Twodimensional Visualization of Discrete Time Domain Intervals Subject to Uncertainty

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Keywords: Possibility Theory, Time Domain Intervals, Visualization, Discrete Time Model, Allen Relationships.

Abstract: One of the most important purposes of information systems is to allow human users to retrieve their data or information or knowledge derived from their data. These data may be subject to imperfections and often represent time indications, as time is an important part of reality. Representations of time indications rely on the information system's time domain. Obviously, the effectiveness of an information system in retrieval context depends greatly on the interpretability of the presentation of its data, information or knowledge. For that reason, such data, information or knowledge is usually visualized. The work presented in this paper proposes a novel approach to visualize time domain intervals subject to uncertainty and also shows how temporal reasoning with these visualizations can be done. The presented novel approach considers gradual confidence in the context of uncertainty and is specifically designed for time domain intervals.

1 INTRODUCTION

Typically, Information Systems (IS) contain data representing properties of real-life objects or concepts. As such objects or concepts often have temporal aspects, many data in IS are used to represent *time indications*, which indicate parts of time (Billiet et al., 2013b), (Billiet et al., 2013a). In existing literature, several proposals have been concerned with the modeling of such time indications (Bolour et al., 1982). The corresponding models are called *time models*. Many of these proposals accept *time intervals* (Dyreson and et al., 1994), (Jensen and et al., 1998), which can intuitively be seen as uninterrupted, bounded periods in time, as primitives (Allen, 1983), (Dyreson and Snodgrass, 1998), (Garrido et al., 2009), (Billiet et al., 2012), (Billiet et al., 2013b), (Billiet et al., 2013a) and approach *instants* (Dyreson and et al., 1994), (Jensen and et al., 1998), which can intuitively be seen as infinitesimally 'short' moments in time, as special cases of time intervals. Usually, the representations of time intervals in time models are called *time domain intervals*. Therefore, in the presented work, the focus will be on time domain intervals. Moreover, as IS usually have finite precision, time models are very often discrete. Therefore, in the presented work, a general discrete time model will be used.

Usually, a lot of the data in IS are made by humans. However, human-made data are prone to imperfections, like uncertainties (Pons and et al., 2012), (Billiet et al., 2013b), (Billiet et al., 2013a). As a consequence, time indications represented in IS may contain such imperfections too. As uncertainty is the most studied imperfection in time indications in current literature, the work presented in this paper will consider time domain intervals subject to uncertainty.

One of the most important purposes of IS is to allow human users to retrieve their data or information or knowledge derived from their data. Obviously, the effectiveness of an IS strongly increases if it presents its data, information or knowledge in such a way that allows easy interpretation or processing by humans. Usually, such interpretability is greatly improved by visualizing the presented data, information or knowledge in a schematic form. This certainly holds for data, information or knowledge related to time intervals (Qiang and et al., 2010), (Qiang and et al., 2012).

Traditional approaches to visualizing time (domain) intervals visually represent time (domain) intervals as line segments (Matkovic and et al., 2007),(Kincaid and Lam, 2006),(Saito et al., 2005),(Aigner and et al., 2005). Such approaches are generally called *linear approaches*. Linear approaches might introduce issues concerning, a.o., visual ordering and scalability (Qiang and et al.,

Billiet C. and De Tré G..
Twodimensional Visualization of Discrete Time Domain Intervals Subject to Uncertainty.
DOI: 10.5220/0005124701370145
In *Proceedings of the International Conference on Fuzzy Computation Theory and Applications* (FCTA-2014), pages 137-145
ISBN: 978-989-758-053-6

2010),(Qiang and et al., 2012), which have a direct, negative impact on the interpretability of the visualization.

In an attempt to deal with such issues, an approach has been introduced in which time intervals are visualized as points in the image plane (Kulpa, 2006). Based on this, Van De Weghe et al. introduced a similar approach called the *Triangular Model* (TM) (Van De Weghe et al., 2007). However, these approaches consider the visualization of time intervals and not time domain intervals, causing them to not be immediately usable in the context of IS. Moreover, these approaches do not account for imperfection. In (Qiang and et al., 2010), an approach is proposed that does consider imperfection in time intervals. However, this approach doesn't consider gradual confidence in the context of uncertainty and doesn't consider time domain intervals. In (De Tré and et al., 2012), an approach is proposed that does consider such gradual confidence, but still doesn't consider time domain intervals. Moreover, this approach has shown to be slightly too modest.

The first contribution of the work presented in this paper is the proposal of a novel way to visualize time domain intervals subject to uncertainty, where the time model involved is a discrete one. This proposal is presented in section 5. A second contribution is the proposal of a novel way of evaluating the temporal relationships between a time domain interval subject to uncertainty and a regular one. This is presented in section 7. The final contribution of this paper is the proposal of a novel way of evaluating the temporal relationships between two time domain intervals subject to uncertainty. This is presented in section 8. Both approaches improve upon the approach introduced in (De Tré and et al., 2012).

2 TIME MODELING IN INFORMATION SYSTEMS

2.1 The Perception of Time by Information Systems

IS usually see time as a totally ordered set of infinitesimally short 'moments', which is the so-called time axis. These 'moments' are called *instants*.

