Supplier Selection Using Fuzzy Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP)

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Abstract: The increasing competition forces companies to use the capital more effectively and using suppliers which operate cheaper and with higher quality. Due to that, it is crucial to select the right suppliers. Supplier selection is a decision making problem that involves quantitative and non-quantitative, conflicting criteria. In The Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP), all the decision data are known precisely or given as crisp values. But the uncertainty in the real life problems makes decision making more difficult. İn these situations, the complex situation varying with respect to decision makers can be solved by Fuzzy logic. Because of that, Fuzzy LINMAP has been used to solve the problem. The main aim of this study is to provide an analytical approach to decision makers for them to make objective decisions. Thus, supplier alternatives and selection criteria are determined. And a fuzzy LINMAP model is developed for supplier evaluation and selection of a company in automotive sector.

1 INTRODUCTION

The quality of goods and performance of organizations and supply chains are affected heavily by the Supplier Selection (SS), one of the most important activities of acquisition. In supply chain management, supplier selection problems have been extensively studied . Since the real-life supplier selection problems often involve multiple different types of attributes (orindices, factors) such as development capability, product quality, technological level and delivery time as well price, they may be ascribed to a kind of multi-attribute decision making (MADM) problems (Wan and Li, 2013). A strong relationship with the suppliers can be constructed by evaluating them through SS. Actually, the initial set of suppliers can be reduced to a final set by supplier selection decision process.

One of the most important steps in the selection process is the formulation of selection criteria. There are several descriptive studies that tried to define criteria used by companies to select suppliers. Dickson found in his study that quality, delivery and performance history are the most important criteria (Junior et al., 2013). It is difficult to make a compromise between quality and delivery related criteria with the purchasing functions that only consider cost minimization objective. But in these days, quality and delivery related criteria are gaining more importance in the purchasing decisions. Lot-sizing and total logistics course are affected a lot by suppliers' quality and delivery performance (Choudhary and Shankar, 2014).

While taking a decision about suppliers, both quantitative and qualitative criteria are important. Depending on the current situation, new suppliers or some of the current suppliers should be selected. In either case, decision process is uncertain, caused by subjective evaluation of criteria, multiple stakeholders and unavailability of previous data (Junior et al., 2014).

Supplier selection methods into two clusters of single model and combined models are illustrated in Fig. 1.

Besides, because the formulation of the decision making process isn't required in Artificial Intelligence based models, they are gaining more popularity. The complexity and uncertainty is better coped in these models. Only performance on the criteria is needed when the models are employed. Furthermore, AI models can do actual trade-off by using what they have learned from experts or cases (Guo and Shi, 2014)

Due to the uncertainty in the evaluation of qualitative criteria and weighing of different criteria by different stakeholders, supplier selection decision

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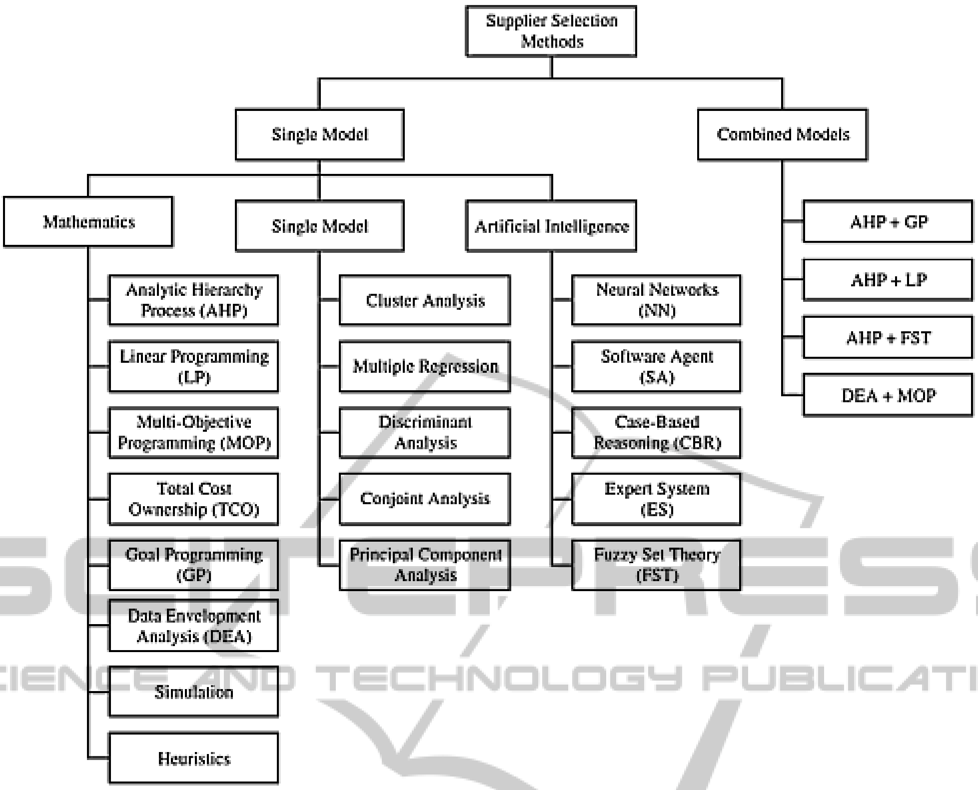

