Fuzzy Inference System to Analyze Ordinal Variables
The Case of Evaluating Teaching Activity

Michele Lalla[1], Davide Ferrari[2] and Tommaso Pirotti[1]
[1]*Marco Biagi Economics Department, University of Modena and Reggio Emilia, Modena, Italy*
[2]*Department of Mathematics and Statistics, University of Melbourne, Melbourne, Australia*

Keywords: Ordinal Scales, Likert Scale, Student Evaluation, Fuzzification, Defuzzification.

Abstract: The handling of ordinal variables presents many difficulties in both the measurements phase and the statistical data analysis. Many efforts have been made to overcome them. An alternative approach to traditional methods used to process ordinal data has been developed over the last two decades. It is based on a fuzzy inference system and is presented, here, applied to the student evaluations of teaching data collected via Internet in Modena, during the academic year 2009/10, by a questionnaire containing items with a four-point Likert scale. The scores emerging from the proposed fuzzy inference system proved to be approximately comparable to scores obtained through the practical, but questionable, procedure based on the average of the item value labels. The fuzzification using a number of membership functions smaller than the number of modalities of input variables yielded outputs that were closer to the average of the item value labels. The Center-of-Area defuzzification method showed good performances and lower dispersion around the mean of the value labels.

1 INTRODUCTION

The purpose of the present paper is twofold. Firstly, it briefly discusses the limits of some statistical indices in representing synthetically ordinal variables, which constitute the current or traditional procedures. Secondly, the fuzzy approach, based on the FIS, is applied to data concerning student evaluations of teaching activity (SETA). In fact, our data set contains prevailingly ordinal information from an online survey conducted by the University of Modena and Reggio Emilia, for the Academic Year 2009/2010. The data analysis was restricted to the evaluated courses in the "Economics and International Management" degree program of the Faculty of Economics. The fuzzy approach through FIS offers a clear advantage over traditional methods because it is highly flexible in handling data and avoids the usual complications related to measurement methodology. For example, the FIS permits handling of both the four- or five-point Likert scale and any other type of scale without theoretical difficulties and great flexibility with a large variety of solutions. A comparison between the results obtained from current procedures and the proposed FIS will be analyzed and described, illustrating the strengths and weaknesses of both. A fuzzy inference model may well be a new and different way to analyze ordinal variables and, specifically, student evaluations of teaching activity.

2 ORDINAL SCALES

The objective of the measurements process is to obtain information that is valid (i.e., it succeeds in evaluating what it is intended to evaluate), reliable (i.e., the results can be reproduced upon replication of the procedure, yielding identical or very similar values), and precise (i.e., the multiples or submultiples of the unit of measurement are contained by the available device). Given that it is not always possible to establish or find the unit of measurement of social concepts, preciseness remains a real difficulty of each intensity concept's evaluation, which is generally classified on the basis of its nature and preciseness (Stevens, 1946), where the lowest level is based on discrimination (nominal) and the subsequent level is based on an order relation (ordinal). The basic assumptions of almost all ordinal scales are (1) the unidimensionality of the surveyed concept, (2) the location of the concept on a *continuum*, (3) the equidistance between the modalities constituting the observable level of the intensity of the concept.

Many techniques of scales have been developed

Lalla M., Ferrari D. and Pirotti T..
Fuzzy Inference System to Analyze Ordinal Variables - The Case of Evaluating Teaching Activity.
DOI: 10.5220/0005054400250036
In *Proceedings of the International Conference on Fuzzy Computation Theory and Applications* (FCTA-2014), pages 25-36
ISBN: 978-989-758-053-6

since the 1920s to study attitudes and, to a lesser extent, psychophysical and psychometric behavior (White, 1926; Thurstone, 1927; Thurstone, 1928). The ordinal scales most used in practice are 'summated' scales and one of the first successful procedures to obtain an ordinal variable, whose values denote the intensity level of its denoted concept, was proposed by Likert (1932) to measure attitudes and opinions through statements. The intensity of each statement was rated with graduated response keys (modalities), originally seven: strongly agree, mildly agree, agree, uncertain, disagree, mildly disagree, and strongly disagree (seven-point Likert scale). Subsequently, the alternatives containing "mildly" were dropped, obtaining a five-point scale. The neutral point presents a theoretical and empirical, unsolved issue because many results do not give strong indications about the advisability of its presence/absence. It is often eliminated (Schuman and Presser, 1996).

Let i be an index denoting the interviewed subject. Let j be an index denoting a concept, and k, a statement or item about the j-th concept. The corresponding score, y_{ijk}, belongs to $\{1,\dots M\} \subset \mathbb{N}$ for any statement favorable to the concept and it belongs to $\{M,\dots 1\} \subset \mathbb{N}$ for any statement not favoring the concept, where M is the number of points of the scale (5 or 7) and $\mathbb{N}$ is the set of natural numbers. The j-th concept is often measured through K_j items (variables), forming a battery and semantically connected to it. Each item, k, has a Likert scale with M_k modalities, in general, but often M_k is the same for all items. The answer of the i-th respondent gives an outcome x_{ijk} in $(1,2,3,4\,[,5,6,7])$. The sum (x_{ij}) or the mean $(\bar{x}_{ij})$ of the K_j natural numbers yields a measure of the intensity of the j-th concept

$$\text{(a)} \quad x_{ij} = \sum_{k=1}^{K_j} x_{ijk} \quad \text{or}$$
$$\text{(b)} \quad \bar{x}_{ij} = (1/K_j)\sum_{k=1}^{K_j} x_{ijk} \qquad (1)$$

The sum is sometimes rescaled to one (or ten), y_{ij}, through the expression $y_{ij} = (x_{ij} - x_{\min\ j})/(x_{\max\ j} - x_{\min j})$, for the i-th individual and the j-th concept, where the $x_{\min\ j}$ and $x_{\max\ j}$ are, respectively, the maximum and the minimum of x_{ij} in the data set. But, this calculus is not admissible as the average and the sum because the device generates only ordinal data.

The semantic differential scale is another ordinal scale (Osgood, 1952; Osgood et al., 1957) and in its usual or standard format, it consists of a set of seven categories, but they may vary in number, associated with bipolar adjectives or phrases. For each bipolar item, the respondent indicates the extent to which one descriptor represents the concept under examination. The semantic differential scale is aimed at measuring direction (with the choice of one of two terms, such as 'useful' or 'useless') and extent/amount (by selection of one of the provided categories expressing the intensity of the choice). The volume of measurements is generally high and the interpretation of the results of word scales is theoretically based on three factors ('evaluation', 'potency', and 'activity'), which involves fairly complex analyses requiring expensive data-processing procedures. Therefore, the objectives of these theoretical scales may necessarily involve long-term research, limiting their applicability or often subjecting them to simplified analysis and thus reducing some of their potential (Yu et al., 2003).

