

Solving Open-Pit Long-Term Production Planning Problems with Constraint Programming

A Performance Evaluation

Ricardo Soto^{1,2}, Broderick Crawford^{1,3}, Boris Almonacid¹, Franklin Johnson^{1,4} and Eduardo Olguín⁵

¹*Pontificia Universidad Católica de Valparaíso, Av. Brasil 2950, Valparaíso, Chile*

²*Universidad Autónoma de Chile, Av. Pedro de Valdivia 641, Santiago, Chile*

³*Universidad Finis Terrae, Av. Pedro de Valdivia 1509, Santiago, Chile*

⁴*Universidad de Playa Ancha, Av. González de Hontaneda 855, Valparaíso, Chile*

⁵*Universidad San Sebastián, Bellavista 7 Recoleta, Santiago, Chile*

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Abstract: Open pit mining problems aims at correctly identifying the set of blocks to be mined in order to maximize the net present value of the extracted ore. Different constraints can be involved and may vary the difficulty of the problem. In particular, the Open-Pit Long-Term Production Planning Problem is one of the variants that better models the real mining operation. It considers, among others, limited processing plant and mining capacity as well as slope and grade blending constraints. During the last thirty years, different techniques have been proposed to solve the multiple variants of the open pit mining problem; however, the resolution via constraint programming has not been reported yet. In this paper, we present a performance evaluation of seven constraint programming solvers for the open pit mining long-term scheduling problem. We illustrate interesting and comparative results on a set of varied open pit mining instances.

1 INTRODUCTION

Open pit mining refers to a method of mineral extraction in which the ore body is reached by opening a large ground surface along a mine. The orebody is commonly discretized to be regarded as a three-dimensional array of blocks, where each block has different attributes, e.g., tonnage, extraction cost, estimated ore content, and expected in-ground value. A main aim of mine planning is to correctly select the blocks to be mined in order to maximize the total profit from the process. Different constraints can be involved and may vary the difficulty of the problem such as, a limited processing plant capacity, the need for a balanced mining flow during a given time horizon, the satisfaction of a given metal demand, or simply to handle the extraction of several predecessors blocks to reach a valuable one. The study of open pit mining problems dates back to the 1960s, and different variants have been reported. The simplest one is the ultimate pit problem (UPIT) [Ahuja et al., 1993] also known as maximum-weight closure problem. This problem aims at finding the set of profitable blocks within the ore body that

maximizes the net present value (NPV). The only constraint involved is about precedence among blocks for extraction, also known as slope constraints. The constrained pit limit problem (CPIT) (Chicoisne et al., 1993) can be seen as the immediate extension of the UPIT, which introduces the time dimension to the problem and the corresponding constraints. The idea is to maximize the NPV in a given time horizon by considering the precedence constraints among blocks, upper and lower bounds for operational resources for each period, and constraints to ensure that blocks are extracted only once during the time horizon. The precedence constrained production scheduling problem (PCPSP) (Espinoza et al., 2012) adds to the CPIT constraints about the destination of blocks. If blocks contain ore they are processed otherwise they are sent to the waste dump. The open-pit mine production scheduling problem with metal uncertainty (MPSP) (Lamghari et al., 2012) introduces mining and processing constraints to the CPIT. The idea is to balance the mining flow through the periods by avoiding exceeding the metal production that can be sold. Processing constraints

ensures a minimum amount of mineral processing but without exceeding the processing plant capacity. Analogously, mining constraints establish lower and upper bounds of mineral tons to be mined. The open pit mining long-term scheduling problem (Caccetta et al., 2003) is another variant and perhaps is the one that better models the real mining operation. It introduces processing, mining, and grade blending constraints to the CPIT. Grade blending constraints ensure that the average grade of the material sent to the mill respect given lower and upper bounds. During the last thirty years, different solving techniques have been proposed to tackle the multiple versions of this problem, mostly belonging from the mathematical programming field and a few from the approximate methods domain. Some examples are the classic linear and mixed-integer linear programming (Caccetta et al., 2003, Chicoisne et al., 2012, Ramazan et al., 2007, Boland et al., 2009), also chance constrained integer programming (Gholamnejad et al., 2006, Gholamnejad et al., 2008), cutting planes (Bley et al., 2010), goal programming (Chanda et al., 1995), stochastic optimization (Marcotte et al., 2013), and genetic algorithms (Denby et al., 1994, Zhang, 2006) among others. However, no report exists about the use of constraint programming (CP) for solving open pit mining problems. In this paper, we present a performance evaluation of seven constraint programming solvers for the open pit mining long-term scheduling problem. We illustrate interesting and comparative results in order to provide a performance overview of constraint programming tackling open pit mining problems.