Figure 1: Existing analytical methods for supplier selection (Kannan et al., 2013).

also comprises of uncertainty. To handle the uncertainty in the supplier selection decision process, fuzzy set theory is one of the most important methods. By using this method, inexact criterion can be dealt and also qualitative and quantitative criterion can be integrated (Junior et al., 2013).

In multiple attribute decision-making problems, the decisionmaker's preference information is used to rank alternatives. Most multiple attribute decision making (MADM) problems include both quantitative and qualitative attributes that use imprecise data and human judgments (Bereketli et al., 2011) The purpose of using Fuzzy LINMAP in the paper is twofold: to deal with the uncertain and imprecise judgment of decision makers, and to express it by fuzzy numbers. Secondly, the linear programming technique for multidimensional analysis of preference (LINMAP) is one of the well-known methods for multiple attribute group decision making (MAGDM). To validate the effectiveness of the methodology, Bereketli et al. (2011), Chen (2013),Wan and Li (2013), Li and Wan (2013) found supportive and reasonable results using Fuzzy LINMAP.

In the LINMAP method, pairwise comparisons of alternatives given by the decision maker are evaluated and the best alternative that has the shortest distance to fuzzy positive ideal solution (FPIS) is selected. In this method, the whole decision data are known for certain or they are given as crisp values. But, crisp data is incorrect or insufficient to model real-life decision problems. Actually, because human judgments are unclear and fuzzy in nature, precise numerical values may not represent them accurately. Instead, to model human judgments, linguistic variables can be used (Xia et al., 2006).

In the second half of the study, detailed information about Fuzzy LINMAP is given. A numerical example is given to clarify the main results developed in Section 3. The fourth section of the study comprises of the application results.

2 FUZZY LINMAP MODEL

2.1 Concepts and Notations of Triangular Fuzzy Numbers

Triangular fuzzy numbers (TFNs) are a subset of fuzzy sets with properties that make them well suited for modelling and design-type activities. Specifically, a TFN has a triangular shape represented by the triple (l, m, n).

Fuzzy number is known as the triangular fuzzy number since its membership function has a triangular form as shown in Fig 2.

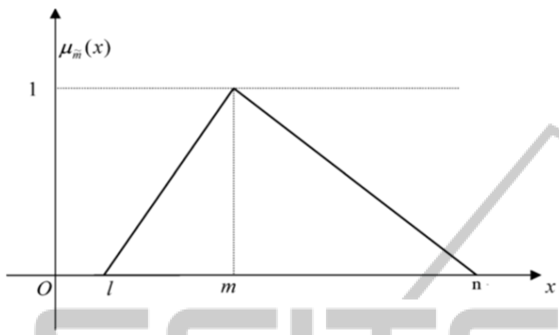

Figure 2: A triangular fuzzy number.

This particular fuzzy number is widely used in both research and practice. With the mode, left endpoint, and right endpoint denoted by m, a, and b respectively, the triangular fuzzy number is defined as

$$t(x;l;m;n)=\begin{cases}1-\dfrac{m-x}{m-l}, & l\le x\le m,\\ 1-\dfrac{x-m}{n-m}, & m<x\le n\\ 0 & , \ elsewhere.\end{cases}$$

The triangular fuzzy conversion scale, given in Table 1, is used in the evaluation model of this paper.

Table 1: Triangular fuzzy conversion scale.

Linguistic expression	Crisp number value	Fuzzy number value
Very good	5	(0.4, 0.5, 0.6)
Good	4	(0.3, 0.4, 0.5)
Medium	3	(0.2, 0.3, 0.4)
Poor	2	(0.1, 0.2, 0.3)
Very Poor	1	(0, 0.1, 0.2)

2.2 Distance between Two Triangular Fuzzy Numbers

Let $\tilde{m}=(m_1,m_2,m_3)$ and $\tilde{n}=(n_1,n_2,n_3)$ be two triangular fuzzy numbers. Then the distance between them using vertex method which is used in this paper can be calculated as

$$d(\tilde{m},\tilde{n})=\sqrt{\begin{matrix}\frac{1}{3}[(m_1-n_1)^2\\ +(m_1-n_1-m_2+n_2)^2\\ +(m_1-n_1+m_3-n_3)^2]\end{matrix}} \quad (1)$$

Two triangular fuzzy numbers $\tilde{m}$ and $\tilde{n}$ are identical if and only if $d(\tilde{m},\tilde{n})=0$.