The Stapel scale is a ten-point non-verbal rating scale, ranging from +5 to -5 without a zero point and measuring direction and intensity simultaneously. It has been stated that *"it cannot be assumed that the intervals are equal or that ratings are additive"* (Crespi, 1961), but the Stapel scale is used under the same assumptions as the Likert scale. With respect to semantic differential, the Stapel scale presents each adjective or phrase separately and the points are identified by number. The use of a ten-point scale is more intuitive and common than the seven-point scale.

The self-anchoring scale is another type of ordinal scale and, in its usual or standard format, it consists of a graphic, non-verbal scale, such as the ten-point ladder scale (Kilpatrick and Cantril, 1960; Cantril and Free, 1962), where respondents are asked to define their own end points (anchors). The best is at the top, if the ladder is in vertical position (case 1), or at the right, if the ladder is in horizontal position (case 2). The worst is at bottom in the first case and on the left in the second case. It is a direct outgrowth of the transactional theory of human behavior in which the 'reality world' of each of us is always to some extent unique, the outcomes of our perceptions being conceived as ongoing extrapolations of the past related to sensory stimulation. The scale may solve some problems and biases typical of category scales, but it is often used as fixed anchoring rating scale, where the anchor of the scale is already defined, assuming, implicitly, the existence of an objective reality.

The feeling thermometer scale was developed by Clausen for social groups and was first used in the American National Election Survey (ANES), 1964. It was later modified by Weisberg and Rusk (1970). Its format is like a segment of a 0-to-100-degree temperature scale, which reports some specific values. In the evaluation of political candidates, it was *"a card listing nine temperatures throughout the scale range and their corresponding verbal meanings as to intensity of 'hot' or 'cold' feelings was handed to the respondent"* (Weisberg and Rusk, 1970).

Roughly speaking, the Stapel, self-anchoring, and feeling thermometer scales are structurally similar to thermometer scales that have a long history, although they are often ascribed to Crespi (1945a, 1945b) as cited, for example, by Bernberg (1952). However, the thermometer scales used in social sciences do not provide values on interval scales, as does a thermometer used to measure temperatures.

Among other ordinal scales, the Guttman scale is a method of discovering and using the empirical intensity structure among a set of given indicators of a concept. The Bogardus social distance scale measures the degree to which a person would be willing to associate with a given class of people - such as an ethnic minority (Babbie, 2010). The Juster scale is an 11-point scale, like a decimal scale, used for predicting the purchase of consumer durables and for each question asking people to assign probabilities to the likelihood of their adopting the described behavior on that question (Juster, 1960; Juster, 1966).

There is no rationale in the practice of handling the figures assigned to the modalities of an ordinal scale as real numbers, also under the assumption of equidistance between the categories. It is possible to envisage the selection of a modality as an output of a normal variable underlying a random discriminatory process, which could justify the use of equations (1) exploiting the properties of the normal random variables. However, if the modalities are subordinate only to a relation order, the use of the sum and the mean remains problematic.

3 STUDENT EVALUATION OF TEACHING ACTIVITY

The students' opinions about teaching activity rose to the attention of the academic administrations in the 1920s and some US universities such as Harvard, the University of Washington and Texas, Purdue University, and other institutions, introduced student evaluations as a standard practice (Marsh, 1987). Since then, many aspects have been investigated, such as the reliability, validity, unbiasedness, efficiency, and efficacy of SETA. Moreover, many more universities in the US and other countries have introduced the practice of evaluating teachers and course organization.

3.1 The Course-evaluation Questionnaire

In Italy, the evaluation of university teaching activities and research is regulated by Law no. 370 (of 19/10/1999, Official Gazette, General Series, no. 252 of 26/10/1999), which also does not allow administrations failing to comply with it to apply for certain grants. The same law established the National Committee for University System Evaluation (Comitato Nazionale di Valutazione del Sistema Universitario, CNVSU), replacing the Observatory for University System Evaluation. A research group of the CNVSU (2002) proposed a standard course evaluation questionnaire with a minimum set (battery) of fifteen items for all universities. Each item includes the following four-point Likert scale: ① Definitely no, ② No rather than yes, ③ Yes rather than no, ④ Definitely yes (CNVSU scale). A traditional item-by-item analysis was generally carried out, using means and variances of numerical values obtained by translating the categories (or labels) into a ten-point scale as follows: ①=2, ②=5, ③=7, ④=10, hereinafter referred to as numeric values of labels. One could argue that the absence of a mid value on this ordinal scale could violate the linearity assumption and the mean and variance analysis cannot validly be used. Moreover, the meaning of the labels might not be clear to all students. Hence, the intensities, or degree of certainty, associated with these labels often correspond to a high level of vagueness; investigations of these topics are reported elsewhere (Lalla et al., 2004).

In the Academic Year 2004/2005, the Committee for Technical Evaluation of the University of Modena and Reggio Emilia adopted the questionnaire proposed by CNVSU (2002) and in the Academic Year 2005/2006, it introduced the online survey for SETA (Lalla and Ferrari, 2011). Some minor changes involved a slight modification of the wording of some items (to make their meaning clearer) and the order of the items (to reduce the halo effect). The overall questionnaire contained seven sections, but Sections I (*personnel data*, containing information about the course, teacher, and some student characteristics) and VII (*remarks and suggestions*, listing nine items with dichotomous choices) are not presented here. Sections from II to VI represent the core of the questionnaire and contain a 15-item battery with the four-point Likert scale to achieve the standard evaluation (Table 1). The current procedure generates the evaluation of a single item or domain, which is performed using the traditional procedure of averaging (over the sample) the numerical labels or the assigned values corresponding to a ten-point scale

$$\bar{x}_{jk} = (1/n)\sum_{i=1}^{n} x_{ijk} \qquad (2)$$

Although the items included in a domain are assumed to have the same importance in the arithmetic mean, one could argue that certain variables indicating the efficiency of the course are more important than oth-

Table 1: Questionnaire items with median (md), mean ($\bar{x}$), standard deviation (σ), and number of valid cases (n) for the "Economics and International Management" degree program: Academic Year 2009/2010.