The remainder of this paper is structured as follows. A CP overview is given in Section 2. The open pit mining long-term scheduling problem is modeled in Section 3. The experiments are illustrated in Section 4, followed by the conclusions and future work.

2 CP BACKGROUND

Constraint programming is a complete search technique devoted to the efficient solving of constraint-based problems. It has its roots on three well-known computer science domains: operational research, artificial intelligence, and programming languages. During the last couple of decades, CP has successfully been employed to solve different real-life problems, e.g., set covering problems (Crawford et al., 2013), sudoku puzzles (Soto et al., 2013), manufacturing cell designs (Soto et al., 2013), nurse

rostering (Pizarro et al., 2011), and water distribution problems (Soto et al., 2012), just to number a few.

Under CP, problems are modeled as Constraint Satisfaction Problems (CSP), which mainly consists of a sequence of variables holding a domain of possible values and a set of constraints over those variables. Formally, a CSP P is defined by a triplet $P = \langle V, D, C \rangle$ where $V = \{v_1, v_2, \dots, v_n\}$ is the set of variables. $D = \{d_{v_i} | v_i \in V\}$, is the set of domains and $d_{v_i} = \{a_{i_1}, a_{i_2}, \dots, a_{i_j}\}$ represents the set of values that variable v_i can take. $C = \{C_R | R \subseteq V, R \neq \emptyset\}$ is the set of constraints, where C_R is a constraint over variables in R . A solution to a CSP is an assignment $\{v_1 \rightarrow a_1, \dots, v_n \rightarrow a_n | a_i \in d_{v_i}, i \in 1..n\}$ that satisfies the whole set of constraints. An optimization problem is simple an extension of a CSP that can be seen as a 4-tuple $P = \langle V, D, C, O \rangle$, where O corresponds to the objective function.

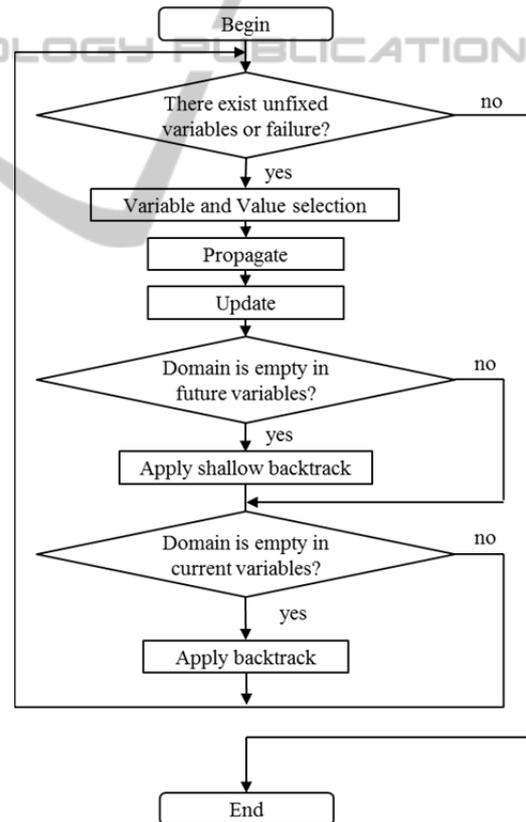


Figure 1: A general algorithm for solving optimization problems under the CP framework.