Definition 1. Instant (Dyreson and et al., 1994), (Jensen and et al., 1998)
An instant *is a time point on an underlying time axis.*

Two instants define a subset of the time axis, which is called a *time interval*.

Definition 2. Time Interval (Dyreson and et al., 1994), (Jensen and et al., 1998)
A time interval *is the subset of the time axis containing all instants between two given instants (and no other).*

Definition 3. Duration (Dyreson and et al., 1994), (Jensen and et al., 1998)
A duration *is an amount of time with known length, but no specific starting or ending instants.*

A time interval is bounded by two instants, whereas a duration is not.

2.2 The Modeling of Time by Information Systems

IS usually model time using *time models*.

Definition 4. Time Model
In a data model used by an IS, a time model *is the collection of definitions, prescriptions and rules that allow describing the structure and behavior of time.*

A time model defines how time-related concepts are represented in IS. To do this, a time model generally uses a *time domain*, which is the set of values used to represent time indications, and a set of rules and operations, which determine the behavior of the elements of the time domain. Existing time models can be categorized as to whether their time domain is a continuous or discrete set. In the presented work, it is assumed that the used time domain always is a discrete set, which means the corresponding time model is called a *discrete time model*.

As the one used in the presented work, a discrete time model usually models an underlying time axis using *chronons*.

Definition 5. Chronon (Dyreson and et al., 1994), (Jensen and et al., 1998)
In a data model, a chronon *is a non-decomposable time interval of some fixed, minimal duration.*

To model a time axis, a time model usually uses a sequence of consecutive chronons. Every such chronon corresponds to exactly one element in the model's time domain, where the ordering of the consecutive time domain elements reflects the temporal ordering of these chronons. These chronons have the same duration and are the smallest time intervals an IS using the time model can distinguish.
An instant is usually modeled as a single element of the time domain, corresponding to the chronon containing the instant, whereas a time interval can be modeled as a *time domain interval*.

Definition 6. Time Domain Interval

In a data model, a time domain interval *is a set of (one or more) consecutive time domain elements, used to represent a set of consecutive chronons which are used to represent a time interval.*

The time model used in the presented work is constructed as follows. Consider a time axis $\mathbb{T}$, which is a totally ordered set of instants. The time model now contains a totally ordered set $\mathbb{D}$ of consecutive chronons $c_i,\ i \in \mathbb{Z}$, in $\mathbb{T}$, equipped with a surjective mapping m from $\mathbb{T}$ to $\mathbb{D}$. Thus, $\mathbb{D}$ is defined by

$$\mathbb{D} = \{c_i | (c_i \ = \ [t_i, t_{i+1}[) \ \wedge \ (i \in \mathbb{Z})\}$$

Here, every $t_i, i \in \mathbb{Z}$ is an instant in $\mathbb{T}$. The mapping m is now defined by

$$\begin{aligned} m : \mathbb{T} &\rightarrow \mathbb{D} \\ : t &\rightarrow c_i = [t_i, t_{i+1}[\text{, for which } t_i \leq t < t_{i+1} \end{aligned}$$

Now, the time model is considered to be used by an IS and to contain a time domain $\mathbb{E}$ to that purpose, where $\mathbb{E}$ is defined as $\mathbb{E} = \{e_i | (i \in \mathbb{Z})\}$.

Every element $e_i, i \in \mathbb{Z}$ of this domain $\mathbb{E}$ now uniquely corresponds to a single chronon $c_i \in \mathbb{D}, i \in \mathbb{Z}$. Two consecutive elements of $\mathbb{E}$ always correspond to two consecutive elements of $\mathbb{D}$, maintaining the ordering. As such, time indications will be represented using values of $\mathbb{E}$:

- an instant $t \in \mathbb{T}$ will be modeled as the element $e \in \mathbb{E}$ for which e corresponds to the chronon $c \in \mathbb{D}$ to which t is mapped by m.
- a time interval $[t_s, t_e] \subseteq \mathbb{T}$ will be modeled as the interval $[e_s, e_e] \subseteq \mathbb{E}$ for which e_s, respectively e_e corresponds to chronon $c_s \in \mathbb{D}$, respectively $c_e \in \mathbb{D}$ to which t_s, respectively t_e is mapped by m.

Any IS can now employ this time model by instantiating $\mathbb{E}$, which is only assumed to be totally ordered. The presented work will only consider closed time domain intervals. This does not limit the applicability of the presented proposal, because of the discrete nature of the time domain.

