A Modified Preventive Maintenance Model with Degradation Rate Reduction in a Finite Time Span

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Abstract: Preventive maintenance (PM) can slow the deterioration process of a repairable system and restore the system to a younger state. The proposed PM model of this paper focuses on the restoration effect of degradation rate reduction which can only relieve stress temporarily and slow the rate of system degradation while the hazard rate is still monotonically increased. This PM model considers a deteriorating but repairable system (or equipment) with a finite life time period. This PM model is modified based on an original degradation-rate-reduction PM model over a finite time span of which the searching range for the optimal solution of the time interval between each PM is limited. It is demonstrated that the proposed degradation-rate-reduction PM model over a finite time period can have a better optimal solution than the original PM model. The algorithm of finding the optimal solution for the modified PM model is developed. Examples are provided and are compared with the corresponding original PM model.

1 INTRODUCTION

For a deteriorating and repairable system, the preventive maintenance (PM) can slow down the aging process and restore the system to a younger state (Pham and Wang 1996). Many PM models shown in the literature assume the PM can restore the system to a younger age or a smaller hazard rate, such as Nakagawa (1986) and Chan and Shaw (1993). However, the PM tasks, such as cleaning, adjustment, alignment, and lubrication work, may not always reduce system's age or hazard rate. Instead, this type of PM tasks may only reduce the degradation rate of the system to a certain level. It can be seen from the literature of the reliability-centered maintenance (RCM) and the total productive maintenance (TPM) (Bertling, Allan and Eriksson 2005, Zhou, Xi, and Lee 2007, McKone, Schroeder and Cua 2001, and Talib, Bon and Karim 2011) that this type of PM tasks is important for keeping a system or equipment in the state of high reliability. Canfield (1986) proposed an infinite-time-span model for the above PM tasks which assumes that the PM can only relieve stress temporarily and slow the rate of system degradation while the hazard rate is still monotonically increased. Based on Canfield's model, Park, Jung and Yum (2000) and Cheng and Chen (2008) developed the optimal periodic PM policy for the deteriorating systems over an infinite time span.

In real world, a system's useful life is normally finite. When an aged system is replaced by a new one, the new system seldom has exactly the same conditions (such as characteristics, investment cost, and maintenance expenses) as those of the system of the previous replacement cycle. However, not many PM models consider the conditon of finite time span. Only some examples are found, such as Pongpech and Murthy (2006), Yeh and Chen (2006) and Ponchet, Fouladirad and Grall (2011). Hence, it is worthwhile to study the PM problem with a finite time span.

Furthermore, it is found from the PM models developed by Pongpech and Murthy (2006) and Cheng and Liu (2010) that a shorter time interval between each PM can result in a better expected total maintenance cost. However, these PM models do limit the possibility of finding a smaller total maintenance cost since the searching range of the time interval between each PM is limited.

In this paper, a new degradation-rate-reduction PM model over a finite time span is proposed by releasing the restriction of the searching range for the time interval between each PM. The algorithm of finding the optimal solution for the proposed new PM model is provided. Examples of Weibull failure

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case are constructed for this new PM model to examine the assumption and to analyze the sensitivity of the optimal solution.

2 MODEL DEVELOPMENT

2.1 Nomenclature

L	The useful life time (finite time span) for the system or equipment
T	The time interval of each periodic PM
N	The number of PM performed in the finite life time span (L)
η	The restoration ratio of the degradation rate corresponding to age in each PM and $0 \le \eta \le 1$ where η=0 represents minimal repair and η=1 represents perfect maintenance. where η is the restored ratio
$\delta(\eta)$	The level of degradation rate reduction after each PM, which is measured by the corresponding age reduction and is function of the restoration ratio η.
π	The time interval between the N^{th} PM and L, i.e., $\pi = L - NT$
$\lambda(t)$	The original hazard rate function (before performing the 1st PM)
$\lambda_i(t)$	The hazard rate function at time t where t is in the i^{th} PM cycle and $\lambda_0(t)=\lambda(t)$
$\Lambda_i(t)$	The expected number of failure at time t of the i^{th} PM cycle and $\Lambda_0(t)=\Lambda(t)$
$C_{pm}(i,\eta)$	Cost of the i^{th} PM which is function of i and $\delta(\eta)$
C_{mr}	The minimal repair cost of each failure
$TC(N,T,\eta)$	The expected total maintenance cost function over the finite life time interval L

2.2 The Assumptions

The following are the assumptions for the proposed PM model.