The most used approach to solve CSP and optimization problems under CP is to combine a backtracking procedure with filtering techniques in

the form of constraint propagation. Constraint propagation attempts to delete from domains the values that do not lead to any solution in order to accelerate the exploration. The constraint propagation is performed by validating a consistency property on the constraints of the problem; the most used one is the arc-consistency (Soto et al., 2014).

Figure 1 illustrates a general procedure for solving optimization problems under the CP framework. The idea is to generate partial solutions to be verified backtracking when inconsistencies are detected until a result is encountered. The first step is to select the variable and its corresponding value to generate a potential solution to be verified. Then, the propagation attempts to delete the unfeasible values. The update instruction is responsible for storing the best optimum value reached at this time. Finally two conditions perform backtracks. The classic backtrack comes back to the most recently tested variable that has still chance to reach a solution. A shallow backtrack jumps to the next value available from the domain of the current variable.

3 PROBLEM FORMULATION

In this section we formulate the Open-Pit Long-Term Production Planning Problem. We proceed by firstly stating the notation followed by the mathematical model.

3.1 Page Setup

- Indices and sets
 - t : time period index $t \in \{1, 2, \dots, T\}$.
 - T : set of periods t .
 - b : time period index $b \in \{1, 2, \dots, B\}$.
 - B : set of blocks b .
 - ex : index of a block considered for extraction.
- Parameters
 - C_b^t : net present value obtained from mining block b in period t .
 - \tilde{g}_b : block grade, which is defined as a random constant.
 - TO_b : The total amount of ore in block b .
 - TW_b : The total amount of waste in block b .
 - MC_{max}^t : The maximum material, including waste and ore, to be mined in period t .

- MC_{min}^t : The minimum material, including waste and ore, to be mined in period t .
- PC_{max}^t : The maximum amount of ore to be mined in period t .
- PC_{min}^t : The minimum amount of ore to be mined in time t .
- G_{max}^t : The maximum average grade of material to be processed in time t .
- G_{min}^t : The minimum average grade of material to be processed in time t .
- d : Discount rate in each period.
- P^t : Selling price of metal unit in time t .
- SP^t : Selling cost of metal unit in time t .
- R : Total metal recovery.
- P_c^t : Unit processing cost of ore in time t .
- M_{co}^t : Mining cost of metal unit in time t .
- M_{cw}^t : Mining cost of waste material in time t .
- e : Total number of blocks overlaying a block.
- Variables
 - x_b^t : a binary decision variable which is set to 1 if the block is mined, 0 otherwise.

3.2 Mathematical Model

The Open pit mining long-term scheduling problem aims at correctly selecting the blocks to be mined in order to maximize the total profit from the process in a given period of time. The corresponding objective function of the problem is depicted below, where C_b^t is computed by Eq. 2.

$$\text{maximize } Z = \sum_{t=1}^T \sum_{b=1}^B C_b^t x_b^t \quad (1)$$

$$C_b^t = \frac{1}{(1+d)^t} \{ [(P^t - SP^t)\tilde{g}_b R - P_c^t - M_{co}^t] TO_b - (M_{cw}^t TW_b) \} \quad (2)$$

The objective function is subjected to several constraints. For instance, the average grade of the material sent to the mill must respect given upper (G_{max}^t) and lower bounds (G_{min}^t). This constraint is known as grade blending constraint.

$$G_{min}^t \leq \frac{\sum_{b=1}^B \tilde{g}_b TO_b x_b^t}{\sum_{b=1}^B TO_b x_b^t} \leq G_{max}^t \quad (3)$$

for $t \in \{1, 2, \dots, T\}$

The total tons of material to be exploited are restricted by processing and mining capacities. The amount of ore to be processed in each period must respect the given upper (PC_{max}^t) and lower bounds (PC_{min}^t) as stated in Eq. 4. Likewise, the total

material mined, involving ore and waste, is bounded by MC_{min}^t and MC_{max}^t as stated in Eq. 5.