As mentioned before, the work presented in this paper aims to reason with time domain intervals. Usually, such reasoning requires the modeling of *temporal relationships*. In current literature, several proposals have been concerned with the modeling and behavior of temporal relationships (Allen, 1983), (Allen, 1991), (Galton, 1990). As opposed to standard mathematical interval relationships, temporal relationships describe relationships with specific semantics because of the temporal nature of the intervals and their connection to time. Allen (Allen, 1983), (Allen, 1991), (Galton, 1990) most notably proposed

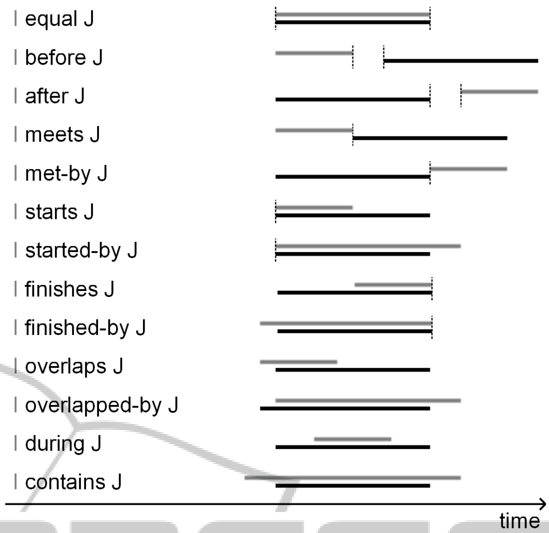

Figure 1: A linear visualization of Allen's relationships. Time interval I is visualized here as a grey line segment, time interval J as a black line segment.

a framework describing such temporal relationships between time (domain) intervals. Figure 1 visualizes the temporal relationships Allen discerned.

3 UNCERTAINTY IN TIME DOMAIN INTERVALS

3.1 Uncertainty and Possibility Theory

Different causes for uncertainty exist. Among others, uncertainty about the outcome of an experiment can be caused by a (partial) lack of knowledge: it could be known that only one outcome may occur, but as the experiment is not perfectly and comprehensively known or controlled, the outcome of the experiment may be unknown and thus uncertain. Confidence in the context of uncertainty caused by a (partial) lack of knowledge is modeled using possibility theory, where possibility is interpreted as plausibility, given all available knowledge (Bronselaer et al., 2013).

Based on prior experiences, it is the belief of the authors that uncertainty concerning time is usually caused by a (partial) lack of knowledge. Therefore, the work presented in this paper only considers uncertainty caused by a (partial) lack of knowledge and uses possibility theory to model confidence in this context. In this paper, possibility is always interpreted as plausibility, given all available knowledge.

3.2 Ill-known (Time Domain) Intervals

The work presented in this paper will allow time domain intervals to be subject to uncertainty by allowing them to be *ill-known time domain intervals*. Before this concept can be explained, the concept of *possibilistic variables* should be introduced.

Definition 7. Possibilistic Variable (Bronselaer et al., 2013)

A Possibilistic variable *X on a universe Ω is a variable taking exactly one value in Ω, but for which this value is unknown. The possibility distribution π_X on Ω, associated with X, models the available knowledge about the value that X takes: for each $u \in \Omega$, $\pi_X(u)$ represents the possibility that X takes the value u.*

When a possibilistic variable is defined on a universe containing intervals, it defines and describes an *ill-known interval* (Billiet et al., 2012), (Billiet et al., 2013b), (Billiet et al., 2013a):

Definition 8. Ill-known Interval (Billiet et al., 2012), (Billiet et al., 2013b), (Billiet et al., 2013a)

Consider a totally ordered set S containing single, atomic values and its powerset $\wp(S)$. Consider the subset $\wp_I(S)$ of $\wp(S)$ and let $\wp_I(S)$ contain every element of $\wp(S)$ that is an interval, but no other elements. Now consider a possibilistic variable $X_{\tilde{I}}$ on $\wp_I(S)$. The unique, exact value $X_{\tilde{I}}$ takes, which is unknown and which is an interval containing single values of S, is called an ill-known interval *in the presented work. Seen as the ill-known interval defines and describes an interval in S, it is also called an* ill-known interval in *S*.

The interpretation is that an ill-known interval in a set S represents a specific, precise interval in S which is unknown. To clarify the difference, an interval not subject to any imperfection (including uncertainty) will be called a *regular interval* in this paper. In the presented work, ill-known intervals will be denoted using upper case letters, with a 'tilde'-sign on top, e.g.: $\tilde{I}$.

The presented proposal will consider *ill-known time domain intervals*.

Definition 9. Ill-known Time Domain Interval

An ill-known subset of a time domain of a time model is called an ill-known time domain interval.

4 TIME DOMAIN INTERVAL VISUALIZATION

The Triangular Model (TM) comprises a set of rules indicating how to visualize intervals as points in an image plane (Kulpa, 2006), (Van De Weghe et al., 2007). In this section, an adaptation of this technique, which is used in the presented work, is presented.

Consider a time domain $\mathbb{E}$. The determination of the point visualizing a time domain interval $I \subseteq \mathbb{E}$ using the TM is illustrated in figure 2. The lower half of this figure contains a traditional linear visualization of $I = [t_s, t_e] \subseteq \mathbb{E}$. The upper half of the figure contains a visualization of $I \subseteq \mathbb{E}$ using the TM. In order to accomplish this visualization, first, an interval in $\mathbb{E}$ should be chosen so that its starting element lies before the starting element of I and its ending element lies after the ending element of I. This chosen interval is visualized as a horizontal straight line segment in the image plane (Qiang and et al., 2010), (Van De Weghe et al., 2007), accommodated with vertical ticks, which visualize the elements of $\mathbb{E}$. This interval is called the *reference interval*. In figure 2, it is given the denotation '$\mathbb{E}$'. To visualize $I = [t_s, t_e] \subseteq \mathbb{E}$ in an image plane equipped with a visualization of this reference interval, first the locations of both t_s and t_e on the line segment representing the reference interval are determined. Next, a straight half-line L_s is drawn on the image plane, its initial point being the aforementioned location point of t_s and another straight half-line L_e is drawn on the image plane, its initial point being the aforementioned location point of t_e. These two half-lines are drawn in such a way that they intersect in a point p and that the size α of the angle formed by L_s and the line segment bounded by the location points of t_s and t_e on the line segment representing the reference interval is exactly the same as the size of the angle formed by L_e and the same line segment (Qiang and et al., 2010), (Van De Weghe et al., 2007). This point p is called the *interval point* (Qiang and et al., 2010), (Van De Weghe et al., 2007) and the size α is traditionally chosen to be $45°$.