If both $\tilde{m}$ and $\tilde{n}$ are real numbers, then the distance measurement $d(\tilde{m},\tilde{n})$ is identical to Euclidean distance.

2.3 The Normalization Method

In this paper we discuss a fuzzy multiattribute decision making problem which can be expressed as follows.

Suppose there exist n possible alternatives $x_1,x_2,\dots,x_n$ from which the decision maker has to choose based on m attributes $f_1,f_2,\dots,f_m$ which are qualitative (Li and Yang, 2004). Suppose that the rating of alternative x_j $(j=1,2,\dots,n)$ on attribute f_i $(i=1,2,\dots,m)$ given by the decision maker is a triangular fuzzy number $\tilde{f}_{ij}=(a_{ijl},a_{ijm},a_{ijn})$. Hence a fuzzy multiattribute decision making problem can be concisely expressed in matrix format as follows:

$$\tilde{F}=(\tilde{f}_{ij})_{m*n}=\begin{matrix} \\ f_1\\ f_2\\ \\ f_3\end{matrix}\begin{matrix}\begin{matrix}x_1 & x_2 & \dots & x_n\end{matrix}\\ \begin{pmatrix}\tilde{f}_{11} & \tilde{f}_{12} & \cdots & \tilde{f}_{1n}\\ \tilde{f}_{21} & \tilde{f}_{22} & \cdots & \tilde{f}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{f}_{m1} & \tilde{f}_{m2} & \cdots & \tilde{f}_{mn}\end{pmatrix}\end{matrix}$$

which is referred to as a fuzzy decision matrix.

Since the physical dimensions and measurements of the m attributes are different, so the fuzzy decision matrix $\tilde{F}$ needs to be normalized. In this paper, we choose the following normalization formula,

$$\tilde{r}_{ij}=\left[\frac{a_{ijl}}{a_{in}^{max}},\frac{a_{ijm}}{a_{im}^{max}},\frac{a_{ijn}}{a_{il}^{max}}\wedge 1\right] \quad (2)$$

where

$$a_{il}^{max}=\max\{a_{ijl}\mid \tilde{f}_{ij}=(a_{ijl},a_{ijm},a_{ijn}),\ j=1,2,\dots,n\} \quad (3)$$

$$a_{im}^{max}=\max\{a_{ijm}\mid \tilde{f}_{ij}=(a_{ijl},a_{ijm},a_{ijn}),\ j=1,2,\dots,n\} \quad (4)$$

and

$$a_{in}^{max} = \max\{a_{ijn} \mid \tilde{f}_{ij} = (a_{ijl}, a_{ijm}, a_{ijn}), \quad j = 1, 2, \ldots, n\} \tag{5}$$

Each $\tilde{r}_{ij} \epsilon [0,1]$ obtained from above equations is a normalized triangular fuzzy number where $\tilde{r}_{ij} = (r_{ijl}, r_{ijm}, r_{ijn})$ for any $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$. Using above equations, fuzzy decision matrix ($\tilde{F}$) can be transformed into the following normalized fuzzy decision matrix.

$$\tilde{R} = (\tilde{r}_{ij})_{m*n} = \begin{matrix} & x_1 & x_2 & \ldots & x_n \\ f_1 & \tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n} \\ f_2 & \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2n} \\ & \vdots & \vdots & \ddots & \vdots \\ f_3 & \tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mn} \end{matrix}$$

2.4 Consistency and Inconsistency Measurements

Let $\tilde{R}_j = (\tilde{r}_{1j}, \tilde{r}_{2j}, \ldots, \tilde{r}_{mj})$ be normalized triangular fuzzy number vector for n alternatives x_j ($j = 1,2,\ldots,n$) where $\tilde{R}_j$ are alternatives

Let the fuzzy ideal solution be $\tilde{a}^* = (\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_m^*)$ which is unknown a priori and must be determined where $\tilde{a}_i^* = (\tilde{a}_{il}^*, \tilde{a}_{im}^*, \tilde{a}_{in}^*)$ ($i = 1,2,,,m$) is a positive triangular fuzzy number for attribute f_i.