$$PC_{min}^t \leq \sum_{b=1}^B To_b x_b^t \leq PC_{max}^t \quad (4)$$

for $t \in \{1, 2, \dots, T\}$

$$MC_{min}^t \leq \sum_{i=1}^B (To_b + Tw_b) x_b^t \leq MC_{max}^t \quad (5)$$

for $t \in \{1, 2, \dots, T\}$

Eq. 6 ensures that all predecessor blocks of a block b must be completely mined in order to have access and be able to mine the block b . This is commonly represented as a cone model as illustrated in Figure 2. Finally Eq. 7 guarantees that any block is mined only once.

$$e x_{ex}^t - \sum_{l=1}^e \sum_{r=1}^t x_l^r \leq 0 \quad (6)$$

for $t \in \{1, 2, \dots, T\}, \quad b \in \{1, 2, \dots, B\}$

$$\sum_{t=1}^T x_b^t = 1 \quad n \in (1, 2, \dots, B) \quad (7)$$

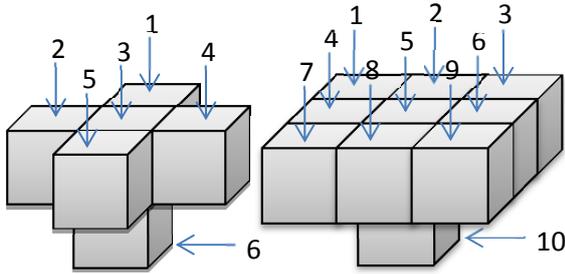


Figure 2: Cone model.

4 EXPERIMENTS

We have performed a set of experiments by using 36 instances of different size in order to compare the performance of the seven CP solvers. The experiments have been performed on an Intel Core i5 with 6 Gb RAM running Windows 7. The description of solvers and instances is detailed in tables 1, 2, and 3, respectively. For each instance, we provide number of periods (T), number of blocks (B), number of precedence, $MC_{min}^t, MC_{max}^t, PC_{min}^t, PC_{max}^t, G_{min}^t$, and G_{max}^t . For space reasons, C_b^t, To_b , and Tw_b for each block b are not included,

but provided in data sets available at <http://inf.ucv.cl/~rsoto/OPM.html>.

The results in terms of solving time are depicted in table 4 and 5. Bold font is used for the best solving time in each instance, and 10:00:00.000 means that no solution was reached after 10 hours of running time. The summary of results is depicted in table 6, which considers as indicators: average solving time for the complete set of instances (Avg.), the difference w.r.t the best average solving time (Δ), the standard deviation (σ), the number of times the solver achieved the best time for a given instance (1st place), the second best time (2nd place), and the third best time (3rd place).

The results illustrate that Gecode, MiniZinc, and Mzn-g12cpx exhibit the best performance by far. Gecode achieves the best average solving time close to the performance of MiniZinc, and Mzn-g12cpx. Likewise, Mzn-g12cpx obtained 20 first places, Gecode obtained 13 first places, and MiniZinc 2 first places. The second places are also taken by these solvers, MiniZinc taking 21, Gecode 14, and Mzn-g12cpx only 1. Finally, MiniZinc takes 13 third places, Mzn-g12cpx takes 8, and Gecode takes 9. The better performance exhibited by those solvers with respect to its competitors can be explained by different reasons. Gecode is a fast solver specially tuned via efficient propagators for solving this kind of problems¹. MiniZinc is rather a modeling language than a solver, but its default solver is also very efficient sharing several solving, search, and filtering features with Gecode. Mzn-g12cpx is a recent solver based on the lazy clause LazyFD solver. A lazy clause solver is a hybrid combining CP and SAT. The idea is to mimic a domain propagation engine by mapping propagators into SAT clauses. As a result, we obtain reduced search by nogood creation, and effective autonomous search. This leads normally to a faster solving process.

On the contrary, Mzn-g12fd is a finite domain solver mostly oriented to satisfaction problems and perhaps not specially tuned for optimization problems.

Mzn-g12fdlp, which is the linear programming version of Mzn-g12fd, slightly improves the results, but the solving times reached remain quite far from the best ones. Finally, Choco is a well-known solver including state-of-the-art CP solving technology but also rather devoted to constraint satisfaction than optimization.

