Dynamical Model of Asphalt-Roller Interaction During Compaction

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Abstract: Proper and uniform compaction during construction is of utmost importance for the long term performance of asphalt pavement. Variations in the conditions of freshly laid pavements require adjustment of the compaction effort in order to obtain uniform and adequate density. One of the goals of on-going research in Intelligent Compaction (IC) is the development of adaptive feedback control mechanism to adjust the compaction effort according to the field and pavement conditions. Such feedback control systems require a good understanding of compaction dynamics. In this study, a dynamical model is developed to study the interaction between a moving vibratory roller and the underlying asphalt pavement during compaction. The asphalt pavement is represented as a lumped element model with visco-elastic-plastic properties. A procedure is presented to estimate the parameters of this model from standard tests on asphalt mix conducted in the laboratory. The combined roller-pavement dynamical model is used to replicate field compaction of an asphalt pavement using a vibratory roller. Numerical simulation results indicate good agreement with results observed during compaction of pavements in the field. Comparison between the simulation results and the results collected from the actual pavement construction job show that the model could be used as a mathematical basis for the development of advanced compaction methods.

1 INTRODUCTION

Compaction is one of the important steps in pavement construction that affect the quality of asphalt pavement. Proper compaction increases the load bearing capacity of the pavement and provides a smooth riding surface. It also increases the useful life of a pavement by reducing its susceptibility to early failure due to fatigue, rutting, low temperature cracking and aging (US Army Corps, 2000). The construction of an asphalt pavement begins with the preparation of the base and subsequent laydown of asphalt mix of desired thickness. Rollers with rotating eccentric masses in the drum are then used to impart static and vibratory energy to the asphalt mat to reduce the air voids and improve the stiffness of the pavement. Several factors like the composition of asphalt mix, its temperature at laydown, thickness of the layer, temperature and stiffness of the underlying layer, temperature, velocity and humidity of air, and solar radiation levels affect the compactbility of asphalt mix (Brown et al., 2009). Unless these factors are addressed during the construction process, they can lead to non-uniform compaction of the pavement. Intelligent Compaction (IC) technologies are being developed to provide continuous real-time quality control by monitoring the level of compaction of the pavement and adjusting the amount of compaction energy applied by the roller in order to obtain uniform stiffness/density.

One of the goals of IC is to develop an automatic feedback control system that can take into account compaction quality and modify in real time, roller parameters such as speed, frequency and amplitude of vibration, to improve the overall quality of the construction (Chang et al., 2012). Intelligent Compaction is based on the hypothesis that the vibratory roller and the underlying pavement layers form a coupled system. The response of the roller is determined by the frequency of its vibratory motors and the natural vibratory modes of the coupled system (Imran et al, 2012). Compaction of the pavement increases its stiffness and as a result, the vibrations of the compactor are altered. The

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knowledge of the field conditions and the vibration spectra of the compactor can, therefore, be used to estimate the properties of the pavement layer (Beainy et al., 2012, Commuri et al., 2009, Commuri and Zaman, 2010, Commuri, 2012). A feedback controller can utilize these estimated properties to adjust the amount of compaction energy applied at any given location on the pavement. These adjustments are designed to ensure that the energy is applied only to those areas of the pavement that are under-compacted while avoiding over-compaction of areas that are adequately compacted. Such a process is expected to improve the uniformity of density/stiffness of the pavement over its entirety and increase its useful life.

Development and analysis of an IC feedback control system requires a good understanding of the interaction of the roller and asphalt pavement and a model that can accurately represent the dynamics of the coupled system. The model has to account for factors such as temperature, density, and thickness of the pavement and loading frequency of the roller that affect the dynamics of the coupled system. Since these factors can change during the course of the compaction process, the model should adapt to these changes.

In recent years, attempts have been made to develop numerical techniques and analytical models to examine the response of asphalt mixes during compaction. Lodewikus (Lodewikus, 2004) developed a Finite Element Method (FEM) approach based on critical state theory to analyze the behavior of asphalt mix. Masad et al. (Masad et al., 2010) developed a thermodynamics-based nonlinear viscoelastic model for capturing the response of asphalt mix during compaction. Pei-Hui Shen and Shu-Wen Lin (Shen and Lin, 2008) derived an asymmetrical model of hysteresis based on the classical Bouc-Wen hysteresis model to investigate the dynamic characteristics of a vibratory compaction system.

