A Catapult. Searching Optima Using Factorial Designs and 2D-Neural Network Mapping Technique
A Tutorial

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Abstract: The goal of this paper is to represent the feed forward bottle neck neural network (FFBN NN) mapping technique in comparison with traditional statistical method like Factorial Design (FD). Application of both methods provides more information about studied process and enable to establish certificate limits more affectively reaching to best quality and selecting the less cost processes. The represented FFBN NN mapping technique is simple in use, not time consuming and gives 2D visualization of multiple optima in studied technological processes. A catapult design was applied to illustrate the cases and purposes where proposed method can be implemented. The FFBN NN mapping technique can be recommended for use in industries including application in Six Sigma improvement phase.

1 INTRODUCTION

The objective of experimental designs and optimization methods is to create the highest quality product, improve quality and reduce the cost of product as well. Many manufacturing and service industries are interested to accomplish this goal.

Statistical design of experiments (DOE) in details is described in the book by Douglas, C., Montgomery, 2012. DOE has very broad application in natural, social science and engineering. For example, DOE can include the surrogate models (based on polynomial response surfaces, Kriging, support vector machines (SVM) and artificial neural network) that mimic the behaviour of simulation model as close as possible. For the references see Jin, Y., 2011 and Loshchilov, et al., 2010.

We have considered design related to the optimization problem. Different algorithms can be used to solve the optimization problem. The type of relationship between input parameters and output response (linear or non-linear) determines the choice of applied technique. Few examples of different approaches for optimization of different processes are given in papers by Pishvaee, M., et al., 2011, Hamdy, M., et al., 2011, Wu, A., et al., 2011, Wang, J., et al., 2010. Optimization of processes using NN with combination of others methods (for example, genetic algorithm) are described in papers by Ozcelik, B., et al., 2006, Zheng, J., et al., 2009, Park, Y.W., et al., 2008, Changyu, S., et al., 2007, Cook, D.F., et al., 2000, and Sette, S., et al., 1997.

This tutorial is written for representing the neural network (NN) method using the feed forward neural network mapping technique for determination of optimal limits for studied process. The article makes comparison between in NN method and traditional technique like factorial design (FD) which is well known for wide range of engineers as well as scientists dealing with statistical research.

The application of feed forward bottle neck neural network (FFBN NN) mapping technique for optimization is relatively new method which is easy in use and not time-consuming. This method enables visualization of process in 2D map in the form of contour plot of response overlapped with setting parameters points. The FFBN NN mapping technique enables finding multiple optimal solutions in an existing or a new technological process. Visualization of process in 2D map enables to set the specification limits more effectively.

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2 METHODS

2.1 Experimental Designs

Statistical Design of Experiment (DOE) is the process of planning experiments so that appropriate data will be collected and then analysed by statistical methods, resulting in valid and objective conclusions. The purpose of DOE is to determine how response (Y) depends on one or more input variables or predictors (x_i) so that future values of response can be predicted from the input variables. DOE is able to account the interactions between variables. DOE includes building a mathematical model for a response as a function of the input parameters. Two level factorial designs for catapult were applied in the study. An introduction of scientific experimentation is represented by Eriksson, 2008 and, Douglas C. Montgomery, 2012. Detailed discussion of design and analysis of industrial experiment can be found in the books written by Davies, 1956 and Natrella, 1963.

2.2 Factorial Designs

Factorial design is a method to determine the effects of multiple variables on a response. There are advantages by combining the study of multiple variables in the same factorial experiment.

In the study full factorial designs in two levels (high/low or `+1' and `-1', respectively) which contains all possible high/low combinations of all the input factors were applied. Two, three and four factor experiments were discussed.

2.3 Neural Network Method

Multidimensional data sets are difficult to interpret and visualize. The feed forward bottle neck neural network (FFBN NN) was used for compression and the visualization of the data in 2D map.

The FFBN NN is a type of auto associative neural network described by Kramer, 1991, Daszykowski, 2003, and Livingstone, 1991. Auto associative neural networks are feed forward nets trained to produce an approximation of the identity mapping between network inputs and outputs using back propagation or similar learning procedures. These types of ANNs can deal with linear and nonlinear correlation among variables.

The architecture of FFBN neural network applied in our work is shown at the left side of Figure 1.

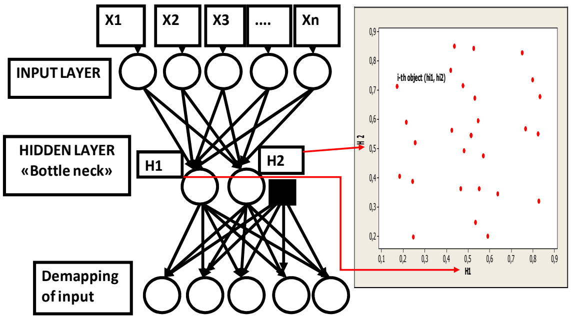

Figure 1: The architecture of FFBN neural network.