The square of the weighted Euclidean distance between the alternative $\tilde{R}_j = (\tilde{r}_{1j}, \tilde{r}_{2j}, \ldots, \tilde{r}_{mj})^T$ and the FPIS $\tilde{a}^* = (\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_m^*)^T$ can be calculated as:

$$S_j = \sum_{i=1}^{m} w_i \left[d(\tilde{r}_{ij}, \tilde{a}_i^*)\right]^2 \tag{6}$$

Using Eq. (1), we get

$$S_j = \sum_{i=1}^{m} \frac{w_i}{3} [(r_{ijl} - a_{il}^*)^2 + (r_{ijl} - a_{il}^* - r_{ijm} + a_{im}^*)^2 + (r_{ijl} - a_{il}^* + r_{ijn} - a_{in}^*)^2] \tag{7}$$

In the same manner the square of the weighted Euclidean distance between the alternative x_k or $\tilde{R}_k = (\tilde{r}_{1k}, \tilde{r}_{2k}, \ldots, \tilde{r}_{mk})^T$ and the FPIS can be calculated as

$$S_k = \sum_{i=1}^{m} \frac{w_i}{3} [(r_{ikl} - a_{il}^*)^2 + (r_{ikl} - a_{il}^* - r_{ikm} + a_{im}^*)^2 + (r_{ikl} - a_{il}^* + r_{ikn} - a_{in}^*)^2] \tag{8}$$

Assume that the decision maker gives his preference relations between alternatives by $\delta = \{(k,j) \mid x_k > x_j \ (k,j = 1,2,\ldots,n)\}$ from his knowledge and experience where the symbol $>$ is a preference relation given by the decision maker. In this set, $x_k > x_j$ means that decision maker prefers the alternative x_k to x_j. If the fuzy positive ideal solution $\tilde{a}^* = (\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_m^*)^T$ and weight vector $w = (w_1, w_2, \ldots, w_n)^T$ are already chosen by the decision maker, using Eq. (7) the decision maker can calculate the square of the weighted Euclidean distance between each pair of alternatives $(k,j)\epsilon\delta$ and the fuzy positive ideal solution $\tilde{a}^* = (\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_m^*)^T$ as follows:

$$S_j = \sum_{i=1}^{m} w_i \left[d(\tilde{r}_{ij}, \tilde{a}_i^*)\right]^2$$

and

$$S_k = \sum_{i=1}^{m} w_i \left[d(\tilde{r}_{ik}, \tilde{a}_i^*)\right]^2$$

If $S_j > S_k$, then the alternative x_k is closer to the FPIS than the alternative x_j for each of alternatives $(k,j) \in \delta$. So the ranking order of alternatives x_k and x_j determined by S_j and S_k based on $(w, \tilde{a}^*)$ is consistent with the preferances given by the decision maker. Conversely, if $S_j < S_k$, then the $(w, \tilde{a}^*)$ is not chosen properly since it result in that the ranking order of alternatives x_k and x_j determined by S_j and S_k based on $(w, \tilde{a}^*)$ is inconsistent with the preferences given by the decision maker. Therefore, $(w, \tilde{a}^*)$ should be chosen so that the ranking order of alternatives x_k and x_j determined by S_j and S_k based on $(w, \tilde{a}^*)$ is consistent with the preferences given by the decision maker.

An index $(S_j - S_k)^-$ to measure inconsistency between the ranking order of alternatives $x_k \ ve \ x_j$ determined by $S_j \ ve \ S_k$ and the preferences given by the decision maker preferring x_k to x_j can be defined as follows.

$$(S_j - S_k)^- = \begin{cases} S_k - S_j & (S_j < S_k), \\ 0 & (S_j > S_k), \end{cases} \tag{9}$$

Obviously, if $S_j > S_k$, the ranking order of alternatives x_k and x_j determined by S_j and S_k based on $(w, \tilde{a}^*)$ is consistent with the preferances given by the decision maker. Then the inconsistency index is defined to be 0. On the other hand, if $S_j < S_k$, the ranking order of alternatives x_k and x_j determined by S_j and S_k based on $(w, \tilde{a}^*)$ is inconsistent with the preferences given by the decision maker. Then the inconsistency index is defined to be $S_k - S_j$. From all of these, the inconsistency index can be rewrite as;

$$(S_j - S_k)^- = \max\{0, S_k - S_j\} \quad (10)$$

Using Eq.(10) we can defined total inconsistency index of decision maker as;

$$\begin{aligned} B &= \sum_{(k,j)\in\delta} (S_j - S_k)^- \\ &= \sum_{(k,j)\in\delta} \max\{0,\ S_k - S_j\} \end{aligned} \quad (11)$$

In the same manner an index $(S_j - S_k)^+$ can be defined to measure consistency between the ranking order of the alternatives x_k and x_j determined by S_j and S_k based on $(w, \tilde{a}^*)$ and the preferances given by the decision maker preferring x_k to x_j as follows;

$$(S_j - S_k)^+ = \begin{cases} S_j - S_k & (S_j > S_k), \\ 0 & (S_j < S_k), \end{cases} \quad (12)$$

The equation (10) which is written for inconsistency can be written for consistency as;

$$(S_j - S_k)^+ = \max\{0, S_j - S_k\} \quad (13)$$

Using Eq.(10) we can defined total inconsistency index of decision maker as;

$$\begin{aligned} G &= \sum_{(k,j)\in\delta} (S_j - S_k)^+ \\ &= \sum_{(k,j)\in\delta} \max\{0,\ S_j - S_k\} \end{aligned} \quad (14)$$