Researchers have utilized analytical models such as Maxwell, generalized Maxwell, Kelvin–Voigt, generalized Kelvin, Huet–Sayegh, and Burger models to represent the asphalt pavement as a combination of springs and dampers (Nillson et al., 2002; Pronk, 2005; Dave et al., 2006; Xu and Solaimanian, 2009). These models were mostly used to study the long term behavior of the pavement under traffic loads. Their applicability in representing the pavement during field compaction is not widely studied. Among the analytical models, Burger's model is simple and can accurately describe the viscoelastic behavior of an asphalt pavement (Liu and You, 2009; Liu et al., 2009). Beainy et al. (2013a, 2013b) showed that Burger's model can be used to develop a Visco-Elastic-Plastic model that can represent the dynamical properties of asphalt pavement during compaction.

Although Beainy's model provides a good insight into the compaction dynamics, it does not consider the longitudinal and lateral movement of the roller. Only the interaction between the drum and the asphalt mat in the vertical direction is taken into acount. However, during compaction, the roller moves at a speed of 4-7 km/h along the pavement as it compacts the mix. Therefore, there is a spatial separation between each impact. In this research, Beiany's visco-elastic-plastic model is enhanced to incorporate the effect of spatial movement of the roller. A mathematical model is developed to describe the asphalt-roller interaction as the roller traverses the pavement using a conventional rolling pattern. Simulation is performed to obtain the density profile for the asphalt pavement after a traditional compaction is complete. Simulation results show that the model can be used to understand the dynamics of field compaction and to represent the densification achieved during field compaction. This understanding will be useful in studying the performance of different types of asphalt mixes and in developing techniques for improving the quality of constructed pavements.

The rest of the paper is organized as follows. The development of the dynamics of the coupled system is detailed in Section 2. A method to determine the parameters of the model based on experimental results from standard laboratory tests is discussed in Section 3. Verification of the proposed approach through numerical simulations is presented in Section 4 and the conclusions and scope of future work are discussed in Section 5.

2 DEVELOPMENT OF THE MODEL

The model developed for this research is an extension of Beainy's model (Beainy et al, 2013a, Beainy et al., 2013b) which is based on the assumption that the roller and the underlying pavement form a coupled system during compaction. Therefore any changes in the stiffness of the asphalt pavement would affect the vibration of the drum of the roller. The roller uses both the static force (weight of the drum and frame) and an impact force to compact the asphalt material. The impact force is a result of an eccentric mass rotating around the axle of the drum. It can be expressed as

$$F_{ec} = m_{ec}r_{ec}\omega_{ec}^{2}\sin(\omega_{ec}t) = m_{ec}r_{ec}\omega_{ec}^{2}\sin(2\pi f_{ec}t) \quad (1)$$

Where, $m_{ec}r_{ec}$ is the moment of the eccentric mass and ω_{ec} is the angular frequency of rotation.

It should be noted that only the movement of the drum in vertical direction is considered in the derivation of the model. The coupling between the drum and the frame of the roller is modelled as a parallel combination of a spring and a dashpot. The spring elements follow Hooke's law ($F = ke$; where F is the applied stress, e is the resulting strain and k is the Young's modulus of the spring). The dashpot elements are assumed to follow the Newton's law ($\sigma = \eta\frac{d\varepsilon}{dt}$, where 'σ' is the applied stress, 'ε' is the strain and 'η' is the viscosity).

The underlying asphalt pavement is modelled as a collection of blocks of Burger's material arranged in a grid. Burger's material can be represented by a series combination of a spring representing the of spring and dashpot representing the delayed (viscous) response, and a dashpot that represents the permanent deformation (Figure 1). It is considered that at any given time the roller drum interacts with adjacent blocks of equal width that are in contact with the drum. The reaction force of the pavement to the drum is the sum of the force exerted by each of the blocks. In the development presented in this paper, the asphalt pavement is considered to be laid on top of a rigid base.