Questionnaire items – Total n=4537	Acronym	md	$\bar{x}$	σ	n
S. II Organization of this course					
I01: Adequacy of the Work Load required by the course	awl	7	6.5	2.2	4500
I02: Adequacy of the Teaching Materials	atm	7	7.3	2.0	4472
I03: Usefulness of Supplementary Teaching Activity (STA)	usta	7	7.2	2.1	2503
I04: Clarity of the Forms and rules of the Exams	cfe	7	7.3	2.2	4429
S. III Elements concerning the teacher					
I05: Reliability of the Official Schedule of Lectures	rosl	7	8.0	2.1	4460
I06: Teacher Availability for Explanations	tae	7	7.8	1.9	4442
I07: Motivation and Interest generated by Teacher	mit	7	7.2	2.2	4449
I08: Clarity and Preciseness of the Teachers Presentations	cptp	7	7.4	2.1	4427
S. IV Lecture room and resource room					
I09: Adequacy of the Lecture Room	alr	7	7.2	2.1	4444
I10: Adequacy of the Room and Equipment for STA	aresta	7	7.2	2.1	2475
S. V Background-interest-satisfaction					
I11: Sufficiency of Background Knowledge	sbk	7	7.0	2.0	4442
I12: Level of Interest in the Subject matter	lis	7	7.4	2.1	4444
I13: Level of Overall Satisfaction with the course	los	7	7.1	2.1	4418
S. VI Organization of all courses in the degree program					
I14: Adequacy of the Total Work Load of current courses	atwl	7	6.0	2.2	4452
I15: Feasibility of the Total Organization (lect. & exams)	fto	7	6.1	2.2	4445

ers. To take into account these differences, one approach is to use a weighted average. Still, the choice of the weights is a controversial point due its arbitrariness. Moreover, it could be noted that the median, which is the correct statistical index for ordinal variables, is less informative of the mean in summarizing the distribution of the answers, as is evident, but questionable, from the data reported in Table 1.

4 FUZZY INFERENCE SYSTEMS

The structure and functioning of a FIS follow defined hierarchical steps (Dubois and Prade, 2000).

4.1 The FIS for Teaching Evaluation

Issue identification (i) involves a stepwise procedure that could be carried out through a top-down or bottom-up strategy. In the first case, it is similar to a scientific inquiry. It starts from the output variables and attempts to identify macro-indicators - possibly including multiple input variables - that can adequately explain the output. The macro-indicators are subsequently broken down into smaller indicators that include fewer variables. The stepwise process continues until the single input variables are isolated. The final product is a modular tree-patterned system, where several fuzzy modules are interlinked. In the bottom-up strategy, the input variables are already available, as is the case examined here, and the goal consists in subsequently aggregating them. Each aggregation generates a fuzzy module. The various modules are interlinked with each other. The final emerging arrangement is like a tree-patterned structure generating a single (or multiple) output(s). An example of the latter is the model considered for SETA, which includes the 15-item battery and item aggregation, from input to final output (Figure 1). The variables enter the system at different levels of importance, which heavily affects the final output. Roughly speaking, the approach corresponds to a weighted average, where the weights are unknown and higher for variables entering the last steps. The aggregating function is generally unknown and not necessarily linear, as in a weighted average. In the FIS, each aggregation of variables gives an intermediate solution, which is a fuzzy set variable and it does not have necessarily a particular meaning attached to it. Sometimes, however, the intermediate variables do have a useful meaning. For example, the aggregation of AWL and ATM generates the new intermediate fuzzy variable OMW (Organization of Materials and Workload), while the aggregation of USTA and CFE generates the new intermediate fuzzy variable OEE (Organization of Exercises and Exams). In a subsequent step, the aggregation of OMW and OEE generates OC (Organization of the Course). The merging of fuzzy modules will continue up to the aggregation of OC with TTC (Total Teaching Capability), obtaining SET (Student Evaluation of Teacher). To account for student satisfaction, SET is aggregated with LOS, the level of overall satisfaction of students, obtaining SETS (Student Evaluation of Teacher plus Satisfaction). The adopted level of inclusion involves a strong influence of satisfaction on SETS, while a mitigation of its effect could be obtained, firstly, by combining TTC or OC with LOS and, secondly, by merging the result with OC or TTC, as reaffirmed in the comments on the results reported below.

The fuzzification of input, (ii), involves the specifications concerning the shapes and the number of the membership functions (mf) for the input variables. A membership function defines the extent to which each value of a numerical variable belongs to some specified categorical labels. In the following, we briefly describe the most popular approaches used to determine their shapes (Smithson, 1988). (1) The survey approach determines the shapes of the membership functions based on information from a specifically designed sample. For this purpose, a common choice is to use empirical sampling distributions from the particular collected data concerning the intensity of the value labels. (2) The comparative judgment approach defines the functions through a comparison of stimuli and some given features or prototypes. (3) The expert scaling approach characterizes the function in accordance with the subjective experience of an expert. (4) The formalistic approach selects functions with specific mathematical properties. (5) Machine-learning builds up the functions from a set of past data (training and testing data set) and transfers the same structure to the present and future.

The aim of SETA is to collect student opinions about the teachers and the course organization. Therefore, approach (1) is preferable because it allows for direct measurement of the meaning that students attribute to the linguistic options/categories on the 15-item battery. In particular, each sampled student should assign scores for each category of the CNVSU scale denoted by a label, for each of the 15-items. This permits construction of frequency polygons, which give approximate representations of the vagueness level of the category choices. The polygons translate the decimal value of each category into a corresponding membership level for the population. This strategy could be costly, as all surveys are costly, and it presents some degree of difficulty due to the nuisance, laboriousness, and repetitiveness of the task. Actually, each student should assign scores for 15×4 elements. In fact, the evaluation of the same four response options is repeated fifteen times, the

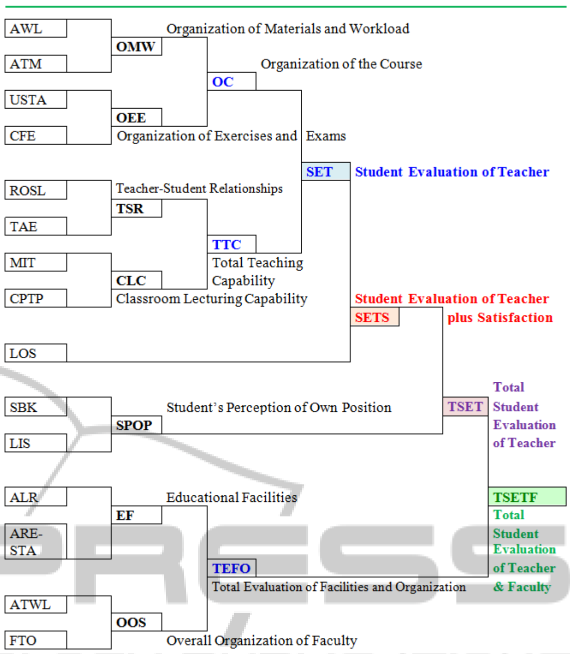


Figure 1: The structure of the Fuzzy Inference System for student evaluation of teaching activity.

number of items on the CNVSU questionnaire. As a consequence, the procedure requires a large sample of students and a well-designed strategy for data collection. Moreover, given that the empirical frequency polygons are somewhat irregular, their final shapes could be determined taking into account method (2) in order to smooth and simplify the forms of the frequency polygons with respect to both the theoretical constraints and the aims of the FIS. In general, the frequency polygons are well fitted by probability distribution, such as normal, gamma, and beta, but they are also well approximated by triangular (a, α, β) or trapezoidal (a, b, α, β) shapes centered about the means of the score distributions for each modality. With respect to other strategies, approach (3) is inadequate for teaching evaluation because the experts' opinions might not match those of students. Approach (4) does not fit our purposes simply because a priori mathematical properties do not necessarily fit the reality of our data (although in many situations they produce smoothed and tractable functions). Finally, approach (5) also appears to be inappropriate because past data on the numerical values for the scale categories are not available and are probably not time-invariant.