5 VISUALIZING ILL-KNOWN TIME DOMAIN INTERVALS

In this section, a construction method is described, which can be used to visualize ill-known time domain intervals as collections of points in an image plane. This method is illustrated in figure 3.

Consider a totally ordered time domain $\mathbb{E}$, exactly as constructed in section 2.2, and its powerset $\wp(\mathbb{E})$. Consider the subset $\wp_I(\mathbb{E})$ of $\wp(\mathbb{E})$ and let $\wp_I(\mathbb{E})$ contain every element of $\wp(\mathbb{E})$ that is an interval, but no other elements. Now consider an arbitrary ill-known interval $\tilde{I} \subseteq \mathbb{E}$, defined by possibilistic variable $X_{\tilde{I}}$ on $\wp_I(\mathbb{E})$, which is defined by possibility distribution $\pi_{X_{\tilde{I}}}$ on $\wp_I(\mathbb{E})$.

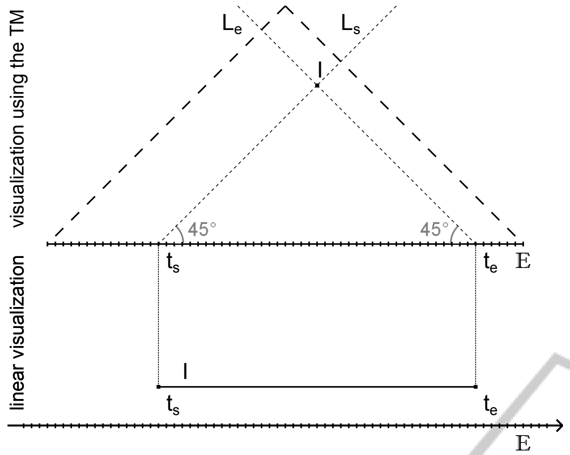

Figure 2: The visualization of an interval in time domain $\mathbb{E}$ using the TM and the construction leading to it.

In order to accomplish the visualization of $\tilde{I}$, first, a reference interval is chosen so that its starting element lies before every element desired to be visualized and its ending element lies after every element desired to be visualized. Now, to visualize $\tilde{I}$ in an image plane equipped with such a visualization of this reference interval, the following steps need to be taken:

1. Consider the subset $\wp_I(\mathbb{E})$ of $\wp(\mathbb{E})$.
2. A set $I_{\tilde{I}}$ should be constructed, where $I_{\tilde{I}}$ contains all regular intervals $K \subseteq \mathbb{E}$ for which $\pi_{X_{\tilde{I}}}(K) > 0$ and only these intervals.
3. For every interval K in $I_{\tilde{I}}$ fully visualizable in the figure, the interval point of K is drawn in the image plane following the TM visualization technique explained in section 4. However, the gray scale color intensity of the interval point of an interval K in $I_{\tilde{I}}$ now visualizes the possibility $\pi_{X_{\tilde{I}}}(K)$ of K of being the interval intended by $\tilde{I}$.

The visualization of $\tilde{I}$ is now the collection of the visualizations of all the intervals in $I_{\tilde{I}}$. Visualizations of different intervals may use different colors.

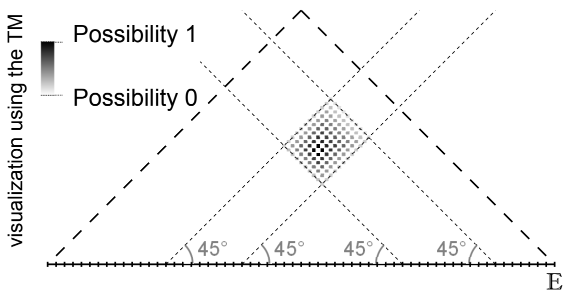

Figure 3: The visualization of an ill-known interval in time domain $\mathbb{E}$ using the TM and the construction leading to it.

Table 1: Allen Relationships corresponding to a DURZ.