Thus, a special architecture of error back-propagation neural network was used (n, 2, n), in which the data are fed into the n-nodes input layer and then transferred through the 2-nodes hidden layer (compared to a bottleneck) to the n-nodes output layer. The n depends on the number of factors used in experiments (X1-Xn). The input data are expressed as n vectors. Each vector represents input parameter from X1 till Xn with m varied data-points in n-dimensional representation space. The number "m" corresponds here to the number of runs (setting points) in DOE.

The driving force of the training in the bottleneck auto association process is to reproduce the input signals in the output nodes, i.e. to obtain in the output nodes the values most similar to the input variables of the samples, after passing the bottle neck of the two-node hidden layer. The signals in the two hidden nodes are then taken as two coordinates for each input object acting as a 2D projection of samples into a map. In other words, the two neurons in the hidden layer produce, for each input object *Xi*, a corresponding pair of coordinates (H={h_1, h_2}). The projection of m objects into h_1/h_2 plot is shown in the right side of Figure 1. Thus, the multidimensional data were transformed into two dimensional map.

For each of m experimental settings the corresponding value of response Y (Y1, Y2, Y3..., Y_k) can be determined in the course of experiment. The projection of Y values into H_1/H_2 coordinate represents the contour plots of Y. Overlapping the projection of m experimental objects (obtained from the FFBN neural network 2D map) with responses contour plots enable to visualize and to detect optimal settings corresponding to the Y optimal values or determine the specification limits.

3 RESULTS AND DISCUSSIONS

3.1 Design and Analysis of Catapult

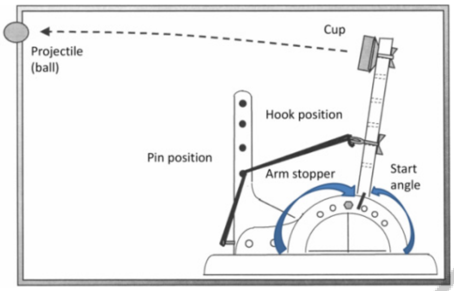

Figure 2: A Catapult.

In ancient world the romans invented advanced machinery, such as ballista (catapult), which could launch stones as heavy as 220 pounds and was used by legionnaires. Nowadays hundreds of companies and universities use the Catapult for applying statistical methods to real problems and designed experiments study. In the present article we demonstrated traditional statistical methods (full, factorial designs) using the MINITAB software as well as neural network mapping technique. We illustrated the capabilities of statistical and neural network methods and showed which kind of problem can be solved by using one or another method. It should be highlighted that the same plan of experiment was applied in both methods.

Figure 2 represents a catapult with indication of different regulated settings related to studied parameters.

In the present article we demonstrated two, three and four factor DOE using traditional statistical methods (with calculations in the MINITAB software) as well as the neural network mapping technique to illustrate prediction ability from simplest to more complex models.

The output Y is a fire distance (in cm). It can be expressed as a function of input X: Y=f(X1, X2, X3,..., Xn. The fire distance can be predicted using this equation.

The following factors (input variables) were considered in the paper (in designs with different number of factors):

X1-hook position (B, D);

X2- start angle, cm (1, 3);

X3- pin position (band guide) (2A, 4A);

X4-cup type (E, G).

The catapult supports both continuous and categorical factors. Start angle is continuous (1-3cm) while others are categorical.

Qualitative factors (categorical variables) assume certain distinct level. We considered such cases where no center level is definable.

The simplest experiments using factors at two levels (low and high) were illustrated. The coded and uncoded values of independent input variables (factors X1-X4) for 4, 3 and 2 factor experiments are represented in Table 1.

Table 1: Coded and uncoded values of independent input variables (factors) for 4, 3 and 2 factor experiments.

Factors	4 factor		3 factor		2 factor	
	DOE Coded levels					
	-1	+1	-1	+1	-1	+1
X1-Hook position	B	D	B	D	B	D
X2- Start angle, cm	1	3	1	3	1	3
X3- Pin position	2A	4A	2A	4A		
X4-Cup type	E	G				

Two level full factorial designs with 2 replicates were performed.

For two factor DOE we have 8 runs, for three factor DOE- 16 and for four factor DOE- 32.

Design matrixes for two, three and four factor DOE are represented in tables 2, 3 and 4, correspondingly.

Table 2: Design matrix (experimental plan) for 2 factor DOE (2 levels 2 replicate experiment).

Run order	X1-Hook position	X2-Start angle, cm	MAX /MIN
1, 5	-1	-1	
2, 6	+1	-1	MAX
3, 7	-1	+1	MIN
4, 8	+1	+1	

Table 3: Design matrix (experimental plan) for 3 factor DOE (2 levels 2 replicate experiment).