The membership functions of some input variables for the FIS, in Figure 1, could be deduced from the survey carried out in October 2000, where the modalities of eight items were evaluated by students using a decimal scale (Lalla et al., 2004; Lalla and Facchinetti, 2004). Only eight items (seven of them about the teacher) were available out of fifteen, but the scores refer to different formats and ten years have already passed. Therefore, a simple fuzzification was adopted, assuming as membership functions trapezoidal or triangular shapes and considering the symmetry about some specific values in the decimal scale range or the value labels of the modalities (Grzegorzewski and Mrówka, 2005; Grzegorzewski and Mrówka, 2007; Grzegorzewski, 2008; Yeh, 2009). In fact, the relative frequency polygons could be well approximated by triangular shapes centered about the means of the score distributions for each category. The triangularization of the membership functions of the input variable is a common practice, but it involves a right-angled triangle for the first and the last modality, i.e., the first and the last triangle. Hence, the following types of fuzzification were considered. The first type used three membership functions (mf3); this number is lower then the number of modalities (4) to allow the activation of two membership functions for the internal modalities. The triangular fuzzy numbers (a, α, β) had peaks $a = (2, 6, 10)$ and the left width α was equal to the right width β, i.e., $\alpha = \beta = 4$, as in Figure 2 generated by "fuzzyTECH" (N.N., 2007). The domain of the membership functions ranged from 2 to 10 and the response of the FIS was restricted to the interval $[2, 10]$ because the traditional means of the numerical values attributed to the CNVSU scale categories, $D = \{2, 5, 7, 10\}$, clearly ranged from 2 to 10. Note that there are also many other possible choices and modifications of the shapes for improvement of the performances of the FIS.

There is also the possibility of using trapezoidal fuzzy numbers. For example, the first membership function is a trapezoidal fuzzy number, (a, b, α, β), with peak $(a = 2, b = 3)$, left width $\alpha = 0$ and right width $\beta = 3$. The second function is triangular shaped (a, α, β), with peaks $a = 6$, left width $\alpha = 3$, and right width $\beta = 3$. The third, which is also the last, would be a trapezoidal fuzzy number again (a, b, α, β), with peak $(a = 9, b = 10)$, left width $\alpha = 3$ and right width $\beta = 0$. The structure with two trapezoids, at the extremes of the decimal scale, could emphasize the FIS scores towards the upper and lower bounds of the support (Lalla et al., 2004) in some defuzzification methods. However, it was not used here.

The second type used four membership functions (mf4). This number was equal to the number of modalities, implying that, in the absence of any other information out of the symmetry and the range of numeric values of the CNVSU scale categories, the triangular fuzzy numbers (a, α, β) had peaks coinciding

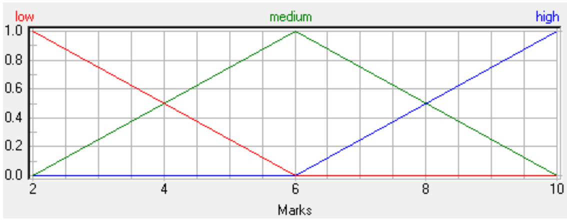


Figure 2: Fuzzification of an input variable using three triangles as membership functions (mf).

with the numeric values $a = (2, 4.\bar{6}, 7.\bar{3}, 10)$ and the left width α was equal to the right width β, i.e., $\alpha = \beta = 2.\bar{6}$, as generated by "fuzzyTECH" (N.N., 2007) in Figure 3. In this case, the most natural pattern may be a fuzzification with peaks in $a = (2, 5, 7, 10)$ and different values of the left width α and the right width β. In other terms, the membership function associated with a fixed modality is represented by a triangle with the peak centered on its value on the scale and the amplitude ranging from the first lower to the first upper modality. Therefore, it tends to confine the results to the selected modalities and the FIS does not work completely, but only through the rule-blocks, thus partially losing its nature.

In formal terms, the indices j and k of x_{ijk} are summarized in a single index l to simplify the formalism. Therefore, let x_{il}, $l = 1, \ldots, L$, be the input variables ($L = 15$ in the examined case) provided by the $i-$th student with range U_l and let y be the output variable with range V. Let $M(l)$ be the number of categories of x_l. Generally, such a number could change from one variable to another, but in this case, $M(l) = 4$ (the number of CNVSU scale options/categories) for all $l = 1, \ldots, L$. Therefore, in general, an effective fuzzification of input requires a number of membership functions greater than one and less than $M(l)$. Moreover, each category of x_l is described by a fuzzy number, $A^l_{j(l)}$, $\forall j(l) \in [1, \ldots, M(l)]$, and the set $A^l = \{A^l_1, \ldots, A^l_{M(l)}\}$ denotes the fuzzy input x_l, while the fuzzy output of y is defined by $B = \{B_1, \ldots, B_{M(y)}\}$ where $M(y)$ denotes the number of membership functions (or categories or modalities) for y. Each set has a membership function:

$$\mu_{A^l_{j(l)}}(x) : U_l \to [0, 1] \qquad \mu_{B_m}(x) : V \to [0, 1] \qquad (3)$$

The construction of rule-blocks, (iii), concerns the relationships between the input linguistic variables and output linguistic variables. It involves a multi-criteria situation, described by a number of rules like:

$$R_s : \text{IF } [x_1 \text{ is } A^1_{j(1)} \otimes \ldots \otimes x_L \text{ is } A^L_{j(L)}] \text{ THEN } (y \text{ is } B_m) \quad (4)$$

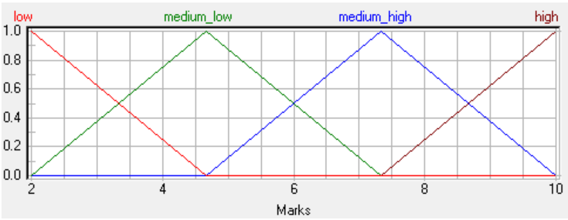


Figure 3: Fuzzification of an input variable using four triangles as membership functions (mf).