- The system is deteriorating over time with increasing failure rate (IFR) in which Weibull failure distribution is assumed in this paper, i.e.

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \tag{1}$$

  where θ is the scale parameter and β is the shape parameter with $\beta > 1$.
- The PM can reduce the system's degradation rate to a younger level.
- The reduced degradation rate of each PM is assumed to be constant and is measured by the restored ratio (η) of the corresponding age of the degradation rate.
- The time interval of each PM (T) is limited in the range of (0, L].
- Minimal repair is performed when a failure occurs between each PM where the system is restored to its condition just prior to the failure.
- The system is disposed at the specified finite time L without replacing a new one.
- The total maintenance cost (TC) is considered as the objective function which includes the minimal repair cost and the PM cost in this new model.
- The minimal repair cost (C_{mr}) is assumed to be constant.
- The cost of the i^{th} PM (C_{pm}) is assumed to be variable, which is affected by the age (expressed by the number of PM already performed) and the reduced level of degradation rate, and is defined in the following equation.

$$C_{pm}(i,\eta) = a + bi + c\delta(\eta)\,, \tag{2}$$

  where coefficient a represents the constant part of the PM cost, b and c represent the unit incremental PM cost of aging and the restored level of degradation rate, respectively.
- The times to perform PM and minimal repair are negligible.

2.3 The Idea of the Modified Model

It is found that the time interval between each PM (T) of the failure-rate-reduction PM models developed by Pongpech and Murthy (2006) and Cheng and Liu (2010) is constrained in the range of $[T_{\min}, T_{\max})$ where $T_{\min} = L/(N+1)$ and $T_{\max} = L/N$ for a specified number of PM (N) in the finite time span (L). It is also known that the optimal value of T be the smallest possible value (i.e., $T_{\min}$) when given a specified N. This result is also seen in the degradation-rate-reduction PM model (no warranty case) proposed by Cheng et al. (2009). Thus, the constraint do limit the possibility of finding the optimal value of T being smaller than $T_{\min}$.

In this paper, the degradation-rate-reduction PM model of no warranty case proposed by Cheng et al. (2009) is called the original PM model. Figure 1 illustrates the hazard rate function of the original PM model where π is the time interval between L and the time of the last PM (NT), i.e., $\pi = L–NT$. It can be seen that $\pi < T$ when T is restricted in the range of $[T_{\min}, T_{\max})$. Next, the modified degradation-rate-reduction PM model over a finite time span is proposed by releasing the restriction of T which is illustrated in Figure 2. Under a specified finite time span (L) and a given number of PM (N), it can be observed that the interval of π shown in Figure 2 be greater than the PM interval T' (i.e., $\pi > T'$) and T' has smaller value than T of the original PM model (as shown in Figure 1). Then, it is desired to presume and verify the modified PM model can provide a better optimal solution.

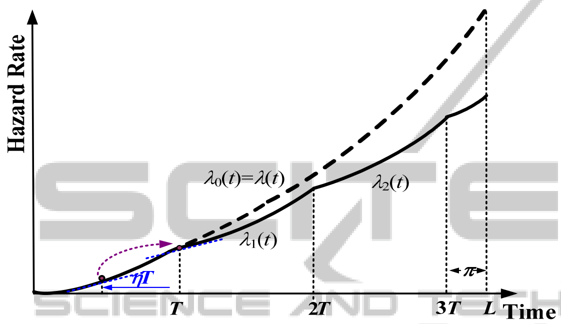

Figure 1: The illustration of the original degradation-rate-reduction PM model.

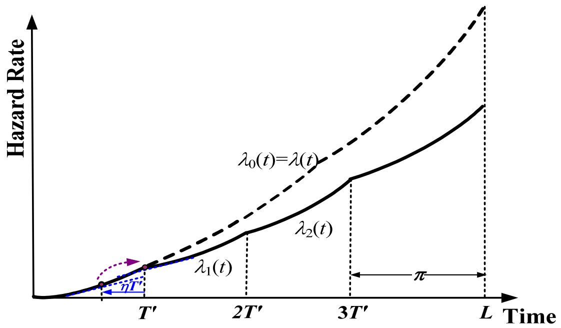

Figure 2: The illustration of the modified degradation-rate-reduction PM model.