Run order	X1-Hook position	X2-Start angle, cm	X3-Pin position	MAX /MIN
1, 9	-1	-1	-1	
2, 10	+1	-1	-1	
3, 11	-1	+1	-1	MIN
4, 12	+1	+1	-1	
5, 13	-1	-1	+1	
6, 14	+1	-1	+1	MAX
7, 15	-1	+1	+1	
8, 16	+1	+1	+1	

Table 4: Design matrix (experimental plan) for 4 factor DOE (2 levels 2 replicate experiment).

Run Order	X1	X2	X3	X4	MIN/MAX
1, 17	-1	-1	-1	-1	
2, 18	+1	-1	-1	-1	
3, 19	-1	+1	-1	-1	
4, 20	+1	+1	-1	-1	
5, 21	-1	-1	+1	-1	
6, 22	+1	-1	+1	-1	
7, 23	-1	+1	+1	-1	
8, 24	+1	+1	+1	-1	MIN
9, 25	-1	-1	-1	+1	MAX
10, 26	+1	-1	-1	+1	
11, 27	-1	+1	-1	+1	
12, 28	+1	+1	-1	+1	
13, 29	-1	-1	+1	+1	
14, 30	+1	-1	+1	+1	
15, 31	-1	+1	+1	+1	
16, 32	+1	+1	+1	+1	

Two level factorial designs were analysed by analysis of variance (ANOVA) and by regression analysis. The model equations describing the relationship between response Y and factors X1, X2, X3 and X4 are represented in Table 5.

Table 5: Model equations for 2, 3 and 4 factor DOE.

DOE type	Models equations
2 factor	Y= 160.3 + 30,62X1 - 24,75X2 -3X1*X2
3 factor	Y= 212,50 + 28,62X1 - 21,75X2 + 62,25X3 - 19,38X1*X2 - 10,37X1*X3 + 4,75X2*X3 - 13,62X1*X2*X3
4 factor	Y= 105,87 - 15,46X1 - 19,28X2 - 27,69X3 + 14,80X4

The goal of many designed experiments is to determine the optimal factors settings that produce the best value for a response of interest. To specify the goal three options were chosen: Maximize, Minimize and target distance 160 cm. These three options were chosen because in industry and science different goals can be met in solving optimization problem (MAX, MIN or definite value).

The projection of factors settings (experimental condition) and response (firing distance values) in H1/H2 coordinates for 2, 3, 4 factor experiments are illustrated in Figures 3, 4 and 5, correspondingly. The experimental running points (corresponding to the setting parameters) are complemented with levels (+1;-1) for factors X1 and X2 in the case of 2 factor experiment as well as for factors X1, X2, X3 in the case of 3 factor experiment. In the case of 4 factors experiment the levels of X1-X4 can be found in Table 4.

The location of MAX, MIN and Target=160 cm are shown in red colour in Figures 3, 4 and 5.

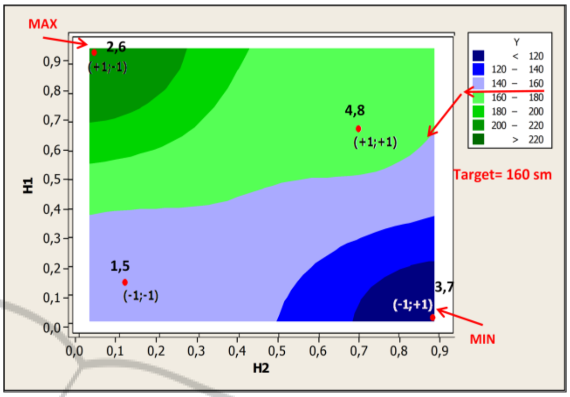

Figure 3: The projection of factors settings (experimental condition) and response (firing distance values) in H1/H2 coordinates for 2 factors experiment.

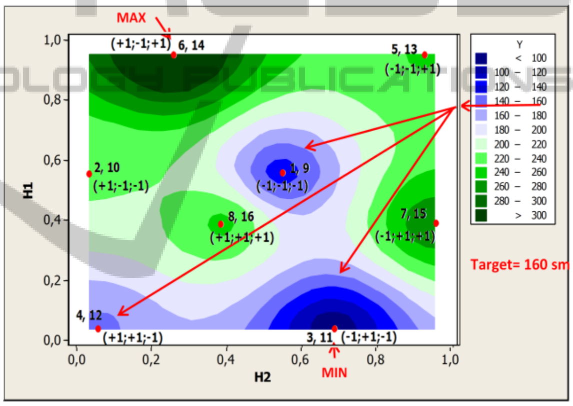

Figure 4: The projection of factors settings (experimental condition) and response (firing distance values) in H1/H2 coordinates for 3 factors experiment.