Name	Allen Relationships
B	{ before }
BM	{ before , meets }
BMS	{ before , meets , starts }
BMO	{ before , meets , overlaps }
BMOS	{ before , meets , overlaps , starts }
BMOSD	{ before , meets , overlaps , starts , during }
BMSD	{ before , meets , starts , during }
MO	{ meets , overlaps }
MOS	{ meets , overlaps , starts }
MOSD	{ meets , overlaps , starts , during }
MSD	{ meets , starts , during }
O	{ overlaps }
OS	{ overlaps , starts }
OSD	{ overlaps , starts , during }
SD	{ starts , during }
D	{ during }
OF-B	{ overlaps , finished-by }
DF	{ during , finishes }
DFM-B	{ during , finishes , met-by }
OF-BC	{ overlaps , finished-by , contains }
E	{ equals }
DFO-B	{ during , finishes , overlapped-by }
DFO-BM-B	{ during , finishes , overlapped-by , met-by }
DFO-BM-BA	{ during , finishes , overlapped-by , met-by , after }
DFM-BA	{ during , finishes , met-by , after }
F-BC	{ finished-by , contains }
FO-B	{ finishes , overlapped-by }
FO-BM-B	{ finishes , overlapped-by , met-by }
FO-BM-BA	{ finishes , overlapped-by , met-by , after }
FM-BA	{ finishes , met-by , after }
C	{ contains }
CS-B	{ contains , started-by }
CS-BO-B	{ contains , started-by }, overlapped-by }
S-BO-B	{ started-by }, overlapped-by }
O-B	{ overlapped-by }
O-BM-B	{ overlapped-by , met-by }
O-BM-BA	{ overlapped-by , met-by , after }
M-BA	{ met-by , after }
A	{ after }

6 DISCRETE UNCERTAIN RELATIONAL ZONES

In this section, the concept of *Discrete Uncertain Relational Zones* (DURZ) will be presented.

Consider a totally ordered time domain $\mathbb{E}$, exactly as constructed in section 2.2. Now consider an arbitrary ill-known interval $\tilde{I} \subseteq \mathbb{E}$ and an image containing a visualization of $\tilde{I}$ using the construction method presented in section 5. It is now possible to discern 39 different collections of points in the image plane, dependent on the visualization of $\tilde{I}$. Each collection corresponds to a single set of Allen relationships. These collections of points are called $\tilde{I}$'s *Discrete Uncertain Relational Zones* (DURZ). They are related to the 'Uncertain Relational Zones' introduced in (De Tré and et al., 2012), but massively expand upon them. Their visualizations are shown in figures **??** and **??** in the appendix. In these figures, each collection is given a unique acronym name. DURZ will be referred to in this paper using these acronyms. Table 1 shows, for each DURZ, the unique set of Allen relationships which corresponds to the DURZ.

Given a totally ordered time domain $\mathbb{E}$, exactly as constructed in section 2.2, an arbitrary ill-known interval $\tilde{I} \subseteq \mathbb{E}$ and a visualization of $\tilde{I}$ using the construction method presented in section 5, $\tilde{I}$'s DURZ in the image containing $\tilde{I}$'s visualization can be constructed as follows.

Consider the visualizations of the earliest and latest starting and ending points of regular intervals in $\mathbb{E}$ with non-zero possibilities of being the interval intended by $\tilde{I}$. For each of these visualizations, draw two straight half-lines starting in the visualization point and having angles with size α with the visualization of the reference interval, but with different orientations. Every collection of points now created by an intersection of these lines, line segments between intersections of these lines and area's bounded by these line segments and intersections is now a DURZ, including the visualization of $\tilde{I}$ itself.

The interpretation of such DURZ is the following. Every interval point in a given DURZ visualizes a regular interval in $\mathbb{E}$ which has a non-zero possibility of being in one of the Allen relationships corresponding to the DURZ, with the interval intended by $\tilde{I}$.

7 TEMPORAL RELATIONSHIPS BETWEEN AN ILL-KNOWN AND A REGULAR TIME DOMAIN INTERVAL

In this section, a technique is presented, which allows to determine, for each existing Allen relationship, the possibility with which an arbitrary regular time domain interval is in this Allen relationship with a given ill-known interval in the same time domain.

Consider a totally ordered time domain $\mathbb{E}$, exactly as constructed in section 2.2. Now consider an arbitrary regular interval $J = [t_s, t_e] \subseteq \mathbb{E}$ and a given ill-known interval $\tilde{I}$ in $\mathbb{E}$. Now consider the visualization of $\tilde{I}$ using the technique described in section 5, its DURZ and the visualization of J using the TM model, all in the same image. Now, let the interval point of J be part of DURZ Z, which corresponds to the set $\{R_i | (0 \leq i \leq n) \wedge (i \in \mathbb{N})\}$ of Allen relationships, where every $R_i, 0 \leq i \leq n \wedge i \in \mathbb{N}$ is an Allen relationship. For every $R_i, 0 \leq i \leq n \wedge i \in \mathbb{N}$, the possibility $\pi\left(JR_i\tilde{I}\right)$ with which J is in Allen relationship R_i with $\tilde{I}$ is now found visually after the following construction.

1. The two straight half-lines L_s and L_e used to construct J's interval point are drawn. L_s's initial point is t_s and L_e's initial point is t_e.
2. Two more straight half-lines L'_s and L'_e are drawn. L'_s has as initial point t_s and is orthogonal to L_s. L'_e has as initial point t_e and is orthogonal to L_e.