2.4 The Modified PM Model

First, we have to obtain the total expected number of failures over the entire finite time interval L, denoted as $\Lambda(L)$, which is shown in Equation (4).

$$\Lambda(L) = \sum_{i=0}^{N-1} \int_{iT}^{(i+1)T} \lambda_i(t)dt + \int_{NT}^{L} \lambda_N(t)dt. \tag{3}$$

Based on Canfied (1986), the hazard rate function of the i^{th} PM ($\lambda_i(t)$) of the degradation-rate-reduction PM model can be presented as

$$\lambda_i(t) = \begin{cases} \lambda(t), \quad 0 < t \le T \quad \text{for } i = 0, \\ \sum_{k=1}^{i} \{\lambda(T + (k-1)(1-\eta)T) - \lambda(k(1-\eta)T)\} \\ +\lambda(t - i\eta T), \quad iT < t \le (i+1)T, \\ \qquad \text{for } i = 1, 2, \ldots, N. \end{cases} \tag{4}$$

For a Weibull failure distribution, $\Lambda(L)$ can be obtained as the following equation.

$$\Lambda(L) = \begin{cases} \left(\frac{L}{\theta}\right)^{\beta}, \; N=0 \\ \left(\frac{1}{\theta}\right)^{\beta} \Big\{ (T)^{\beta} + T\beta \sum_{i=1}^{N-1}\sum_{k=1}^{i}\left[(kT-(k-1)T\eta)^{\beta-1} - (kT-kT\eta)^{\beta-1}\right] \\ \quad + \sum_{i=1}^{N-1}\left[((i+1)T - i\eta T)^{\beta} - (iT - i\eta T)^{\beta}\right] \\ \quad + (L-NT)\beta \sum_{k=1}^{N}\left[(kT-(k-1)T\eta)^{\beta-1} - (kT-kT\eta)^{\beta-1}\right] \\ \quad + \left[(L-N\eta T)^{\beta} - (NT-N\eta T)^{\beta}\right] \Big\}, \; N>0 \end{cases} \tag{5}$$

Next, the expected total maintenance cost (TC) of the modified PM model is shown as follows.

$$TC(N,T,\eta) = \begin{cases} C_{mr}\Lambda(L), \; N=0 \\ C_{mr}\Lambda(L) + \sum_{i=1}^{N} C_{pm}(i,T,\eta), \; N>0 \end{cases} \tag{6}$$

By substituting Equations (4) and (5) into Equation (6), we can obtain the expected TC cost in Equation (7). Note that the equation of TC cost of the modified PM model is same as the original PM model. The difference of the two PM models is the searching range of T.

$$TC(N,T,\eta) = \begin{cases} C_{mr}\int_0^L \lambda(t)dt, \; N=0 \\ C_{mr}\Big\{\int_0^T \lambda(t)dt \\ \quad + \sum_{i=1}^{N-1}\sum_{k=1}^{i} T\left[\lambda(kT-(k-1)T\eta) - \lambda(kT-kT\eta)\right] \\ \quad + \sum_{i=1}^{N-1}\left[\Lambda((i+1)T - i\eta T) - \Lambda(iT - i\eta T)\right] \\ \quad + \sum_{k=1}^{N}(L-NT)\left[\lambda(kT-(k-1)T\eta) - \lambda(kT-kT\eta)\right] \\ \quad + \Lambda(L-N\eta T) - \Lambda(NT-N\eta T)\Big\} \\ +N(a + \frac{N+1}{2}b + c\eta T), \; N>0 \end{cases} \tag{7}$$

For a Weibull failure distribution, the expected total maintenance cost becomes

$$TC(N,T,\eta)=\begin{cases} C_{mr}\left(\dfrac{L}{\theta}\right)^{\beta}, \ N=0 \\ C_{mr}\left(\dfrac{1}{\theta}\right)^{\beta}\Bigg\{(T)^{\beta}+T\beta\displaystyle\sum_{i=1}^{N-1}\sum_{k=1}^{i}\begin{bmatrix}(kT-(k-1)T\eta)^{\beta-1} \\ -(kT-kT\eta)^{\beta-1}\end{bmatrix} \\ \quad +\displaystyle\sum_{i=1}^{N-1}\left[((i+1)T-i\eta T)^{\beta}-(iT-i\eta T)^{\beta}\right] \\ \quad +(L-NT)\beta\displaystyle\sum_{k=1}^{N}\left[(kT-(k-1)T\eta)^{\beta-1}-(kT-kT\eta)^{\beta-1}\right] \\ \quad +\left[(L-N\eta T)^{\beta}-(NT-N\eta T)^{\beta}\right]\Bigg\} \\ +N(a+\dfrac{N+1}{2}b+c\eta T) \ , \ N>0 \end{cases} \quad (8)$$