¹ See results of different competitions at <http://www.gecode.org/>

Table 1: Solver Description.

MiniZinc	It is a state-of-the-art high level CP modeling language that can be interfaced with several solvers via the FlatZinc low-level solver input language. For the experiments, we employ the default solver for MiniZinc.
Flatzinc	FlatZinc is the interface of MiniZinc to derive models to target solvers. For the experiments, we employ the default solver for FlatZinc.
Mzn-g12cpx	It is the successor of the LazyFD solver (lazy clause generation) involving Constraint Programming with eXplanations.
Mzn-g12fd	It is the finite domain solver of the G12 project, to be used with the MiniZinc modeling language.
Mzn-g12fdlp	It is the linear programming solver of the G12 project, to be used with the MiniZinc modeling language.
Choco	It is another state-of-the-art CP solver implemented on top of Java. It is built on a event-based propagation mechanism with backtrackable structures.
Gecode	It is a well-known CP solver, implemented as a C++ library. It can also be interfaced to several languages such as MiniZinc, Alice, Ruby, and Lisp.

Table 3: Lower and upper bounds for MC, PC per periods, for all instances.

	MC_{min}^t	MC_{max}^t	PC_{min}^t
t = 1	2000	0	200
t = 2	2000	0	20000
t = 3	200000	0	20000
t = 4	200000	0	200000
t = 5	200000	0	200000
t = 6	200000	0	20000
t = 7	200000	0	20000

Table 2: Instance Description.

Instance	T	B	Precedences
1	3	27	98
2	5	27	98
3	7	27	98
4	3	36	140
5	5	36	140
6	7	36	140
7	3	45	182
8	5	45	182
9	7	45	182
10	3	48	200
11	5	48	200
12	7	48	200
13	3	54	224
14	5	54	224
15	7	54	224
16	3	60	260
17	5	60	260
18	7	60	260
19	3	64	300
20	5	64	300
21	7	64	300
22	3	80	390
23	5	80	390
24	7	80	390
25	3	90	416
26	5	90	416
27	7	90	416
28	3	96	480
29	5	96	480
30	7	96	480
31	3	120	624
32	5	120	624
33	7	120	624
34	3	150	832
35	5	150	832
36	7	150	832

Table 4: Lower and upper bounds for PC , and G per periods, for all instances.

	PC_{max}^t	G_{min}^t	G_{max}^t
t = 1	0	0.5	0
t = 2	0	5	0
t = 3	0	5	0
t = 4	0	5	0
t = 5	0	5	0
t = 6	0	0.5	0
t = 7	0	0.5	

Table 5: Solving times of the seven tested solvers for instances 1 to 30 using the hh:mm:ss format. Part 1.