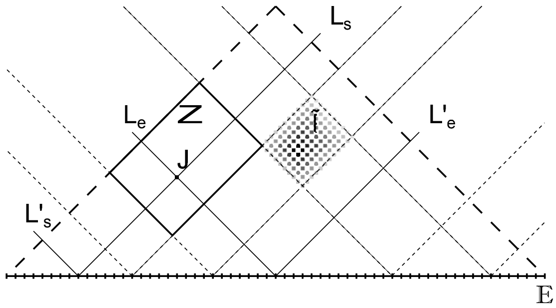

Figure 4: The evaluation of the temporal relationships between a regular time domain interval J and an ill-known time domain interval $\tilde{I}$, where J is part of DURZ 'O'.

3. If none of the half-lines L_s, L'_s, L_e or L'_e contain any point in the visualization of $\tilde{I}$, then Z corresponds to a singleton of Allen Relationships $\{R_0\}$. In this case, $\pi\left(JR_0\tilde{I}\right) = 1$, because J is in Allen relationship R_0 with $\tilde{I}$, regardless of which regular interval $\tilde{I}$ is intended to be. An example of this situation is shown in figure 4. If one or more of the half-lines L_s, L'_s, L_e or L'_e contain any point in the visualization of $\tilde{I}$, these will divide the collection of points which is the visualization of $\tilde{I}$ in as many sub-collections $SC_i, (0 \leq i \leq n) \wedge (i \in \mathbb{N})$ as there are Allen relationships in the set corresponding to Z. For this, any set of points of $\tilde{I}$'s visualization all contained by the same half-line L_s, L'_s, L_e or L'_e is also counted as a sub-collection. In fact, every sub-collection $SC_i, 0 \leq i \leq n \wedge i \in \mathbb{N}$ corresponds to a single Allen relationship R_i in this set: the sub-collection SC_i contains every point $sc_{i,j}, 0 \leq j \leq m_i \wedge j \in \mathbb{N}$, where m_i is the amount of points in SC_i, for which J is in this Allen relationship R_i with the regular interval having $sc_{i,j}$ as interval point.
4. For every $R_i, 0 \leq i \leq n \wedge i \in \mathbb{N}$, it is now easy to determine $\pi(JR_i\tilde{I})$:

$$\pi\left(JR_i\tilde{I}\right) = \sup_{sc_{i,j} \in SC_i}\left(\pi_{\tilde{I}}(sc_{i,j})\right)$$

Here, $\pi_{\tilde{I}}(sc_{i,j})$ is the possibility that $sc_{i,j}$ is the interval point of the regular interval intended by $\tilde{I}$. An example of this situation is shown in figure 5.

8 TEMPORAL RELATIONSHIPS BETWEEN TWO ILL-KNOWN TIME DOMAIN INTERVALS

In this section, a technique is presented, which allows to determine, for each existing Allen relationship, the possibility with which an arbitrary ill-known time domain interval is in this Allen relationship with a given ill-known interval in the same time domain.

Consider a totally ordered time domain $\mathbb{E}$, exactly as constructed in section 2.2. Now consider an arbitrary ill-known interval $\tilde{J}$ in $\mathbb{E}$ and a given ill-known interval $\tilde{I}$ in $\mathbb{E}$. Now consider an image containing the visualizations of $\tilde{I}$ and $\tilde{J}$ and the DURZ of $\tilde{I}$. Two possibilities may be discerned:

- The visualization of $\tilde{J}$ is completely contained in a single DURZ corresponding to a singleton of Allen relationships $\{R_0\}$. In this case, independent of which regular interval $\tilde{J}$ is intended to be and independent of which regular interval $\tilde{I}$ is intended to be, the possibility $\pi\left(\tilde{J}R_0\tilde{I}\right)$ that $\tilde{J}$ is in Allen relationship R_0 with $\tilde{I}$ is 1. This is because every regular interval with a non-zero possibility of being the regular interval intended by $\tilde{J}$ is in Allen relationship R_0 with every regular interval with a non-zero possibility of being the regular interval intended by $\tilde{I}$. An example of this situation is shown in figure 6.
- The visualization of $\tilde{J}$ either is completely contained in a single DURZ corresponding to a set $\{R_{i,0}|(0 \leq i \leq n) \wedge (i \in \mathbb{N})\}$ of Allen relationships, where every $R_{i,0}, 0 \leq i \leq n \wedge i \in \mathbb{N}$ is an Allen relationship, or the visualization of $\tilde{J}$ is partially contained in different DURZ Z_j corresponding to sets $\{R_{i,j}|(0 \leq i \leq n) \wedge (0 \leq j \leq m) \wedge (i \in \mathbb{N}) \wedge (j \in \mathbb{N})\}$ of Allen relationships, where every $R_{i,j}, 0 \leq i \leq n \wedge 0 \leq j \leq m \wedge 0 \leq j \leq n \wedge i \in \mathbb{N} \wedge j \in \mathbb{N}$ is an Allen relationship. In these cases, the possibility $\pi\left(\tilde{J}R_{i,j}\tilde{I}\right)$ that $\tilde{J}$ is in Allen relationship $R_{i,j}$ with $\tilde{I}$ is given by:

  $$\pi\left(\tilde{J}R_{i,j}\tilde{I}\right) = \sup_{\forall J \in \tilde{J}} \min\left(\pi_{\tilde{J}}(J), \pi\left(JR_{i,j}\tilde{I}\right)\right)$$

  Here, all J are arbitrary interval points in $\tilde{J}$ and $\pi_{\tilde{J}}(J)$ is the possibility that the regular interval intended by $\tilde{J}$ is J. The formula above illustrates that for every regular interval J considered, both the possibility that J is intended by $\tilde{J}$ and the possibility that J is in the Allen relationship with $\tilde{I}$ should be accounted for. This conjunction is modeled by using the minimum-operator. Examples of these situations are shown in figures 7 and 8.