3 THE OPTIMAL PM POLICY

The optimal solution for the modified PM model can be obtained by minimizing the expected total maintenance cost (TC). The decision variables are the number of PM (N) over the finite time span (L), the time interval of each PM (T_N), and the restoration ratio (η_N). It requires an algorithm with numerical method to search for the optimal solution. In this paper, we modify the algorithm provided by Cheng and Liu (2010) with the Nelder-Mead searching method which is a commonly used nonlinear optimization technique for minimizing the objective function in a multi-dimensional space. The modified algorithm is presented as follows.

1. Let $N = 0$, $T_N = L$, $\eta_N = 0$.
2. Calculate $C_{\min} = TC(N, T_N, \eta_N)$ using Equation (7) or (8). (Note: $C_{\min}$ equals to the expected total maintenance cost of no PM.)
3. Let $N = 1$.
4. Calculate $T_U = L/N$.
5. Use Nelder-Mead method to search the values of T_N in the range of $(0, T_U]$ and η_N in the range of $[0, 1]$ such that $TC(N, T_N, \eta_N)$ shown in (7) or (8) is minimized; let C_0 = minimal value of $TC(N, T_N, \eta_N)$.
6. If $C_0 \geq C_{\min}$ then obtain the optimal solution: $N^*=N-1$, $T^*=T_{N^*}$, $\eta^* = \eta_{N^*}$, $TC(N^*,T^*,\eta^*)$, and stop; else let $N = N+1$ and $C_{\min} = C_0$; go to Step 4.

4 NUMERICAL EXAMPLES

Numerical examples are performed and the optimal solutions of the modified PM model are compared with those of the original PM model proposed by Cheng et al. (2009). The system's life is assumed to follow Weibull distribution with scale parameter θ =1 and shape parameter β = 2.5, and 3. Let L=5, C_{mr}=1 and the coefficients a, b, and c of C_{pm} are assigned with different values which satisfy $C_{pm} \geq C_{mr}$ as shown in Table 1.

Table 1: The comparison of the optimal solutions of the modified and the original models.