The total deformation occurring in ith block of the asphalt pavement due to stress applied by the roller is comprised of an instantaneous elastic deformation 'ε_{ei}', a delayed viscous deformation 'ε_{vi}', and a permanent deformation 'ε_{pi}'. The constitutive equation of the strain can be expressed as

$$\varepsilon_i(t) = \varepsilon_{vi} + \varepsilon_{ei} + \varepsilon_{pi} = \left(\frac{1}{\eta_{avi}}\int \sigma_i e^{\frac{K_{avi}}{\eta_{avi}}t}\,dt + C_{1i}\right)e^{-\frac{K_{avi}}{\eta_{avi}}t} + \frac{\sigma_i}{K_{aei}} + \frac{\int \sigma_i\,dt + C_{2i}}{\eta_{api}} \quad (2)$$

where σ_i is the force experienced by the ith block. The viscous property of the block is represented as using a spring of stiffness K_{avi} and a dashpot of damping η_{avi}. The elastic strain is represented by the displacement of a spring of stiffness K_{aei} and the permanent deformation is modelled as a dashpot of coefficient η_{api} for ith block. C_{1i}, and C_{2i} are constants that represent the boundary conditions (Beainy et al, 2013a, Beainy et al., 2013b).

The dynamics of interaction between the roller and the asphalt mat can be formulated as

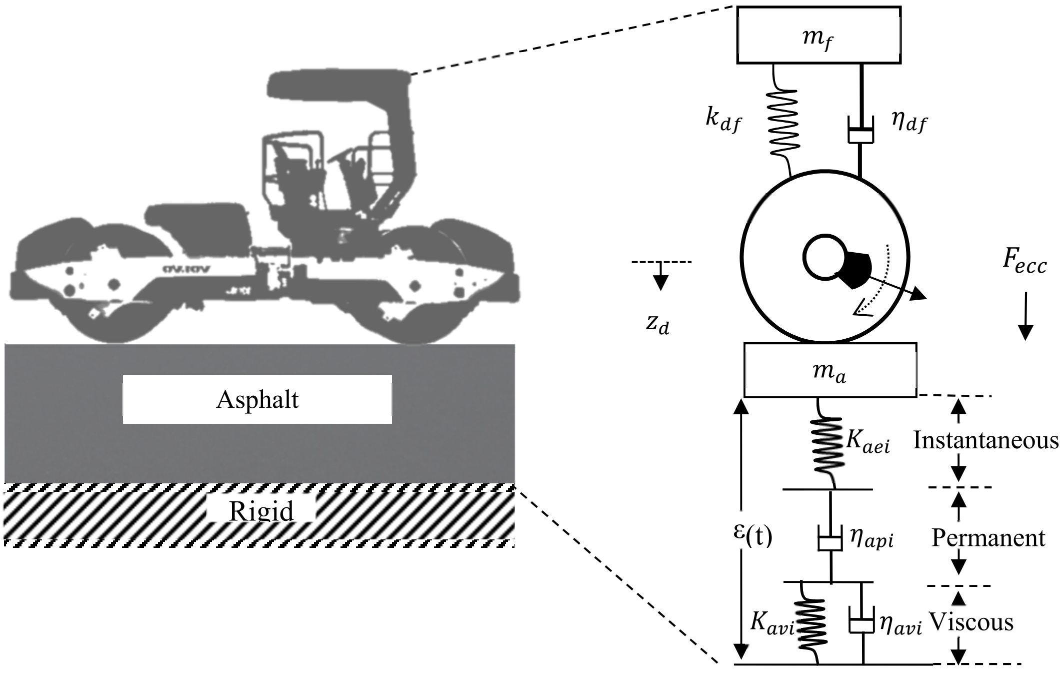

Figure 1: Viscoelastic-plastic model of asphalt-roller interaction.