2.5 Fuzzy LINMAP Model Based on Consistency and Inconsistency Indices

For ranking alternatives, it is necessary to know $(w, \tilde{a}^*)$, and to determine $(w, \tilde{a}^*)$. Because of this, the following mathematical programming model is constructed as follows;

$$\begin{gathered} \text{Max } \{G\} \\ G - B \geq h \\ \sum_{i=1}^{m} w_i = 1 \\ w_i \geq \varepsilon \quad (i = 1, 2, \dots, m) \end{gathered} \quad (15)$$

where $h \geq 0$ is given by the decision maker a priori and $\varepsilon > 0$ is sufficiently small. $\varepsilon > 0$ is written in the model for ensuring that the weights generated are not zero as it may be the case in the LINMAP method (Srinivasan, 1973). The objective of the Eq. (14) is to maximize the total consistency index G of the decision maker under the condition in which the total consistency index G is greater than the total inconsistency index B by given value h > 0.

Combining Eqs. (11) - (14),

$$\begin{aligned} G - B &= \sum_{(k,j)\in\delta} (S_j - S_k)^+ - \sum_{(k,j)\in\delta} (S_j - S_k)^- \\ &= \sum_{(k,j)\in\delta} \left[(S_j - S_k)^+ - (S_j - S_k)^- \right] \\ &= \sum_{(k,j)\in\delta} (S_j - S_k) \end{aligned} \quad (16)$$

Using Eqs. (14) - (16) in the Eq. (15), the new model can be written as;

$$\begin{gathered} \max\left\{ \sum_{(k,j)\in\delta} \max\{0,\ S_j - S_k\} \right\} \\ \sum_{(k,j)\in\delta} (S_j - S_k) \geq h, \\ \sum_{i=1}^{m} w_i = 1 \\ w_i \geq \varepsilon \quad (i = 1, 2, \dots, m) \end{gathered} \quad (17)$$

Let $\lambda_{kj} = \max\{0, S_j - S_k\}$, for each pair of $(k, j) \in \delta$ then, for each $(k, j) \in \delta$

$$\lambda_{kj} \geq 0$$

and

$$\lambda_{kj} \geq S_j - S_k$$

Thus, the above equation (17) can be transformed into the following mathematical programming model

$$\begin{gathered} \max\left\{ \sum_{(k,j)\in\delta} \lambda_{kj} \right\} \\ \sum_{(k,j)\in\delta} (S_j - S_k) \geq h, \\ \sum_{i=1}^{m} w_i = 1 \\ w_i \geq \varepsilon \quad (i = 1, 2, \dots, m) \\ S_k - S_j + \lambda_{kj} \geq 0 \qquad (k, j) \in \delta \\ \lambda_{kj} \geq 0 \qquad (k, j) \in \delta \end{gathered} \quad (18)$$

The S_j, S_k and $S_j - S_k$ can be written explicitly using Eqs. (7) – (8) as;

$$S_j=\sum_{i=1}^{m}\frac{w_i}{3}\Big[\big(r_{ijl}-a_{il}^*\big)^2+\big(r_{ijl}-a_{il}^*-r_{ijm}+a_{im}^*\big)^2+\big(r_{ijl}-a_{il}^*-r_{ijn}+a_{in}^*\big)^2\Big]$$
$$=\sum_{i=1}^{m}\frac{w_i}{3}\big[r_{ijl}^2+a_{il}^*-2r_{ijl}a_{il}^*+r_{ijl}^2+a_{il}^{*2}+r_{ijm}^2+a_{im}^{*2}-2r_{ijl}a_{il}^{*2}-2r_{ijl}r_{ijm}+2r_{ijl}a_{im}^{*2}+2a_{il}^*r_{ijm}-2a_{il}^*a_{im}^*-2r_{ijm}a_{im}^*+r_{ijl}^2+a_{il}^{*2}+r_{ijn}^2+a_{in}^{*2}-2r_{ijl}a_{il}^*+2r_{ijl}r_{ijn}-2r_{ijl}a_{in}^*-2a_{il}^*r_{ijn}+2a_{il}^*a_{in}^*-2r_{ijn}a_{in}^*\big]$$