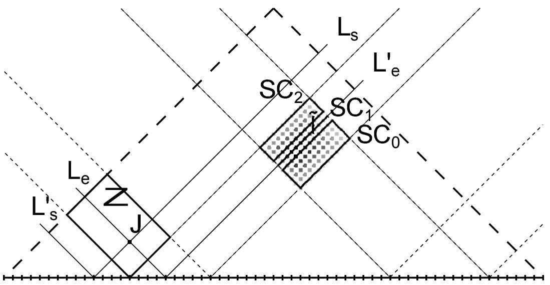

Figure 5: The evaluation of the temporal relationships between a regular time domain interval J and an ill-known time domain interval $\tilde{I}$, where J is part of DURZ 'BMO'.

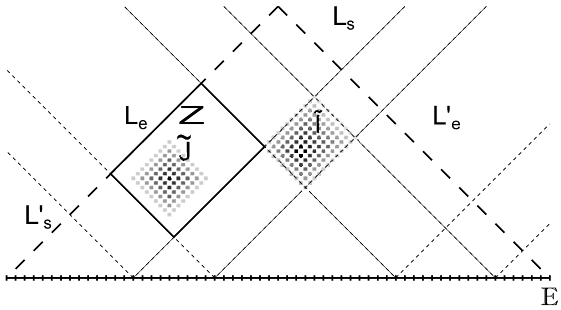

Figure 6: The evaluation of the temporal relationships between ill-known time domain intervals $\tilde{J}$ and $\tilde{I}$, where $\tilde{J}$ is part of DURZ 'O'.

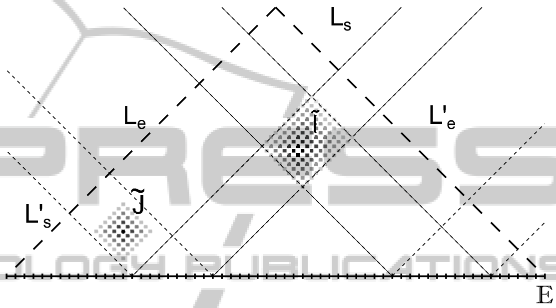

Figure 7: The evaluation of the temporal relationships between ill-known time domain intervals $\tilde{J}$ and $\tilde{I}$, where $\tilde{J}$ is completely part of DURZ 'BMO'.

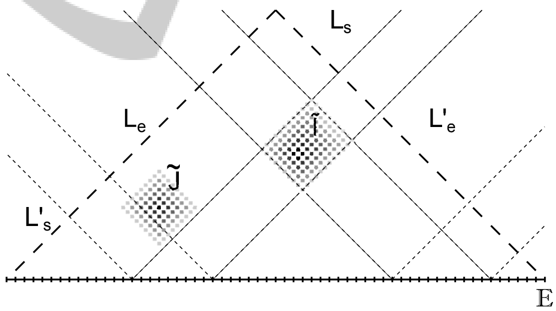

Figure 8: The evaluation of the temporal relationships between ill-known time domain intervals $\tilde{J}$ and $\tilde{I}$, where $\tilde{J}$ is partially part of DURZ 'BMO', 'MO' and 'O'.

9 CONCLUSIONS AND FUTURE WORK

In this paper, a novel method is presented to visualize and temporally reason with ill-known time domain intervals. This method is specifically designed for time domain intervals and accounts for graded confidence in the context of uncertainty. Based on the belief that the source of uncertainty in time usually is a (partial) lack of knowledge, possibility theory is used to model confidence. Future work is expected to focus on advanced querying of such temporal data, using the introduced novel methods for temporal reasoning, and eventually on data mining in the context of such temporal data.