$$(m_d + m_a)\ddot{z}_d = (m_d + m_a)\ddot{z}_a = m_{ec}r_{ec}\omega_{ec}{}^2\sin(\omega_{ec}t) + m_d g + m_a g - k_{df}(z_d - z_f) - \eta_{df}(\dot{z}_d - \dot{z}_f) - F_a \tag{3}$$

$$m_f\ddot{z}_f = m_f g + k_{df}(z_d - z_f) + \eta_{df}(\dot{z}_d - \dot{z}_f) \tag{4}$$

$$F_c = m_{ec}r_{ec}\omega_{ec}{}^2\, sin(\omega_{ec}t) + m_d g + m_f g - m_d\ddot{z}_d - m_f\ddot{z}_f = m_a\ddot{z}_d - m_a g + F_a \tag{5}$$

where z_a is the displacement of the asphalt layer; F_a is the reaction force of the asphalt layer; F_c is the drum-asphalt contact force; z_d is the displacement of the drum; z_f is the displacement of the frame; k_{df} is the drum-frame stiffness coefficient; $\dot{z}_d$ is the velocity of the drum; $\dot{z}_f$ is the velocity of the frame; η_{df} is the drum-frame damping coefficient; m_a is the asphalt weight; $\ddot{z}_d$ is the vertical acceleration of the drum; $\ddot{z}_a$ is the vertical acceleration of the asphalt pavement; $\ddot{z}_f$ is the acceleration of the frame.

The reaction force of the asphalt mat F_a can be expressed as

$$F_a = \sum_{i=1}^{6}\sigma_i = \sum_{i=1}^{6}K_{aei}\varepsilon_{ei} = \sum_{i=1}^{6}\left[z_a - K_{aei}\left\{\left(\frac{1}{\eta_{avi}}\int \sigma_i e^{\frac{K_{avi}}{\eta_{avi}}t}\,dt + C_{1i}\right)e^{-\frac{K_{avi}}{\eta_{avi}}t} + \frac{\int \sigma_i\,dt + C_{2i}}{\eta_{api}}\right\}\right] \tag{6}$$

For simplicity of calculation, the drum is considered to be in constant contact with the asphalt mat. The bouncing of the drum due to excessive vibrations is being studied in on-going research.

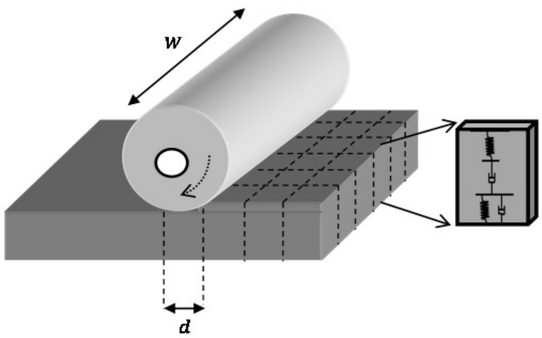

Figure 2: Block representation of asphalt pavement.

In this study, the pavement is considered as a grid of small blocks. At each impact during compaction, the compactor interacts with only one set of 6 adjacent blocks along the width of the drum. The size of each block is dependent on the geometry of the drum and asphalt layer, the velocity of the roller and the frequency of the vibration. Its height is equal to the thickness of the pavement layer. The width corresponds to 1/6 th of the width of the drum. The length is the distance the roller travels along the pavement while making one impact. At the beginning of each impact cycle the drum moves on top of a new set of blocks. The interaction between the roller and each set of block continues for one cycle of the vibration. Then in the next cycle the roller moves to the next block set. The parameter values of each block are dependent on the density and temperature of the block.

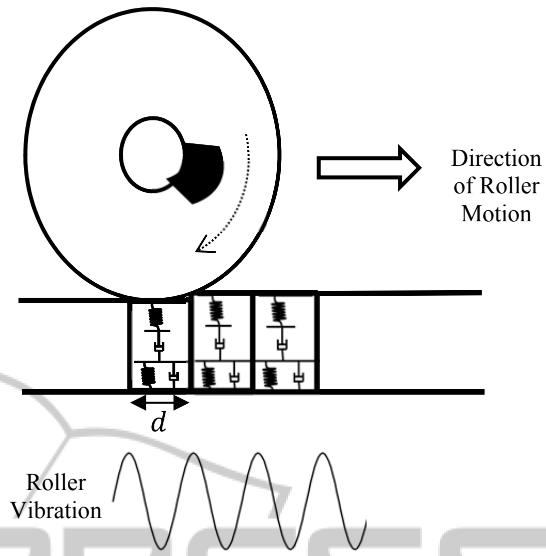

Figure 3: Asphalt block-drum interaction model for a moving roller.