$$S_k=\sum_{i=1}^{m}\frac{w_i}{3}\big[(r_{ikl}-a_{il}^*)^2+(r_{ikl}-a_{il}^*-r_{ikm}+a_{im}^*)^2+(r_{ikl}-a_{il}^*-r_{ikn}+a_{in}^*)^2\big]$$
$$=\sum_{i=1}^{m}\frac{w_i}{3}\big[r_{ikl}^2+a_{il}^*-2r_{ikl}a_{il}^*+r_{ikl}^2+a_{il}^{*2}+r_{ikm}^2+a_{im}^{*2}-2r_{ikl}a_{il}^{*2}-2r_{ikl}r_{ikm}+2r_{ikl}a_{im}^{*2}+2a_{il}^*r_{ikm}-2a_{il}^*a_{im}^*-2r_{ikm}a_{im}^*+r_{ikl}^2+a_{il}^{*2}+r_{ikn}^2+a_{in}^{*2}-2r_{ikl}a_{il}^*+2r_{ikl}r_{ikn}-2r_{ikl}a_{in}^*-2a_{il}^*r_{ikn}+2a_{il}^*a_{in}^*-2r_{ikn}a_{in}^*\big]$$

$$S_j-S_k=\sum_{i=1}^{m}\frac{w_i}{3}\big[3\big(r_{ijl}^2-r_{ikl}^2\big)+\big(r_{ijm}^2-r_{ikm}^2\big)+\big(r_{ijn}^2-r_{ikn}^2\big)\big]-\frac{2}{3}v_{il}\big[3\big(r_{ijl}-r_{ikl}\big)-\big(r_{ijm}-r_{ikm}\big)+\big(r_{ijn}-r_{ikn}\big)\big]+\frac{2}{3}v_{im}\big[\big(r_{ijl}-r_{ikl}\big)-\big(r_{ijm}-r_{ikm}\big)\big]-\frac{2}{3}v_{ir}\big[\big(r_{ijl}-r_{ikl}\big)+\big(r_{ijn}-r_{ikn}\big)\big]-2r_{ijl}r_{ijm}+2r_{ijl}r_{ijn}+2r_{ikl}r_{ikm}-2r_{ikl}r_{ikn} \tag{19}$$

Combining Eqs.(18) and (19), we constructed the following linear programming model:

$$\max\left\{\sum_{(k,j)\in\delta}\lambda_{kj}\right\}$$

$$\sum_{i=1}^{m}w_i\sum_{(k,j)\in\delta}\big[3\big(r_{ijl}^2-r_{ikl}^2\big)+\big(r_{ijm}^2-r_{ikm}^2\big)+\big(r_{ijn}^2-r_{ikn}^2\big)\big]-2\left[\sum_{i=1}^{m}v_{il}\big[3\big(r_{ijl}-r_{ikl}\big)-\big(r_{ijm}-r_{ikm}\big)+\big(r_{ijn}-r_{ikn}\big)\big]\right]+2\left[\sum_{i=1}^{m}v_{im}\big[\big(r_{ijl}-r_{ikl}\big)-\big(r_{ijm}-r_{ikm}\big)\big]\right]-2\left[\sum_{i=1}^{m}v_{ir}\big[\big(r_{ijl}-r_{ikl}\big)+\big(r_{ijn}-r_{ikn}\big)\big]\right]-\sum_{(k,j)\in\delta}6r_{ijl}r_{ijm}+\sum_{(k,j)\in\delta}6r_{ijl}r_{ijn}+\sum_{(k,j)\in\delta}6r_{ikl}r_{ikm}-\sum_{(k,j)\in\delta}6r_{ikl}r_{ikn}\geq 3h$$

$$\sum_{i=1}^{m}w_i\big[3\big(r_{ikl}^2-r_{ijl}^2\big)+\big(r_{ikm}^2-r_{ijm}^2\big)+\big(r_{ikn}^2-r_{ijn}^2\big)\big]-2\left[\sum_{i=1}^{m}v_{il}\big[3\big(r_{ikl}-r_{ijl}\big)-\big(r_{ikm}-r_{ijm}\big)+\big(r_{ikn}-r_{ijn}\big)\big]\right]+2\left[\sum_{i=1}^{m}v_{im}\big[\big(r_{ikl}-r_{ijl}\big)-\big(r_{ikm}-r_{ijm}\big)\big]\right]-2\left[\sum_{i=1}^{m}v_{in}\big[\big(r_{ikl}-r_{ijl}\big)+\big(r_{ikn}-r_{ijn}\big)\big]\right]-6r_{ikl}r_{ikm}+6r_{ikl}r_{ikn}+6r_{ijl}r_{ijm}-6r_{ijl}r_{ijn}+3\lambda_{kj}\geq 0\quad for\ \forall(k,j)\in\delta$$

$$\sum_{i=1}^{m}w_i=1$$
$$w_i\geq\varepsilon\quad(i=1,2,\dots,m)$$
$$v_{il}\geq 0,\ v_{im}\geq 0,\ v_{in}\geq 0\quad(i=1,2,\dots,m)$$
$$\lambda_{kj}\geq 0\quad(k,j)\in\delta \tag{20}$$

where
$$v_{il}=w_i\,a_{il}^*,\quad v_{im}=w_i\,a_{im}^*,\quad v_{in}=w_i\,a_{in}^* \tag{21}$$

w_i, v_{il}, v_{im} and v_{in} can be obtained by solving the above linear programming (20) using simplex method. Then the best values of a_{il}^*, a_{im}^*, a_{in}^* are calculated using Eq. (21) and which are denoted as the triangular fuzzy numbers.