Instance	MiniZinc	Flatzinc	Mzn-g12cpx	Mzn-g12fd	Mzn-g12fdlp	Choco	Gecode
2	00:00:00.150	00:00:00.812	00:00:00.930	00:00:02.040	00:00:01.850	00:00:01.280	00:00:00.169
2	00:00:12.997	00:01:00.980	00:02:16.290	00:01:21.120	00:01:11.950	00:01:10.360	00:00:13.043
3	00:03:50.798	00:14:55.507	00:13:13.350	00:15:39.960	00:16:04.700	00:17:04.000	00:03:44.042
4	00:00:03.635	00:00:22.105	00:00:26.360	00:00:23.070	00:00:20.290	00:00:34.540	00:00:02.694
5	00:12:40.846	00:48:20.136	01:40:26.190	00:49:18.220	00:43:28.680	01:07:13.220	00:12:37.014
6	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000
7	00:00:00.560	00:00:03.681	00:00:01.770	00:00:03.990	00:00:03.610	00:00:04.401	00:00:00.548
8	00:02:29.646	00:10:36.109	00:05:20.740	00:10:32.120	00:10:17.210	00:12:03.390	00:02:13.807
9	01:58:58.975	07:40:50.599	01:29:02.750	07:38:07.250	07:30:34.710	08:45:14.445	01:59:26.708
10	00:00:00.220	00:00:00.015	00:00:00.500	00:00:02.030	00:00:01.770	00:00:02.710	00:00:00.271
11	00:01:08.555	00:05:22.735	00:00:11.530	00:05:17.390	00:04:54.750	00:05:22.110	00:01:12.042
12	00:16:27.507	00:53:22.070	00:00:46.120	00:56:47.420	00:56:30.670	01:07:10.120	00:22:02.862
13	00:00:00.411	00:00:02.824	00:00:00.630	00:00:03.400	00:00:03.410	00:00:03.004	00:00:00.390
14	00:00:56.978	00:04:19.585	00:00:10.010	00:04:29.830	00:03:44.840	00:04:54.403	00:01:01.748
15	00:16:59.469	01:16:28.910	00:00:48.780	01:24:09.280	01:10:18.450	01:15:40.358	00:15:58.783
16	00:00:04.884	00:00:25.412	00:00:14.000	00:00:23.690	00:00:23.530	00:00:23.710	00:00:04.415
17	01:01:42.276	04:15:41.200	04:53:15.860	04:03:15.000	03:43:18.280	10:00:00.000	01:00:33.329
18	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000
19	00:00:00.150	00:00:00.765	00:00:00.800	00:00:01.100	00:00:01.010	00:00:02.030	00:00:00.143
20	00:00:09.617	00:00:43.010	00:00:31.910	00:00:43.630	00:00:41.910	00:01:01.520	00:00:09.604
21	00:01:10.816	00:06:21.000	00:01:57.930	00:06:22.240	00:06:06.300	00:09:03.706	00:01:08.862
22	00:00:16.200	00:01:19.014	00:00:12.740	00:01:19.540	00:01:12.000	00:01:38.060	00:00:16.192
23	03:40:37.626	10:00:00.000	01:11:47.810	10:00:00.000	10:00:00.000	10:00:00.000	03:32:05.568
24	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000	10:00:00.000
25	00:00:00.812	00:00:04.601	00:00:00.771	00:00:04.640	00:00:04.910	00:00:05.010	00:00:00.770
26	00:02:44.479	00:10:14.520	00:00:18.270	00:10:17.310	00:09:40.130	00:10:36.211	00:02:37.258
27	00:31:02.560	02:09:37.945	00:00:47.930	02:06:37.440	02:05:32.460	02:31:24.025	00:30:35.024
28	00:00:00.906	00:00:05.133	00:00:00.750	00:00:05.440	00:00:05.070	00:00:05.457	00:00:00.894
29	00:01:34.900	00:05:27.212	00:00:10.480	00:05:33.400	00:05:03.880	00:05:33.007	00:01:33.349
30	00:30:43.134	02:15:08.105	00:01:01.720	02:48:42.800	02:44:18.000	02:58:12.070	00:31:12.581

5 CONCLUSIONS

In this paper, we have solved the open-pit long-term production planning problem by using constraint programming. This problem aims at maximizing the net present value of the extracted ore from the mining operation by considering limited processing plant and mining capacity as well as slope and grade blending constraints. We have solved this problem

by means of seven well-known CP solvers: MiniZinc, Mzn-g12cpx, Gecode, Flatzinc, Mzn-g12fd, Mzn-g12fdlp, and Choco. MiniZinc, Mzn-g12cpx, and Gecode obtained the best results, which can be explained by different reasons such as the incorporation of efficient propagators, and state-of-the-art search and filtering techniques.

As future work, we expect to study additional variants of this problem in order to solve them with constraint programming or related complete and

Table 6: Solving times of the seven tested solvers for instances 31 to 36 using the hh:mm:ss format. Part 2.