During the first pass of the roller each grid element representing the pavement is assumed have no initial permanent or visco-elastic deformation. The elastic deformation comes into play the moment the roller drum comes into contact with the grid element. Therefore,

$$\varepsilon_{vi} = \varepsilon_{pi} = 0 \tag{7}$$

$$z_d = z_a = \frac{1}{6}\sum_{i=1}^{6}\varepsilon_{ei} \tag{8}$$

Using these boundary conditions, the constants C_{1i} and C_{2i} can be determined. Setting

$$\frac{1}{\eta_{avi}}\int \sigma_i e^{\frac{K_{avi}}{\eta_{avi}}t}\,dt + C_{1i} = 0 \tag{9}$$

$$C_{1i} = -\frac{\left(\int \sigma_i e^{\frac{K_{avi}}{\eta_{avi}}t}\, dt\right)_{at\ t=t_{sn}}}{\eta_{avi}}$$

Similarly, setting

$$\frac{\int \sigma_i\, dt + C_{2i}}{\eta_{api}} = 0$$
$$C_{2i} = -\left(\int \sigma_i\, dt\right)_{at\ t=t_{sn}}. \quad (10)$$

Where, t_{sn} is the time at which the drum starts compacting the nth block of the stretch.

However, from the second pass of the roller, the constant C_{2i} of each block is equal to the resultant permanent deformation after the previous pass. The viscous deformation is considered to be recovered between consecutive passes. Therefore C_{1i} is calculated in the same manner for all the passes.

3 DETERMINATION OF MODEL PARAMETERS

3.1 Roller Parameters

Roller parameters include mass of the drum, mass of the frame, drum-asphalt contact area, width and diameter of the drum, stiffness coefficient and damping coefficient of drum-frame, rotational frequency of eccentrics and the eccentric moment. These parameters can be determined from the manufacturer's specifications of the roller. The IR DD 118HF vibratory compactor is considered in this study in order to illustrate the procedure.

The drum-asphalt contact area is dependent on the drum width, weight of the compactor, stiffness of asphalt pavement, and total eccentric force applied to the pavement (Kröber et al. 2001). However, in this study, since the roller is assumed to be interacting with only 6 blocks that are underneath the drum for any given cycle period. The total contact force is assumed to be evenly distributed in the whole surface area of the block. The contact area is also assumed to be equal to the surface area of the blocks. The contact area between the drum and each block is calculated according to the following equation,

$$A_c = d \times w/6 \quad (11)$$

where, A_c is the drum-asphalt block contact area, d is the length of the block underneath and w is the width of the drum. The mass of the blocks in contact with the roller can then be determined using the contact area and the thickness of the pavement layer as well as the asphalt mix properties.

3.2 Pavement Parameters

Liu et al. (2009) developed a procedure to determine Burger's model parameters by fitting the parameters with the complex modulus test data. This procedure is enhanced to estimate the four parameters ($K_{ma}, \eta_{ma}, K_{ka}$, and η_{ka}) of the asphalt pavement section of the model (Beainy et al., 2013a, Beainy et al., 2013b). A laboratory testing according to AASHTO TP-62 standard procedure (AASHTO 2007) is first performed to determine the dynamic modulus |E* | and the phase angle ϕ of the asphalt mix at different temperatures, air void contents, and loading frequencies. The relationship between the dynamic modulus and phase angle of the mix and the model parameters can be expressed as follows

$$^1/_{E^*} = {^{\Delta\varepsilon}/_{\Delta\sigma}} = \sqrt{^1/_{K_{ae}^2} + {^1/_{\eta_{ap}^2\omega^2}} + {^{1 + 2(^{K_{av}}/_{K_{ae}} + ^{\eta_{av}}/_{\eta_{ap}})}/_{K_{av}^2 + \eta_{av}^2\omega^2}}} \quad (12)$$