for all combinations of $j(l) \in [1,\ldots,M(l)]$ and $m \in [1,\ldots,M(y)]$. The left-hand side of THEN is the antecedent (premise) and the right-hand side is the consequent (conclusion). The symbol $\otimes$ (otimes) denotes an aggregation operator, one of several t-norms (if the aggregation is an AND operation) or t-conorms (if the aggregation is an OR operation). The aggregation operator, AND, that was chosen, produces a numerical value $\alpha_{s,m} \in [0,1]$ and the latter represents the execution of the antecedent in rule R_s. The $\alpha_{s,m}$ number should be applied to the consequent membership function of B_m, in order to calculate the output of each rule. The AND aggregation operator was used once again, but in a slightly different context: $\otimes$ works on a number and the membership function of a fuzzy set B_m, whereas in the case of the R_s rule, it is applied on two numbers (Von Altrock, 1997). An example of the rule-block is presented for the fuzzy module OMW (organization of material and workload) aggregating AWL and ATM (Figure 1) and using the numeric values instead of labels, for the sake of brevity (three membership functions):

- IF AWL is mf1 and ATM is mf1, THEN OMW is mf1
- IF AWL is mf1 and ATM is mf2, THEN OMW is mf2
- IF AWL is mf1 and ATM is mf3, THEN OMW is mf3
- IF AWL is mf2 and ATM is mf1, THEN OMW is mf2
- IF AWL is mf2 and ATM is mf2, THEN OMW is mf3
- IF AWL is mf2 and ATM is mf3, THEN OMW is mf4
- IF AWL is mf3 and ATM is mf1, THEN OMW is mf3
- IF AWL is mf3 and ATM is mf2, THEN OMW is mf4
- IF AWL is mf3 and ATM is mf3, THEN OMW is mf5.

It is possible to generate these rules automatically through an algorithm, but an expert may express them in a form that more closely fits the reality.

The aggregation of rule-blocks, (iv), is the step of the evidential reasoning incorporating the process of unification of the outputs of all the rules in a single output Y. For every rule (R_s) involved in the numerical inputs, $\mu(\alpha_{s,m} \otimes B_m)$, a different output is obtained. These membership functions of fuzzy sets have to be aggregated by an *OR* operation using a t-conorm. The most frequently used are the max conorm, the probabilistic conorm, and the Lukasiewicz conorm usually known as the bounded sum. Now, the response of a module is ready, but it is still in a fuzzy form. If the module needs to be aggregated with other modules, the aggregation process continues. Otherwise, it is an output module, even if it is not the last output module, implying that it needs to be changed back into a number to provide an easy understanding of the system response.

Defuzzification of output, (v), is the process that maps the output fuzzy set $\mu_B(y)$ into a crisp value, y_{ij}, for the i-th student and j-th fuzzy module or concept; i.e., it concentrates the vagueness expressed by the polygon resulting from the activated output membership functions into a single summary figure that best describes the central location of an entire polygon. There is no universal technique to perform defuzzification, i.e., to summarize this output polygon by a number, as each algorithm exhibits suitable properties for particular classes of applications (Van Leekwijck and Kerre, 1999). The selection of a proper method requires an understanding of the process that underlies the mechanism generating the output and the meaning of the different possible responses on the basis of two criteria: the "best compromise" and the "most plausible result". Moreover, in an ordinal output, with modalities described by linguistic expressions, their corresponding real values are always given by the membership definitions, where the understanding of their meaning plays a key role.

For the first criterion, one of the most popular methods is the Center-of-Maxima (CoM), which yields the best compromise between the activated rules (Von Altrock, 1997). Given that more than one output membership function could be activated or evaluated as a possible response for the i-th student and j-th fuzzy module or concept, let $M_{F;ij}$ be the number of output- activated membership functions. Let y_{ijm} be the abscissa of the maximum in the m-th activated output membership function. If the latter has a maximizing interval, y_{ijm} will be the median of this interval. The final output crisp value, $y_{CoM;ij}$, is given by an average of membership maxima weighted by their corresponding level of activation, $\mu_{out;k}$,

$$y_{CoM;ij} = \sum_{m=1}^{M_{F;ij}} \mu_{out;m} y_{ijm} \Big/ \sum_{m=1}^{M_{F;ij}} \mu_{out;m} \qquad (5)$$

The method of the center of area/gravity (CoA/G) was excluded because it cannot reach the extremes of the range $[2,10]$ without fuzzification of input and output on an interval wider than $[2,10]$, which may seem unnatural. In fact, it would have been possible to pick a more suitable fuzzification of the input, but, in general, FIS might produce an output greater than the maximum or lower than the minimum of the scale.

For the second criterion, the Mean-of-Maxima (MoM) method yields the most plausible result, determining the system output only for the membership function with the highest resulting degree of the support. If the maximum is not unique, i.e., it is a maximizing interval, the mean of the latter is the response

$$y_{MmM;ij} = \max_{l<=m<=M_{F;ij}} (y_{ijm}) \quad (6)$$

This approach selects the typical value of the terms that is most valid, instead of balancing out the different inference results (Von Altrock, 1997). Therefore, it is often used in pattern recognition and classification applications, as in the case of an ordinal output whose modalities are described by linguistic expressions, because the most plausible solution is more appropriate instead of the mean.

In addition, the sensitivity analysis, (vi), is a possible sixth step that may be carried out to adapt the FIS to the real situations that it would represent. The FIS is handled as a parametric model, relating input variables to membership functions, to fuzzy rules, to hedges operations, to aggregations, and so on.

5 EMPIRICAL RESULTS

The academic year 2009/2010 was fixed as the reference date and the degree program in Economics and International Management was selected out of three undergraduate degree programs of the Faculty of Economics. Some restrictions were imposed to the overall dataset of 4537 (evaluating) students (Table 1), even if some analyses suggesting such restrictions are not reported here for the sake of brevity. The course-teacher is a unique combination because the same teacher may teach more than one course and in the same course there may be more than one teacher. Five courses had less than twenty evaluating students and these courses were thus eliminated, leading to a reduction of 55 cases.

The complete elimination of nonresponses is not the ideal strategy as it is too costly, it implies a loss of cases and biases the estimates because nonresponses do not randomly occur. However, considering the nature of SETA data, if all items concerning the teacher (I01-I08, I13), or all items concerning the organization (I09, I10, I14, I15), were missing in a case, that case was dropped. Fifty-four cases were lost through this control, prevailingly owing to missing values for all items referring to the teacher. Background knowledge (I11) was used to replace the level of interest in the subject matter (I12) when the latter was missing and vice versa; if both were missing, they were replaced with the mean of teacher items. Therefore, they did not involve a loss of cases. If in the 15-item battery there were more than 8 (threshold) missing values in a single case (student), that case was dropped: 17 cases crossed the threshold. In the end, a total of 4411 cases were used.