Instance	MiniZinc	Flatzinc	Mzn-g12cpx	Mzn-g12fd	Mzn-g12fdlp	Choco	Gecode
31	00:00:01.261	00:00:06.224	00:00:01.010	00:00:06.760	00:00:06.140	00:00:06.210	00:00:01.262
32	00:02:17.578	00:08:07.190	00:00:11.260	00:08:11.050	00:07:46.920	00:08:49.040	00:02:08.285
33	00:35:49.456	02:37:20.354	00:00:55.500	02:39:53.180	02:37:34.020	02:57:41.408	00:32:59.947
34	00:00:00.676	00:00:03.494	00:00:01.020	00:00:03.420	00:00:03.420	00:00:03.588	00:00:00.595
35	00:04:12.895	00:14:03.637	00:00:16.040	00:14:04.330	00:13:31.680	00:14:22.753	00:03:54.553
36	01:00:00.734	02:41:58.363	00:00:44.660	02:48:25.340	02:49:57.390	03:05:42.428	00:50:05.400
28	00:00:00.906	00:00:05.133	00:00:00.750	00:00:05.440	00:00:05.070	00:00:05.457	00:00:00.894
29	00:01:34.900	00:05:27.212	00:00:10.480	00:05:33.400	00:05:03.880	00:05:33.007	00:01:33.349
30	00:30:43.134	02:15:08.105	00:01:01.720	02:48:42.800	02:44:18.000	02:58:12.070	00:31:12.581
31	00:00:01.261	00:00:06.224	00:00:01.010	00:00:06.760	00:00:06.140	00:00:06.210	00:00:01.262
32	00:02:17.578	00:08:07.190	00:00:11.260	00:08:11.050	00:07:46.920	00:08:49.040	00:02:08.285
33	00:35:49.456	02:37:20.354	00:00:55.500	02:39:53.180	02:37:34.020	02:57:41.408	00:32:59.947
34	00:00:00.676	00:00:03.494	00:00:01.020	00:00:03.420	00:00:03.420	00:00:03.588	00:00:00.595
35	00:04:12.895	00:14:03.637	00:00:16.040	00:14:04.330	00:13:31.680	00:14:22.753	00:03:54.553
36	01:00:00.734	02:41:58.363	00:00:44.660	02:48:25.340	02:49:57.390	03:05:42.428	00:50:05.400

Table 7: Summary of performance.

	MiniZinc	Flatzinc	Mzn-g12cpx	Mzn-g12fd	Mzn-g12fdlp	Choco	Gecode
Avg.	00:19:02.570	01:05:31.917	00:17:44.134	01:06:40.831	01:04:56.483	01:22:28.139	00:17:08.711
Δ	00:01:53.859	00:48:23.206	00:00:35.423	00:49:32.120	00:47:47.772	01:05:19.428	00:00:00.000
σ	00:44:50.159	02:16:14.323	00:55:37.507	02:16:16.958	02:14:44.799	02:48:26.598	00:43:20.211
1 st place	2	1	20	0	0	0	13
2 nd place	21	0	1	0	0	0	14
3 rd place	13	3	8	0	3	0	9

incomplete search techniques. Another interesting further work would be the introduction of autonomous search in the solving process. As detailed in (Crawford et al., 2013, Monfroy et al., 2013, Soto et al., 2013), the incorporation of autonomous search in a CP search engine can clearly speed up the resolution, especially in the presence of harder instances.

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REFERENCES

Ahuja, R. K, Magnanti, T. L., Orlin, J. B. Network flows, 1993. Theory, Algorithms, and Applications. Prentice Hall, Englewood.

Boland, N., Dumitrescu, I., Froyland, G., Gleixner, A., 2009. LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. *Computers & Operations Research*, 36 (4), pp. 1064–1089.