$$\tan(\phi) = {^1/_{\tan(\theta)}} = {^{K_{ae}[K_{av}^2 + \eta_{av}(\eta_{ap} + \eta_{av})\omega^2]}/_{\eta_{ap}\omega(K_{av}^2 + K_{ae}K_{av} + \eta_{av}^2\omega^2)}} \quad (13)$$

$$K_{ae} = \lim_{\omega\to\infty} |E^*| \quad (14)$$

and

$$\eta_{ap} = \lim_{\omega\to 0} {^{|E^*|}/_{\omega}} \quad (15)$$

An iterative procedure is adopted to determine the model parameter values at the different loading frequencies and temperature values used in laboratory testing. The procedure is shown in the flow chart (Figure 4) where ω_{max}, ω_{min} are the maximum and minimum values of angular frequencies in laboratory tests.

The parameters $K_{ae}, \eta_{ap}, K_{av}$, and η_{av} are obtained for different test loading frequencies and temperatures. These values are then extrapolated using a power curve fitting method to get estimated values at operating frequency of the roller. The pavement properties such as temperature (T) and air void content (V_a) of the mix vary during compaction.

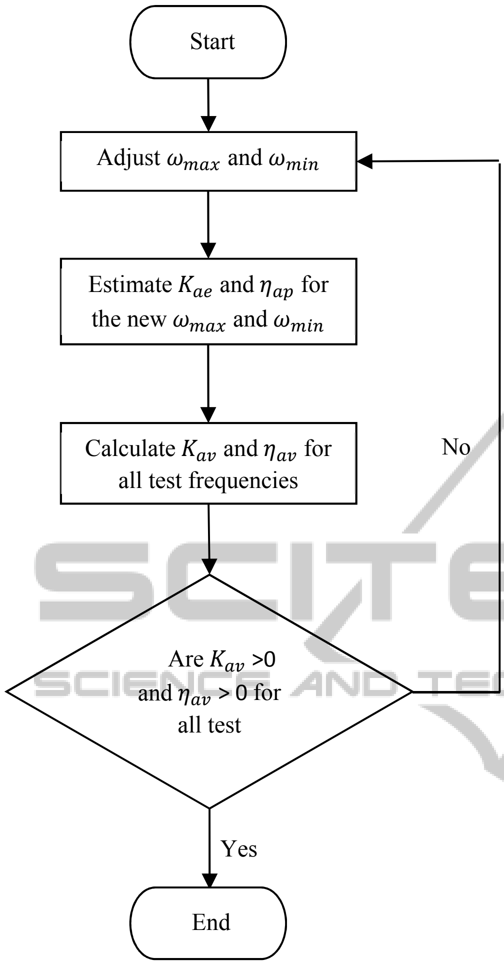

Figure 4: Flow chart of the iteration process to determine the Burger's model parameters.

This affects the stiffness and viscosity of the pavement. Equations are developed to estimate the parameter values taking into account the pavement temperature and air void content. These are expressed as follows.

$$K_{ae} = 5340 + 84V_a - 54T - 28.5V_a^2 + 0.23V_aT + 0.31T^2 + 0V_a^3 + 0.15V_a^2T - 0.0085V_aT^2 - 0.00054T^3 \quad (16)$$

$$\eta_{ap} = 7129 + 1373V_a - 125.1T - 106.9V_a^2 - 5.1V_aT + 0.831T^2 + 0V_a^3 + 0.5673V_a^2T - 0.01493V_aT^2 - 0.001496T^3 \quad (17)$$

$$K_{av} = 4088 - 384.4V_a - 41.53T + 12.7V_a^2 + 2.677V_aT + 0.1767T^2 + 0V_a^3 - 0.0657V_a^2T - 0.00446V_aT^2 - 0.0002987T^3 \quad (18)$$

$$\eta_{av} = 349.4 - 87.32V_a - 0.9664T + 7.825V_a^2 + 0.0933V_aT + 0.003178T^2 - 0.2273V_a^3 - 0.003858V_a^2T - 0.000065V_aT^2 - 0.0000056T^3 \quad (19)$$

4 NUMERICAL SIMULATION

Simulation is performed to study and evaluate the response of the model. The physical model includes an IR DD 118HF smooth drum vibratory roller compacting a 76.2 mm thick layer of asphalt mix with a nominal maximum aggregate size of 12.5mm and PG 76 -28 binder. The roller is assumed to operate at rated frequency of 50 Hz and moving at a constant speed of 6.4km/h throughout the process.

