pareto set, and  $K$  is the average number of iterations (4) necessary to obtain the next point. In practice,  $K \leq 3$ .

It should also be mentioned that another important design goal, i.e.,  $|S_{21}| = |S_{31}|$  at the operating frequency (here, 1 GHz), ensuring equal division of the signal power between ports 2 and 3 of the circuit, is handled implicitly. More specifically, it is enforced for each design obtained in the optimization process by applying an additional penalty function to  $F_1$  in (4) that penalizes designs for which  $||S_{21}| - |S_{31}|| > ds$  at the operating frequency (here, we use  $ds = 0.1$  dB, as an acceptable inaccuracy level).

## 4 NUMERICAL RESULTS

The coupler structure of Section 2 has been designed using the multi-objective design methodology described in Section 3. The first design obtained by using (4) without any area constraints resulted in -20 dB bandwidth of 281 MHz and the layout area of 570 mm<sup>2</sup>. Nine other designs have been obtained by setting up  $F_2^{(j)}$  to 540, 500, 475, 450, 425, 400, 375, 350, and 325 mm<sup>2</sup>, respectively. Figure 4 shows the obtained representation of the Pareto front.

For the layout area of 300 mm<sup>2</sup>, it was impossible to obtain a design with positive value of -20 dB bandwidth, which essentially means that 300

mm<sup>2</sup> is a lower limit (in terms of layout area) for practically useful designs for this particular coupler topology. This is—from the designer standpoint—an important information regarding miniaturization limitations, which may be utilized, e.g., to discriminate structures suitable for a given (in particular, space-limited) application.

Table 1 and Figure 5 show the numerical data and frequency characteristics for the selected designs. It can be observed that the coupler size can be reduced by over 40 percent with respect to its original size (corresponding to the best possible electrical performance), while maintaining acceptable performance.

Table 1: Multi-objective design optimization of rat-race coupler: selected results.

Design Variables [mm]					Objectives	
$l_1$	$l_2$	$l_3$	$d$	$W$	-20 dB Bandwidth [MHz]	Layout Area [mm <sup>2</sup> ]
4.18	13.20	20.68	0.994	0.865	281	570
3.83	11.76	20.44	0.825	0.877	270	500
4.10	13.78	21.14	0.581	0.887	260	450
4.25	12.17	22.12	0.400	0.923	202	400
3.95	10.87	21.71	0.350	0.936	174	375
4.37	12.33	22.52	0.350	0.820	151	350

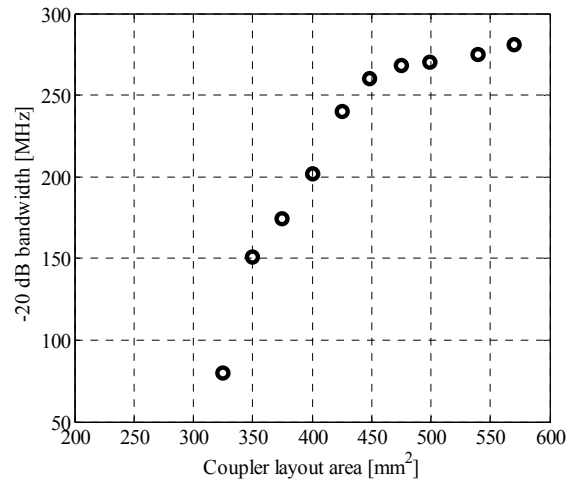


Figure 4: Pareto set obtained using the proposed multi-objective design optimization methodology.

Both Figure 4 and Table 1 indicate the conflicting nature of the considered objectives: reduction of the layout area of the circuit inevitably results in degrading its electrical performance, here, -20 dB bandwidth.