Bley A., Boland, N., Fricke, C., Froyland, G., 2010. A strengthened formulation and cutting planes for the

- open pit mine production scheduling problem. *Computers & OR* 37 (9), pp. 1641-1647.
- Caccetta, L., Hill, S., 2003. An application of branch and cut to open pit mine scheduling. *Journal of Global Optimization*, 27 (2-3), pp. 349-365.
- Chanda, E.K.C., Dagdelen, K., 1995. Optimal blending of mine production using goal programming and interactive graphics systems, *International Journal of surface Mining, Reclamation and the Environment*, Vol. 9, pp. 203-208.
- Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno E., Rubio E., 2012. A New Algorithm for the Open-Pit Mine Production Scheduling Problem. *Operations Research* 60 (3), pp. 517-528.
- Crawford, B., Soto, R., Monfroy, E., Palma, W., Castro, C., Paredes, F., 2013. Parameter tuning of a choice-function based hyperheuristic using particle swarm optimization. *Expert Syst. Appl.* 40 (5), pp. 1690-1695.
- Crawford, B., Soto, R., Monfroy, E., Castro, C., Palma, W., Paredes F., 2013. A hybrid soft computing approach for subset problems *Mathematical Problems in Engineering*, Vol. 2013, Article ID 716069, 12 pages.
- Denby, B., Schofield, D., 1994. Open-pit design and scheduling by use of genetic algorithms. *Transactions of the Institution of Mining and Metallurgy, Section A: Mining Industry*, 103, A21-A26.
- Espinoza, D., Goycoolea, M., Moreno, E., Newman, A., 2012. MineLib: A Library of Open Pit Mining Problems. *Ann. Oper. Res.* 206 (1), pp. 91-114.
- Lamghari, A., Dimitrakopoulos, R., 2012. A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. *European Journal of Operational Research* 222 (3), pp. 642-652.
- Gholamnejad, J., Osanloo, M., Karimi, B., 2006. A Chance-Constrained Programming Approach for Open Pit Long-Term Production Scheduling in Stochastic Environments. *The Journal of the South African Institute of Mining and Metallurgy*, Vol. 106, pp.105-114.
- Gholamnejad, J., Osanloo, M., Khorram, E., 2008. A Chance Constrained Integer Programming Model for Open Pit Long-Term Production Planning. *International Journal of Engineering Transactions A: Basics* (21) 4, pp. 407-418.
- Marcotte, D., Caron, J., 2013. Ultimate open pit stochastic optimization. *Computers & Geosciences* Vol. 51, pp. 238-246.
- Monfroy, E., Castro, C., Crawford, B., Soto, R., Paredes, F., Figueroa, C., 2013. A reactive and hybrid constraint solver. *J. Exp. Theor. Artif. Intell.* 25 (1), pp. 1-22.
- Pizarro, P., Rivera, G., Soto, R., Crawford, B., Castro, C., Monfroy, E., 2011. Constraint-Based Nurse Rostering for the Valparaiso Clinic Center in Chile. In: Stephanidis, C. (ed.) *Posters, HCII 2011, Part II. CCIS*, vol. 174, pp.448-452, Springer.
- Ramazan, S., 2007. The new Fundamental Tree Algorithm for production scheduling of open pit mines. *European Journal of Operational Research* 177(2), pp.1153-1166.
- Soto, R., Crawford, B., Galleguillos, C., Monfroy, E., Paredes, F., 2013. A hybrid AC3-tabu search algorithm for solving Sudoku puzzles. *Expert Syst. Appl.* 40(15), pp. 5817-5821.
- Soto, R., Crawford, B., Misra, S., Palma, W., Monfroy, E., Castro, C., Paredes, F., 2013. Choice functions for autonomous search in constraint programming: *GA vs. PSO*. *Tehnicki Vjesnik*. Vol. 20(3), pp. 525-531.
- Soto, R., Crawford, B., Misra, S., Monfroy, E., Palma, W., Castro, C., Paredes, F., 2014. Constraint Programming for optimal design of architectures for water distribution tanks and reservoirs: a case study. *Tehnicki Vjesnik*. Vol. 21(1).
- Soto, R., Kjellerstrand, H., Duran, O., Crawford, B., Monfroy, E., Paredes, F., 2012. Cell formation in group technology using constraint programming and Boolean satisfiability. *Expert Syst. Appl.* 39(13), pp. 11423-11427.
- Zhang, M., 2006. Combining genetic algorithms and topological sort to optimize open-pit mine plans. In *proceedings of the 15th mine planning and equipment selection*, pp. 1234-1239.