The parameters of the grid elements representing the asphalt pavement are varied with the variation of operating frequency of the roller, and the temperature and density of the mix. The parameter values are calculated using the set of equations (Eq 16-19) discussed in the previous section. During simulation, the model continuously monitors the roller operating frequency, temperature and air void contents of the pavement for each block and adjusts its parameter values accordingly. The roller is assumed to be initially at rest and in contact with the asphalt mix.

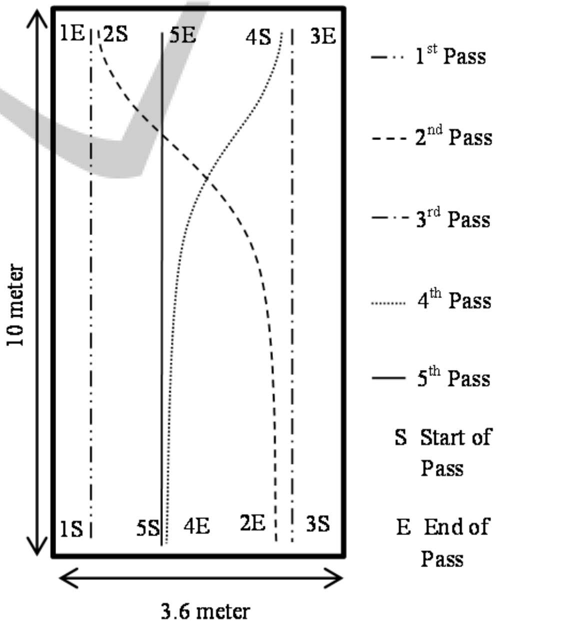

Figure 5: Rolling passes performed by the roller.

Simulations are performed for 5 consecutive passes of the roller using a conventional rolling pattern on the compaction stretch. The pavement is considered to be 10 meter long and 3.6 meter wide. The rolling pattern is shown in Figure 5. Since the drum width is 2 meters, there is some overlap between adjacent passes as the roller moves back and forth while executing the rolling pattern. Therefore, some grid elements representing the pavement are compacted with only one pass while other regions are compacted during each roller pass.

In the simulation examples presented, the temperature of the asphalt mix at laydown is considered to be 150^0 C and the material is assumed to comprise of 12% air voids over the entire stretch. This is done in order to replicate conditions commonly encountered in the field. Therefore, the parameter values are same for each block at laydown. After simulation of each roller pass, the model parameters of each grid element is stored and used as initial condition for the next pass. Figure 6 shows the density profile of the pavement after the compaction of the roller. In this figure, darker shading indicates higher density (represented as a percentage of maximum theoretical density) of the pavement.

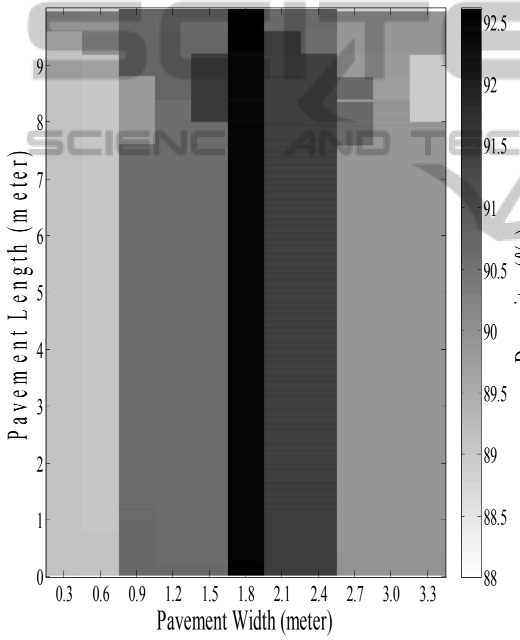

Figure 6: Density profile of the pavement after rolling passes.