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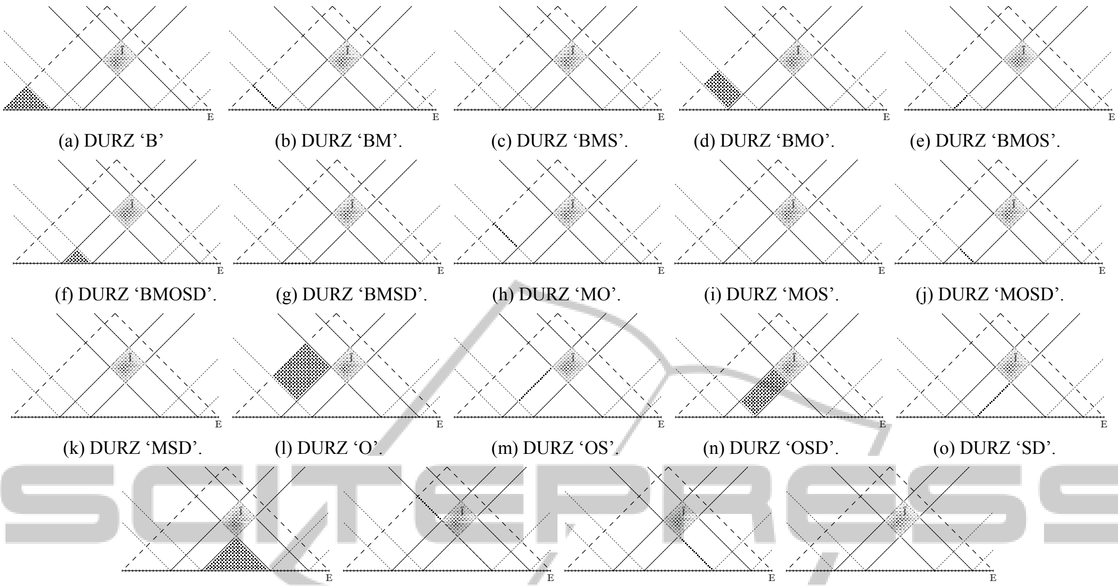
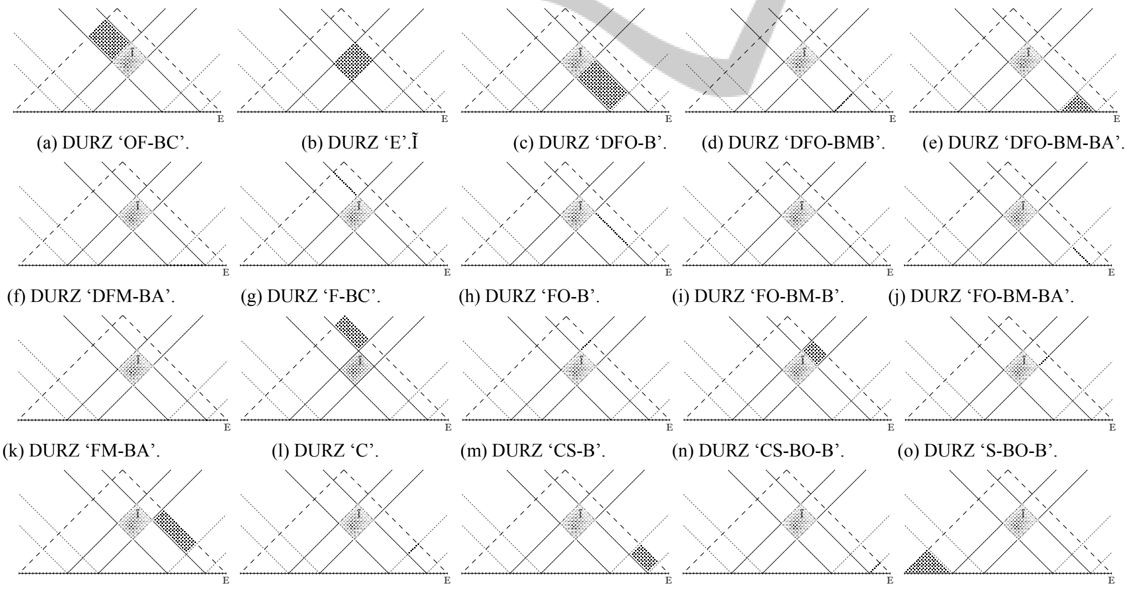
APPENDIX



(a) DURZ ‘B’ (b) DURZ ‘BM’. (c) DURZ ‘BMS’. (d) DURZ ‘BMO’. (e) DURZ ‘BMOS’.
(f) DURZ ‘BMOSD’. (g) DURZ ‘BMSD’. (h) DURZ ‘MO’. (i) DURZ ‘MOS’. (j) DURZ ‘MOSD’.
(k) DURZ ‘MSD’. (l) DURZ ‘O’. (m) DURZ ‘OS’. (n) DURZ ‘OSD’. (o) DURZ ‘SD’.
(p) DURZ ‘D’. (q) DURZ ‘OF-B’. (r) DURZ ‘DF’. (s) DURZ ‘DFM-B’.

Figure 9: The First 19 Collections of Points in the Image Plane Corresponding to the DURZ in a Visualization of the Ill-Known Interval $\tilde{I}$.



(a) DURZ ‘OF-BC’. (b) DURZ ‘E’.Ĩ (c) DURZ ‘DFO-B’. (d) DURZ ‘DFO-BMB’. (e) DURZ ‘DFO-BM-BA’.
(f) DURZ ‘DFM-BA’. (g) DURZ ‘F-BC’. (h) DURZ ‘FO-B’. (i) DURZ ‘FO-BM-B’. (j) DURZ ‘FO-BM-BA’.
(k) DURZ ‘FM-BA’. (l) DURZ ‘C’. (m) DURZ ‘CS-B’. (n) DURZ ‘CS-BO-B’. (o) DURZ ‘S-BO-B’.
(p) DURZ ‘O-B’. (q) DURZ ‘O-BM-B’. (r) DURZ ‘O-BM-BA’. (s) DURZ ‘M-BA’. (t) DURZ ‘A’.

Figure 10: The following 20 collections of points in the image plane corresponding to the DURZ in a visualization of the ill-known interval $\tilde{I}$.