The remaining missing values were replaced on the basis of the available data for each single case and considering that an evaluating student expressed an opinion about three main areas (Figure 1): Student evaluation of teacher plus satisfaction (SETS), student's perception of own position (SPOP), and total evaluation of facilities and organization (TEFO). For each student, i, the k-th item belonging to a certain area with a missing value was replaced with the mean of the values for the non-missing items of the same area provided by the same student and not by the mean of the k-th item for the total sample, as is usual. For example, let I02(i), which belongs to SETS, be missing; it was then replaced by the mean of [I01(i), I03(i), I04(i), I05(i), I06(i), I07(i), I08(i), I13(i)]. Let I02(i) and I13(i) be missing; they were then replaced by the mean of [I01(i), I03(i), I04(i), I05(i), I06(i), I07(i), I08(i)]. The rationale of this procedure relies on the core of the evaluation process, which is the evaluator. Therefore, the value used in the substitution is anchored to his/her average level of judgment and not to the average level of the total sample. The number of replaced values varied from one item to another, ranging from 0.1% to 1%, except for supplementary teaching activity (I03) and adequacy of the room and equipment for the supplementary teaching activity (I10), as those activities were not always present in a course, implying an obvious high rate of absence of evaluations for that particular item.

5.1 Student Evaluations of Teachers

The analysis has been prevailingly restricted to the subsystems concerning the student evaluations of teachers (SET) and SET plus satisfaction (SETS), as indicated in Figure 1, for the sake of brevity and owing to the possibility to simulate the input data, as indicated below. The traditional evaluation of teachers currently in use, from the i-th student, is given by the mean of the value labels assigned to the four modalities of each item:

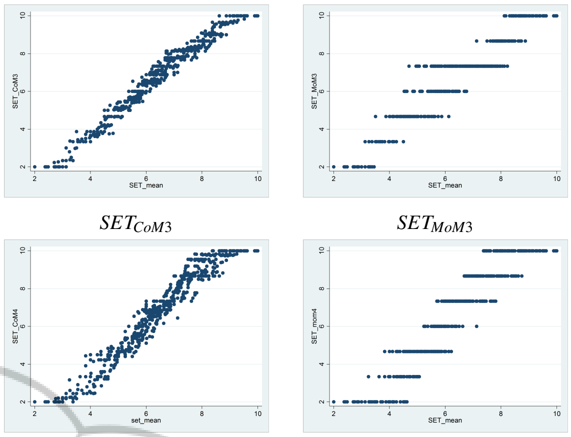
$$\text{(a)} \quad \bar{x}_{set;i} = (x_{01;i} + \cdots + x_{08;i})/8$$
$$\text{(b)} \quad \bar{x}_{sets;i} = (x_{01;i} + \cdots + x_{08;i} + x_{13;i})/9 \quad (7)$$

The output of a FIS, $x_{FIS;i}$, depends on the decisions taken at each step. Specifically, the CoM method used in the defuzzification step, with 3 or 4 membership functions in the fuzzification of input, CoM3 or CoM4, generated $x_{CoM3;i}$ and $x_{CoM4;i}$, respectively. Analogously, the MoM method used in the defuzzification step, with 3 or 4 membership functions in the fuzzification of input, MoM3 or MoM4, generated $x_{MoM3;i}$ and $x_{MoM4;i}$, respectively.

The rank of course-teachers may be a useful tool to identify critical situations, where to offer suggestions to the teacher or to urge him/her to improve his/her behavior, the scope of the program, the teaching materials, and so on. For this purpose, an item-by item analysis could help persons in charge of academic organization and/or teachers, but here the results are limited only to the overall evaluation of the teacher. The first and last positions of the rank are reported in Table 2. The mean of the value labels, $\bar{x}_{set}$, was lower than the fuzzy outputs ($\bar{x}_{CoM3}$, $\bar{x}_{MoM3}$, $\bar{x}_{CoM4}$, $\bar{x}_{MoM4}$). The CoM3 fuzzy evaluations were higher than those obtained by the mean of the value labels and the mean of differences was 0.55, with the lowest standard deviation (sd) being 0.45. The MoM3 provided crisp values that were close to CoM3 evaluations and closer to $\bar{x}_{set;i}$. In fact, the mean difference was 0.41 (sd=0.63). Assuming the mean of the value labels as a benchmark, $\bar{x}_{set;i}$, the differences proved to be slightly higher than 5%, on the average. Moreover, the fuzzification with 4 membership functions did not work as well as the fuzzification with 3 membership functions because the outputs generally showed an increase in the differences with respect to $\bar{x}_{set;i}$. Opposite results were obtained by CoM4 and MoM4, i.e., the CoM4 fuzzy evaluations yielded crisp values closer to $\bar{x}_{set;i}$ than those yielded by MoM4. In fact, the means of the differences with respect to $\bar{x}_{set;i}$ were 0.49 (sd=0.64) and 0.86 (sd=0.99), respectively.

The fuzzy outputs, $x_{FIS;i}$, and the mean of the value labels, $x_{set;i}$, for the i-th student measure the performance of a teacher. Therefore, they should be correlated and an analysis of the relationships between the different fuzzy outputs and $\bar{x}_{set;i}$ clarifies the structure of some differences. The scatter-plots of fuzzy evaluations of teachers (SET) against the mean of the corresponding value labels are reported in Figure 4. The estimates of the linear regression parameters between the four dependent variables ($x_{CoM3;i}$, $x_{MoM3;i}$, $x_{CoM4;i}$, $x_{MoM4;i}$) on $\bar{x}_{set;i}$, as the independent variable, are reported in Table 3. If $x_{FIS;i}$ and $\bar{x}_{set;i}$ are the same, one can expect a slope (β_1) of the regression line equal to 1 and an intercept (β_0) equal to 0. The correlative t-tests showed that these hypotheses were always rejected, but the relationships were always approximately linear. The assumption of constant variance was refused in all models and the coefficients of determination were sufficiently high.

The result closer to the hypotheses, notwithstanding their rejection, was given by CoM3, which showed residuals with the lowest standard deviation



SET_{CoM3} SET_{MoM3} SET_{CoM4} SET_{MoM4}

Figure 4: Output fuzzy variables against the mean of the value labels.

[sd(res-CoM3)=0.45] and a bimodal shape. MoM3 provided residuals with a notable dispersion [sd(res-MoM3)=0.63] and a bell-shaped distribution. Correspondence between the mean of the value labels and the fuzzy outputs by CoM4 and MoM4 was poorer in terms of the slope and the shapes or dispersion of residuals: sd(res-CoM4)=0.58 and sd(res-MoM4)=0.95. For the sake of brevity the figures were not reported here. Therefore, as expected, FIS works better when the number of membership functions for each x_{il} input variable is lower than its number of modalities, $M(l)$. Reasonably, the number of input membership functions for the x_{il} input variable should range from 2 to $[M(l)-1]$. Moreover, for fuzzy output ordinal variables, MoM is more suitable than CoM because it chooses the most plausible result among the possible $M_{F;ij}$ results.