From simulation results (Figure 6) it can be seen that up to 2% variability in density is possible between locations a few inches apart on the pavement. Previous field investigation performed by researchers at the University of Oklahoma (Beainy et al., 2011) on interstate I 86 near Hornell, New York also observed variability in density for up to 1.8% between cores with spatial distance of just a few inches.

Results of the simulations of the model are compared with actual data collected during the construction of Interstate I-35 in Norman, Oklahoma using a IR DD 118HF vibratory compactor. The vibrations of the roller were captured using a 13,200C uniaxial accelerometer from Summit Instruments is mounted on the axle of the drum. These vibrations were recorded during compaction of a 50.8 mm thick asphalt pavement comprising of a 12.5 mm PG 76 -28 OK Superpave asphalt mixture. Model parameters were determined for this asphalt mix and used to simulate the compaction in the field. The spectrogram of the vibrations obtained using numerical simulations is shown in Figure 7. Figure 8 shows the spectrogram of the vibrations of the roller observed during field compaction. In these figures, the normalized power in each frequency band is depicted using a color coded map. It is evident that the model presented in this paper captures not only the response of the roller at the fundamental frequency (frequency of the eccentrics), but also the response at the harmonics.

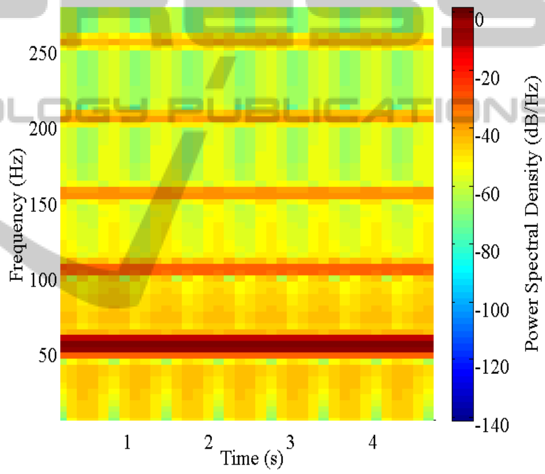

Figure 7: Simulated roller vibration spectrum.

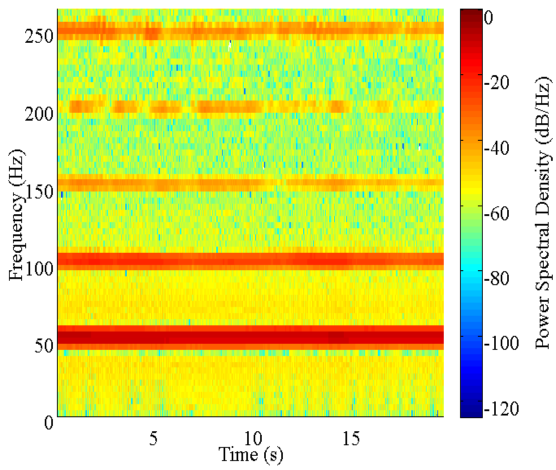

Figure 8: Measured roller vibration spectrum.

5 CONCLUSIONS

The development and validation of a dynamical model that can emulate the field compaction of asphalt mixes during construction of asphalt pavements was presented in this paper. A method to determine the parameters of the model using laboratory test data and roller information was developed and the use of this model in replication field compaction was studied. The model was developed using established visco-elastic-plastic representation of asphalt mixes and for the first time, extended such modelling to the study of field compaction of asphalt pavements.

Numerical simulations show that this approach captures the compaction process well and can be used to study the performance of different type of mixes used for the construction of asphalt pavements. The approach presented in this paper is a first step towards the development and testing of closed-loop techniques for intelligent compaction of pavements.

The model used in this paper was derived assuming rigid base, fixed contact area between roller drum and asphalt pavement, and constant speed of the roller during compaction. The effects of shear flow of asphalt as well as the effects of confinement at the edges were also not considered. Future research is aimed at relaxing these assumptions.

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