$$\tilde{a}_i^*=(\tilde{a}_{il}^*,\tilde{a}_{im}^*,\tilde{a}_{in}^*)\qquad(i=1,2,,,,m)$$

Therefore the ranking order of the alternative set $x=(x_1,\ x_2,\ \dots,x_n)$ is generated based on the increasing order of distances S_j $(j=1,2,\dots,n)$ calculated using Eq. (1).

3 AN APPLICATION OF FUZZY LINMAP ON A REAL LIFE SUPPLIER SELECTION PROBLEM

This application is made to solve supplier selection problem of a company which is operating in automotive supply industry in Kocaeli, Turkey. A fuzzy LINMAP model is developed for this supplier evaluation and selection problem. In the model, to evaluate the alternative supplier which are represented as X_1, X_2, X_3, X_4, X_5, four criteria is used which are cost (C_1), technical (C_2), delivery (C_3) and quality (C_4). These criteria are expressed with fuzzy triangular numbers using linguistic variables. The preferences of the purchasing expert between the alternative suppliers are given below as a set:

$$\delta = \{(2,1),(1,4),(3,4),(5,3)\} \qquad (22)$$

The fuzzy decision matrix obtained by evaluation of alternatives based on determined criteria is presented as follows (Eq. (23)):

$$\tilde{F} = \begin{matrix} & X_1 & X_2 & X_3 & X_4 & X_5 \\ C_1 & (0.4,0.5,0.6) & (0.1,0.2,0.3) & (0.3,0.4,0.5) & (0.2,0.3,0.4) & (0,0.1,0.2) \\ C_2 & (0.1,0.2,0.3) & (0.4,0.5,0.6) & (0.2,0.3,0.4) & (0,0.1,0.2) & (0.3,0.4,0.5) \\ C_3 & (0.1,0.2,0.3) & (0.3,0.4,0.5) & (0.2,0.3,0.4) & (0,0.1,0.2) & (0.4,0.5,0.6) \\ C_4 & (0.2,0.3,0.4) & (0.3,0.4,0.5) & (0.1,0.2,0.3) & (0,0.1,0.2) & (0.4,0.5,0.6) \end{matrix} \qquad (23)$$

Then the fuzzy decision matrix is transformed into the normalization positive triangular fuzzy number matrix as seen in Eq.(24)

$$\tilde{R} = \begin{matrix} & X_1 & X_2 & X_3 & X_4 & X_5 \\ C_1 & (0.667,1,1) & (0.167,0.4,0.75) & (0.5,0.8,1) & (0.333,0.6,1) & (0,0.2,0.5) \\ C_2 & (0.167,0.4,0.75) & (0.667,1,1) & (0.333,0.6,1) & (0,0.2,0.5) & (0.5,0.8,1) \\ C_3 & (0.167,0.4,0.75) & (0.5,0.8,1) & (0.333,0.6,1) & (0,0.2,0.5) & (0.667,1,1) \\ C_4 & (0.333,0.6,1) & (0.5,0.8,1) & (0.167,0.4,0.75) & (0,0.2,0.5) & (0.667,1,1) \end{matrix} \qquad (24)$$

Then the linear programming model constructed by combining Eqs. (20), (22) and (24). $Max\,\{\lambda_{21} + \lambda_{14} + \lambda_{34} + \lambda_{53}\}$

Such that

$$+0{,}8425w_1 - 0{,}7667v_{1l} - 0{,}0667v_{1m} - 0{,}5v_{1n} - 0{,}8425w_2 + 0{,}7667v_{2l} + 0{,}0667v_{2m} + 0{,}5v_{2n} - 0{,}5286w_3 + 0{,}5667v_{3l} + 0{,}0445v_{3m} + 0{,}3889v_{3n} - 0{,}232w_4 + 0{,}2v_{4l} + 0{,}0223v_{4m} + 0{,}111v_{4n} + 0{,}217 - \lambda_{21} \le 0$$

$$-0{,}5467w_1 + 0{,}4v_{1l} + 0{,}044v_{1m} + 0{,}222v_{1n} - 0{,}1719w_2 + 0{,}3667v_{2l} + 0{,}0222v_{2m} + 0{,}2778v_{2n} - 0{,}172w_3 + 0{,}3667v_{3l} + 0{,}022v_{3m} + 0{,}278v_{3n} - 0{,}4678w_4 + 0{,}7333v_{4l} + 0{,}0444v_{4m} + 0{,}5556v_{4n} - 0{,}2334 - \lambda_{14} \le 0$$