5.2 Simulated Data: All Possible Inputs

The mean of the value labels and of the FIS outputs yielded measurements that often did not coincide, as noted regarding the results in Tables 2-3. The differences between the mean of the FIS outputs and the mean of the value labels were statistically different from zero for both the total sample and the single course teacher. However, the surveyed data did not present all the possible combinations of input values because many evaluation patterns were frequently repeated and others were never expressed by students. Therefore, the previous analysis was repeated using a simulated dataset, which contained all the possible combinations of the values of the input variables.

The generation of the dataset considered the output termed SETS in Figure 1, although the more attention was focused on SET. Given that for SETS there

Table 2: First and last three teachers in the rank obtained through the mean of the value labels ($\bar{x}_{set}$) with fuzzy outputs for different conditions (3 or 4 membership functions in the fuzzification of input, CoM and MoM methods in the defuzzification of output).

Order	Teacher	n	$\bar{x}_{set}$	$\bar{x}_{CoM3}$	$\bar{x}_{MoM3}$	$\bar{x}_{CoM4}$	$\bar{x}_{MoM4}$
1	Xy01	109	8.38	8.89	8.70	8.89	9.23
2	Xy02	21	8.33	8.71	8.60	8.72	9.05
3	Xy03	168	8.30	8.87	8.81	8.93	9.19
...	...	...	...	...	...	...	...
39	Xy39	90	6.33	6.78	6.68	6.56	6.92
40	Xy40	90	6.22	6.59	6.77	6.38	6.53
41	Xy41	121	6.16	6.61	6.56	6.32	6.73
	Total	4411	7.32	7.87	7.73	7.81	8.18

Table 3: Parameter estimates for the regression of fuzzy outputs on the mean of the value labels.

Dependent	β_0	$SE(\beta\_0)$	$t(b_0 = 0)$	β_1	$SE(\beta\_1)$	$t(b_1 = 1)$	R^2	Het^*
SET CoM3	0.367	0.031	11.72	1.026	0.004	6.12	0.932	0
SET MoM3	0.848	0.044	19.34	0.940	0.006	-10.28	0.854	0
SET CoM4	-0.711	0.041	-17.53	1.164	0.005	30.38	0.913	0
SET MoM4	-0.426	0.066	-6.41	1.176	0.009	19.83	0.799	0

$*$ Breusch-Pagan / Cook-Weisberg test for heteroskedasticity, where H_0 is constant variance.

were nine input variables and there were four modalities for each variable, the various possible combinations were given by four raised to nine (or 4 to the 9^{th} power) equal to 262144. Each combination corresponded to an evaluation of a potential student, which was different from the other 262143.

In the simulated dataset, the input variables are perfectly uncorrelated to each other, as each pattern appears once, while in the surveyed datasets there are often a correlation because the input variables are like paired variables or repeated measurements. The differences between the fuzzy outputs and the mean of the value labels have been plotted in Figure 5. Differing from the above results, CoM3 showed a distribution of residuals with an acceptable shape and it was more concentrated than other fuzzy outputs. The resulting differences were less marked than those observed in the surveyed data: $\bar{x}_{CoM3} - \bar{x}_{set} = 0.22$ (sd=0.44), $\bar{x}_{MoM3} - \bar{x}_{set} = 0.28$ (sd=0.65), $\bar{x}_{CoM4} - \bar{x}_{set} = 0.20$ (sd=0.78), $\bar{x}_{MoM4} - \bar{x}_{set} = 0.20$ (sd=0.93).

Analogously, the parameters of the regression between the fuzzy outputs ($x_{CoM3;i}$, $x_{MoM3;i}$, $x_{CoM4;i}$, $x_{MoM4;i}$), as dependent variables, on the mean of the value labels, $\bar{x}_{set;i}$, as the independent variable, were estimated (Table 4). The usual hypotheses about the parameters were tested, with the slope equal to 1 and the intercept equal to 0, and rejected. Again, the assumption of constant variance was refused in all models and the coefficients of determination were sufficiently high. In a different direction, the result closest to the hypotheses, notwithstanding their rejection, was given by MoM3, which showed a distribution of residuals with an acceptable shape, though less concentrated [sd(res-MoM3)=0.61] than in the case of CoM3 [sd(res-CoM3)=0.32]. Moreover, CoM3 showed a bimodal distribution. As for the surveyed data, the correspondence between the mean of the value labels and the fuzzy outputs by CoM4 and MoM4 was poorer in the slope, but CoM4 showed a better coefficient of determination and shape of the histogram of residuals than MoM4: sd(res-CoM4)=0.50, sd(res-MoM4)=0.66. However, this was not the same for SETS, in which the coefficients of determination decreased by about 50%. These substantial differences mainly depended on the structure of the tree reported in Figure 1, where satisfaction (LOS) is combined directly with SET involving a high weight of LOS on the fuzzy output for SETS and the absence of correlation between LOS and $\bar{x}_{set;i}$ increased the reduction of the determination coefficients. In fact, with the surveyed data, this reduction was not observed because it was negligible, for in that case, LOS was correlated with other input variables and the output of various fuzzy modules.

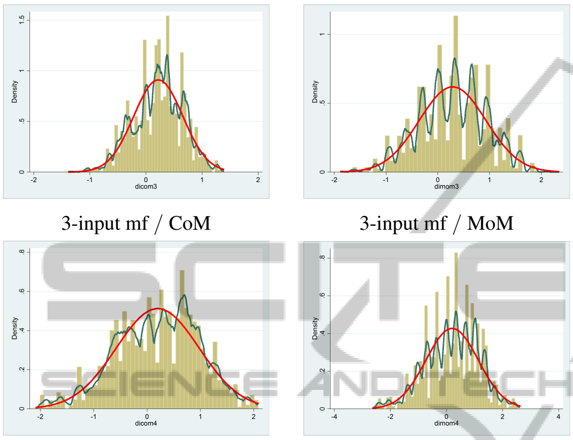
6 COMMENTS AND REMARKS

FIS offers the possibility of handling verbal terms via approximately quantitative values and avoids some methodological issues inherent in traditional procedures concerning the measurement of concepts and

Table 4: Parameter estimates for the regression of fuzzy outputs on the means of the value labels.

Dependent	β_0	SE(β_0)	t($b_0 = 0$)	β_1	SE(β_1)	t($b_1 = 1$)	$\mathcal{R}^2$	Het^*
SET CoM3	-1.54	0.004	-420.2	1.293	0.001	487.1	0.946	0
SET MoM3	-0.947	0.007	-134.6	1.205	0.001	177.7	0.806	0
SET CoM4	-3.622	0.006	-562.6	1.577	0.001	605.5	0.913	0
SET MoM4	-3.623	0.008	-473.1	1.637	0.001	506.7	0.866	0

$*$ Breusch-Pagan / Cook-Weisberg test for heteroskedasticity, where H_0 is constant variance.