a	b	c	Model*	β= 2.5						β= 3					
				N	T	π	η	TC	Type**	N	T	π	η	TC	Type**
1	0.1	0.1	M	6	0.52	1.88	1	32.31	P	8	0.45	1.4	1	29.89	P
			O	6	0.71	0.74	1	34.19	F	9	0.5	0.5	1	32.08	F
1	0.8	0.1	M	3	0.85	2.45	1	38.84	P	4	0.76	1.96	1	43.08	P
			O	3	1.25	1.25	1	41.37	F	5	0.83	0.85	1	46.93	F
1	1.5	0.1	M	2	1.09	2.82	1	41.7	P	3	0.92	2.24	1	49.92	P
			O	2	1.67	1.66	1	44.49	F	4	1	1	1	54.4	F
1	0.1	0.8	M	6	0.49	2.06	1	34.45	P	8	0.44	1.48	1	32.37	P
			O	6	0.71	0.74	1	37.19	F	8	0.56	0.52	1	35.22	F
1	0.8	0.8	M	3	0.8	2.6	1	40.58	P	4	0.74	2.04	1	45.18	P
			O	3	1.25	1.25	1	43.99	F	5	0.83	0.85	1	49.85	F
1	1.5	0.8	M	2	1.03	2.94	1	43.18	P	3	0.91	2.27	1	51.84	P
			O	2	1.67	1.66	1	46.82	F	4	1	1	1	57.2	F
1	0.1	1.5	M	5	0.54	2.3	1	36.4	P	8	0.43	1.56	1	34.79	P
			O	6	0.71	0.74	1	40.19	F	8	0.56	0.52	1	38.33	F
1	0.8	1.5	M	3	0.76	2.72	1	42.23	P	4	0.73	2.08	1	47.24	P
			O	3	1.25	1.25	1	46.62	F	5	0.83	0.85	1	52.76	F
1	1.5	1.5	M	2	0.97	3.06	1	44.58	P	3	0.89	2.33	1	53.72	P
			O	2	1.67	1.66	1	49.15	F	4	1	1	1	60	F
1.5	0.1	0.1	M	5	0.6	2	1	34.94	P	7	0.5	1.5	1	33.65	P
			O	5	0.83	0.85	1	36.99	F	8	0.56	0.52	1	36.11	F
1.5	0.8	0.1	M	3	0.85	2.45	1	40.34	P	4	0.76	1.96	1	45.08	P
			O	3	1.25	1.25	1	42.87	F	4	1	1	1	49.4	F
1.5	1.5	0.1	M	2	1.09	2.82	1	42.7	P	3	0.92	2.24	1	51.42	P
			O	2	1.67	1.66	1	45.49	F	4	1	1	1	56.4	F
1.5	0.1	0.8	M	5	0.57	2.15	1	36.98	P	7	0.49	1.57	1	36.06	P
			O	5	0.83	0.85	1	39.91	F	8	0.56	0.52	1	39.22	F
1.5	0.8	0.8	M	2	1.03	2.94	1	42.08	P	4	0.74	2.04	1	47.18	P
			O	3	1.25	1.25	1	45.5	F	4	1	1	1	52.2	F
1.5	1.5	0.8	M	2	1.03	2.94	1	44.18	P	3	0.91	2.27	1	53.34	P
			O	2	1.67	1.66	1	47.82	F	4	1	1	1	59.2	F
1.5	0.1	1.5	M	4	0.63	2.48	1	38.85	P	7	0.48	1.64	1	38.42	P
			O	5	0.83	0.85	1	42.83	F	7	0.63	0.59	1	42.32	F
1.5	0.8	1.5	M	2	1.09	2.82	1	43.48	P	4	0.73	2.08	1	49.24	P
			O	2	1.67	1.66	1	48.05	F	4	1	1	1	55	F
1.5	1.5	1.5	M	2	0.97	3.06	1	45.58	P	3	0.89	2.33	1	55.22	P
			O	2	1.67	1.66	1	50.15	F	4	1	1	1	62	F

*: In the 4th column, M represents the modified PM model while O represents the original PM model.

**: In the 10th and 16th columns, P represents the partially-periodic PM interval while F represents the fully-periodic PM interval.

Table 1 shows the optimal solutions for the examples. It can be found that the η^* = 1 for each example. It can also be seen from these examples that the new (modified) PM model has smaller PM interval (T) and better optimal total maintenance cost (TC) than the original PM model where the original PM model has the fact: $T^* = T_{\min} = L/(N^*+1)$. When further examining the optimal policies of the examples, it can be noticed that the modified PM model has partially-periodic PM interval while the original PM model has fully-periodic PM interval. It can also be seen that a PM model with a shorter PM interval does provide a better optimal solution. This makes the presumption of this research acceptable.

We also analyze the sensitivity of each parameter to the optimal solution for the modified PM model by using the ANOVA method as shown in Table 2. It can be found that β and c are significantly sensitive to the optimal total maintenance cost *TC*. The results indicate that the optimal *TC* is significantly affected by a system's failure rate (or aging process) and the unit incremental PM cost of the restored level of degradation rate.

Table 2: The Sensitivity Analysis for the Optimal Solution of the New PM Model.

Response: *TC*					α=0.05
Source	Sum of Square	DF	Mean Square	F Value	Prob>F
Model	18730.60	9	2081.18	122.56	< 0.0001
β	16656.19	2	8328.09	490.43	< 0.0001
a	70.26	1	70.26	4.14	0.0450
b	1873.76	3	624.59	36.78	< 0.0001
c	130.39	3	43.46	2.56	0.0603
Residual	1460.39	86	16.98		
Cor Total	20190.99	95			

5 CONCLUSIONS

For the PM problem in a finite time span, based on the fact that a shorter time interval of PM can result in a better expected total maintenance cost, a modified degradation-rate-reduction PM model is developed with no constraint on the PM interval (*T*). The algorithm of finding the optimal solution for the new PM model is also constructed in this paper. It is shown from the examples that the modified PM model can provide better optimal solution than the original PM model. This indicates that the modified PM model is more suitable for the deteriorating and repairable systems. For the future work, the theoretical proof of the existence of the optimal solution for the modified PM model needs to be explored.

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