$$-0{,}232w_1 + 0{,}2v_{1l} + 0{,}0223v_{1m} + 0{,}1111v_{1n} - 0{,}4678w_2 + 0{,}7333v_{2l} + 0{,}0444v_{2m} + 0{,}5556v_{2n} - 0{,}4678w_3 + 0{,}7333v_{3l} + 0{,}044v_{3m} + 0{,}5556v_{3n} - 0{,}1719w_4 + 0{,}3667v_{4l} + 0{,}0222 * v4m + 0{,}2778 * v4n - 0{,}5832 - \lambda_{34} \le 0$$

$$+0{,}7w_1 - 0{,}933v_{1l} - 0{,}0667v_{1m} - 0{,}6667v_{1n} - 0{,}2322w_2 + 0{,}2v_{2l} + 0{,}0223v_{2m} + 0{,}1111v_{2n} - 0{,}5467w_3 + 0{,}4v_{3l} + 0{,}044v_{3m} + 0{,}2222v_{3n} - 0{,}8425w_4 + 0{,}7667v_{4l} + 0{,}0667v_{4m} + 0{,}5v_{4n} + 0{,}6503 - \lambda_{53} \le 0$$

$$+0{,}7636w_1 - 1{,}10v_{1l} - 0{,}0667v_{1m} - 0{,}8333v_{1n} - 4{,}5222w_2 + 4{,}933v_{2l} + 0{,}4v_{2m} + 3{,}333v_{2n} - 3{,}8933w_3 + 4{,}5333v_{3l} + 0{,}3556v_{3m} + 3{,}1111v_{3n} - 4{,}485w_4 + 5{,}2667v_{4l} + 0{,}4v_{4m} + 3{,}6667v_{4n} - 0{,}1831 \ge 0$$

$$\sum_{i=1}^{m} w_i = 1$$

$$w_i \ge 0.01 \quad (i = 1, 2, \dots, m)$$

$$v_{il},\ v_{im},\ v_{in},\ \lambda_{21},\ \lambda_{14},\ \lambda_{34},\ \lambda_{53} \ge 0 \quad (i = 1, 2, \dots, m) \qquad (25)$$

Solving Eq. (25) using the existing Simplex method software, we can obtain the optimal solutions as follows:

$$w = (w_1, w_2, w_3, w_4)^T = (0.437, 0.200, 0.131, 0.232)^T \qquad (26)$$

and

$$\tilde{v} = (v_1, v_2, v_3, v_4) = \big((0,0,1.187),(0,0,1.346),(0,0,0),(0,0,0)\big)$$

Using Eq. (21) and combined with Eqs. (26) and (27), the fuzzy positive ideal solution can be calculated as:

$$\tilde{a}^* = (\tilde{a}_1^*,\ \tilde{a}_2^*,\ \tilde{a}_3^*,\ \tilde{a}_4^*)^T = \big((0,0,2.716),(0,0,6.73),(0,0,0),(0,0,0)\big)^T$$

And square distance of each alternative from FPIS can be calculating using Eq. (7) as follows:

$S_1 = 2.6916$, $S_2 = 2.5431$, $S_3 = 2.3756$, $S_4 = 2.9303$, $S_5 = 2.9707$

So the ranking order of five supplier is generated as;

$$x_3 > x_2 > x_1 > x_4 > x_5$$

Obviously, the best alternative is x_3.

4 CONCLUSION

The supplier selection problem has been extensively studied by the researchers due to being a very critical activity in Logistics and Supply Chain Management. In this study, the Fuzzy LINMAP method is applied to evaluate suppliers in terms of 4 criteria which are cost, delivery, quality and technical. Supplier selection is a decision making process which involves uncertainty. The criteria are defined as the fuzzy numbers and the linguistic variables to overcome the uncertainty and evaluate the suppliers in a systematic way. As an illustrative example, 5 suppliers of a firm in the automotive sector are assessed based on the proposed algorithm. The main objective of this paper is to bring a different aspect to the applications of decision making techniques on supplier selection by using The Fuzzy LINMAP approach.

LINMAP method generally requires a set of decision makers' pairwise preference information between two alternatives and a decision matrix. If the number of pairs in the collective set δ is small, the optimal criteria weights obtained by the LINMAP method will be less reliable. If the number of conflicting preference relations in δ is large, the LINMAP model may become infeasible. Therefore, collecting the preference information over the alternatives is an important issue for the sake of effectively implementing the LINMAP procedure. Therefore, future studies may be conduct on this issue. Another issue on which can be studied is the h value which is subjective and determines the dominance of consistence over inconsistence in the model. What would be the value of h can be examined with experiments.

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