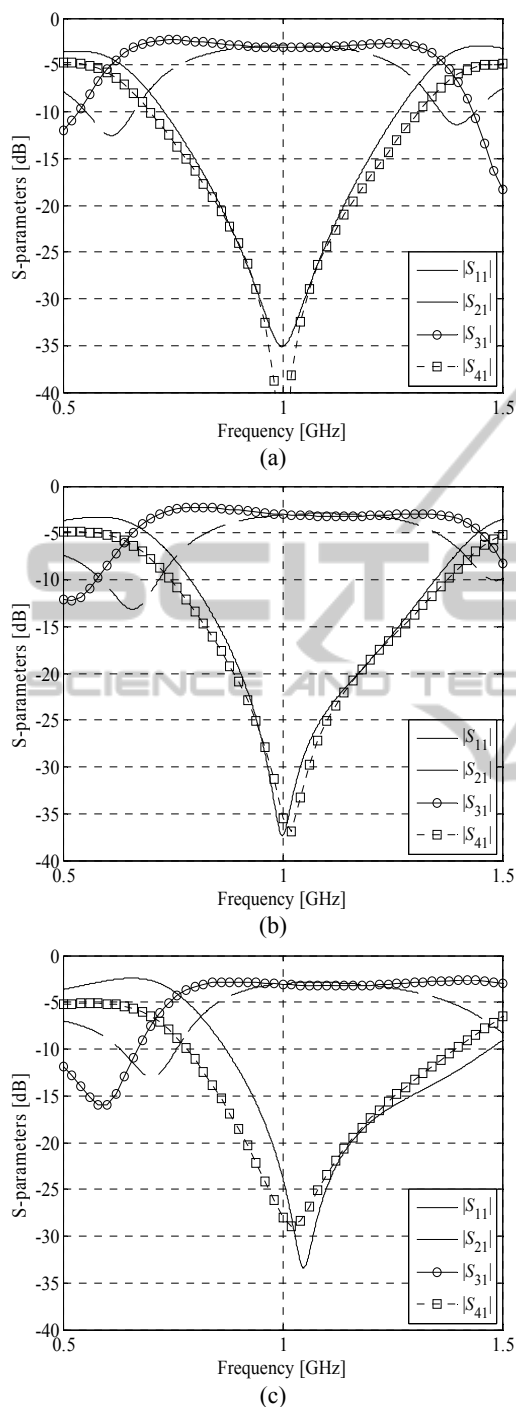


Figure 5: Frequency characteristics for selected coupler designs, corresponding to the layout area 570 mm<sup>2</sup> (a), 448 mm<sup>2</sup> (b), and 375 mm<sup>2</sup> (c).

All the designs along the Pareto front are nominally satisfying the basic design goal (i.e., both  $|S_{11}|$  and  $|S_{41}|$  are lower than  $-20$  dB and centred around the operating frequency of 1 GHz). However,

the designs with wider  $-20$  dB bandwidth (such as the one shown in Fig. 5(a) versus that in Fig. 5(c)) are electrically better because of higher chance of satisfying design specifications in case of unavoidable manufacturing tolerances (a consequence of which will be a deviation of actual characteristics of the fabricated circuit with respect to the nominal ones).

The total cost of the design process corresponds to less than 30 high-fidelity model evaluations ( $\sim 7.5$  hours), including the overhead related to multiple evaluations of the circuit model  $R_c$  (the latter does not exceed 20 percent of the overall EM simulation cost). It should be noted that direct multi-objective optimization of the high-fidelity EM antenna model  $R_f$  would not be possible (the expected cost of a few thousand of model evaluations is practically prohibitive).

## 5 CONCLUSIONS

In this paper, a technique for low-cost multi-objective design optimization of miniaturized microwave structures has been proposed. The design speedup has been obtained through the usage of appropriately corrected, fast equivalent circuit model of the structure under design. Another important component was point-by-point Pareto set identification through constrained single-objective optimizations. As a result, the number of high-fidelity EM simulations of the structure was greatly reduced (to less than three per identified Pareto set point). As demonstrated using a compact RRC coupler, a set of designs corresponding to best possible trade-offs between conflicting objectives (here, electrical performance and the layout area of the structure) has been obtained at a low computational cost, corresponding to less than thirty EM simulations of the coupler. According to our knowledge, this is a first successful attempt to solve the low-cost multi-objective design problem of compact structures exploiting surrogate-based optimization.

The future work will aim at extending the presented methodology to cases with larger number of conflicting design objectives, as well as applying it to other classes of structures, especially antennas, where fast equivalent circuit models are normally unavailable.

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