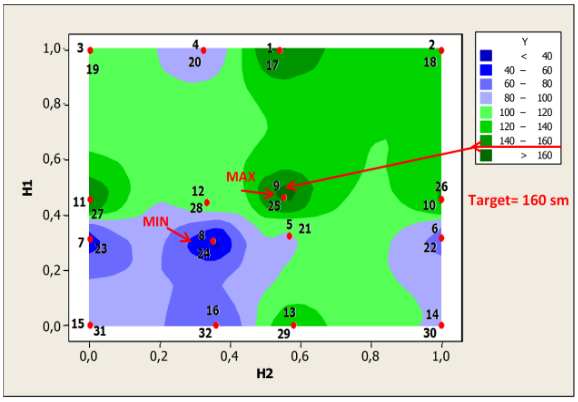

Figure 5: The projection of factors settings (experimental condition) and response (firing distance values) in H1/H2 coordinates for 4 factors experiment.

Table 6: Minitab response optimizer and NN mapping summary results for 2, 3 and 4 factor full factorial DOEs.

Type of DOE	Goal, cm	Global solution (FD, Minitab)	NN map results (levels of factors in opt. point)	Run order (see Tables 2,3,4)	Predicted Y, cm	Desir-ability
2 factor DOE	Max 221	X1=D; X2=1 (+1;-1)	(+1;-1)	(2, 6)	Y=218,5	D=0,978
	Min 106	X1=B; X2=3 (-1;+1)	(-1;+1)	(3,7)	Y=107,8	D=0,985
	Target =160	X1=B; X2=1 (-1;-1)	NA	NA	Y=151,3	D=0,838
3 factor DOE	Max 344	X1=D; X2=1; X3=4A (+1;-1;+1)	(+1;-1;+1)	(6, 14)	Y=343	D=0,996
	Min 90	X1=B; X2=3; X3=2A (-1;+1;-1)	(-1;+1;-1)	(3, 11)	Y=90,5	D=0,998
	Target =160	X1=D; X2=2,93≈3; X3=2A (+1;+1;-1)	NA	NA	Y=160	D≈1,0
4 factor DOE	Max 170,5	X1=B; X2=2A; X3=1; X4=G (-1;-1;-1;+1)	(-1;-1;-1;+1)	(9, 25)	Y=170,5	D=1,00
	Min 32,2	X1=D; X2=4A; X3=3; X4=E (+1;+1; +1;-1)	(+1;+1; +1;-1)	(8, 24)	Y=32,25	D=0,99
	Target =160	X1=B; X2=2A; X3=1,5; X4=G ≈ (-1;-1;-1+1)	(-1;-1;-1+1)	Close to (9, 25)	Y=160	D=1,00

Table 6 represents the summary optimization results including calculations using Minitab response optimizer for FD as well as NN mapping visualisation results.

In the case of MAX and MIN values we have got concordance of results obtained using both methods. Thus, MAX for global solution as well as for NN mapping (see Figure 3) in the case of 2 factor DOE corresponds to setting X1,X2 equal to (+1;-1) and MIN corresponds to the opposite setting (-1;+1). In the case of 3 factor DOE we have got MAX at setting X1,X2,X3 (+1;-1;+1) and MIN at setting (-1;+1;-1). 4 factor DOE indicates the MAX at setting (-1;-1;-1;+1) and MIN at setting (+1;+1; +1;-1).

The target equal to 160 cm for 2 factor DOE was found at the setting X1,X2 (-1;-1) as a global solution (Minitab calculations) with desirability 0,838. The NN map in Figure 3 shows the line between blue and green light colours. Additional calculation should be done for finding the desired levels for this target which is not in scope of present article. In the Table 6 it was marked as not available (NA). For 3 factor DOE Minitab response optimizer offered the setting X1,X2,X3 (+1;+1;-1) with desirability 1,0. The NN map in Figure 4 shows the round lines between blue zone colours marked with red arrows. Additional calculation should be also done for finding the desired levels here (not available (NA) in Table 6). In the case of 4 factor DOE we have got target close to maximal value in both methods.

4 CONCLUSIONS

In the paper we demonstrated how different methods (particularly, factorial designs and neural network mapping) provide information about optima or target. No additional experiments are required to perform both methods. (The same data were used).

The model equations obtained using FD were replaced by an equivalent NN. The transformation of multidimensional data into two dimensional maps enables the full mapping of the objective function and identification of multiple optima easily. This is an important feature not presented by conventional optimization methods like FD or others statistical methods.

NN mapping technique enables the visualisation of studied process (response) in 2D map. In some cases the target can be represented as a region (area). Engineers can use such areas for determination of specification limits.

The FFBN NN mapping technique is simple in use, non- time consuming and can be recommended for wide use in different industries.

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