3-input mf / CoM — 3-input mf / MoM — 4-input mf / CoM — 4-input mf / MoM

Figure 5: Distributions of differences between the fuzzy outputs and the mean of the value labels ($\bar{x}_{set;i}$).

the consequent limitation of statistical data analysis of ordinal, but also nominal variables. For example, the use of the mean (sample average) becomes irrelevant because the response of the FIS could be maintained as ordinal. However, if a numerical output is desired, as in the case-study presented here, then many problems still hold conceptually, but some of them are operatively irrelevant as the vagueness weakens the sharpness. In other words, by construction, the nature of the fuzzy inputs mitigates the certainty that we would normally have about the distances of the numbers on a given scale. Therefore, the issue concerning the value attributed to a modality (e.g. 7 assigned to "Yes rather than no" leading to questions like "Why 7 and not 7.5 or 6.5 or 8 or 6") is less restrictive because the fuzzification spreads the choice over the support, even if all choices affect the output. In any case, the FIS-based approach could represent a bridge between qualitative and quantitative analysis for a consistent treatment of concepts that are measured differently. This potentiality would be useful in many fields of application.

FIS, however, also presents difficulties at each construction step. In the identification of the issue (step *i*), the order in which the input variables are aggregated in the system affects the output. Particularly, input variables in the first nodes of the tree affect the output less than those forming the subsequent nodes. Moreover, the exponential explosion of the number of rules limits the input of fuzzy modules to two or three variables. The fuzzification of input (step *ii*) is not a straightforward step and leaves a kind of indeterminacy. The construction of block rules (step *iii*) is a subjective process open to criticism by all, as there is no rule to make rules. In fact, the heuristic fuzzy rules constitute a controversial issue. Certainly, the flexibility derived from these rules allows for adequately representing the actual phenomenon, but for this same reason, the choices of the decision-maker play a key role in the pattern of combinations involving the wording of the items. There are many methods and possibilities for the aggregation of block rules (step *iv*), but they must be selected on the bases of the knowledge of their functioning. Defuzzification (step *v*) also offers a large variety of techniques that might puzzle final users, although it can be seen as not being a part of the core of a FIS or of the fuzzy set theory (Van Leekwijck and Kerre, 1999).

Overall, the FIS generates reasonable and reliable results, showing remarkable flexibility and more manageability than the official evaluation systems, in spite of discrepancies with respect to the means of the value labels, which are the official results used by the persons in charge of academic organization. However, part of this manageability could originate from the arbitrary choices required by the construction steps, especially from the heuristic fuzzy rules (if – then rules) and from the fuzzy inference method (selection of aggregation's operators for precondition and conclusion). Despite some unavoidable degree of arbitrariness in some modeling choices, the results were satisfactory. The final outcomes resembled those of the traditional procedure, but the values were slightly higher than those of the official evaluations.

REFERENCES

Babbie, E. R. (2010). *Introduction to social research*. Cengage learning, Wadsworth (Belmont, CA), 12th edition.

Bernberg, R. E. (1952). Socio-psychological factors in industrial morale: I. the prediction of specific indicators. *The Journal of Social Psychology*, 36(1):73–82.

Cantril, H. and Free, L. A. (1962). Hopes and fears for self and country. *American Behavioral Scientist*, 6(2):4–30.

CNVSU (2002). Proposta di un insieme minimo di domande per la valutazione della didattica da parte degli studenti frequentanti. *Doc 09/02*, Retrieved from http://www.cnvsu.it:Accessed 28 July 2011.

Crespi, I. (1961). Use of a scaling technique in surveys. *The Journal of Marketing*, 25(July):69–72.

Crespi, L. P. (1945a). Public opinion toward conscientious objectors: Ii. measurement of national approval-disapproval. *The Journal of Psychology*, 19(2):209–250.

Crespi, L. P. (1945b). Public opinion toward conscientious objectors: Iii. intensity of social rejection in stereotype and attitude. *The Journal of Psychology*, 19(2):251–276.

Dubois, D. and Prade, H. E. (2000). *Fundamentals of fuzzy sets*. Kluwer Academic Publishers, Boston, MA.

Grzegorzewski, P. (2008). Trapezoidal approximations of fuzzy numbers preserving the expected interval – algorithms and properties. *Fuzzy Sets and Systems*, 159(11):1354–1364.

Grzegorzewski, P. and Mrówka, E. (2005). Trapezoidal approximations of fuzzy numbers. *Fuzzy Sets and Systems*, 153(1):115–135.

Grzegorzewski, P. and Mrówka, E. (2007). Trapezoidal approximations of fuzzy numbers – revisited. *Fuzzy Sets and Systems*, 158(7):757–768.

Juster, F. T. (1960). Prediction and consumer buying intentions. *The American Economic Review*, 50(2):604–617.

Juster, F. T. (1966). Consumer buying intentions and purchase probability: An experiment in survey design. *Journal of the American Statistical Association*, 61(315):658–696.

Kilpatrick, F. P. and Cantril, H. (1960). Self-anchoring scaling: A measure of individuals' unique reality worlds. *Journal of Individual Psychology*, 16(2 Nov):158–173.

Lalla, M. and Facchinetti, G. (2004). Measurement and fuzzy scales. *Atti della XLII Riunione Scientifica*, pages 351–362.

Lalla, M., Facchinetti, G., and Mastroleo, G. (2004). Ordinal scales and fuzzy set systems to measure agreement: An application to the evaluation of teaching activity. *Quality and Quantity*, 38(5):577–601.

Lalla, M. and Ferrari, D. (2011). Web-based versus paper-based data collection for the evaluation of teaching activity: empirical evidence from a case study. *Assessment & Evaluation in Higher Education*, 36(3):347–365.

Likert, R. (1932). A technique for the measurement of attitudes. *Archives of psychology*, monograph no. 140:1–55.

Marsh, H. W. (1987). Students' evaluations of university teaching: Research findings, methodological issues, and directions for future research. *International journal of educational research*, 11(3):253–388.

N.N. (2007). *fuzzyTECH 5.7 Users Manual*. INFORM GmbH, Aachen, D.

Osgood, C. E. (1952). The nature and measurement of meaning. *Psychological bulletin*, 49(3):197–237.

Osgood, C. E., Suci, G. J., and Tannenbaum, P. H. (1957). *The measurement of meaning*. University if Illinois Press, Urbana, IL.

Schuman, H. and Presser, S. (1996). *Questions and answers in attitude surveys: Experiments on question form, wording, and context*. Sage Publications, Thousand Oaks, CA.

Smithson, M. (1988). Fuzzy set theory and the social sciences: the scope for applications. *Fuzzy Sets and Systems*, 26(1):1–21.

Stevens, S. S. (1946). On the theory of scales of measurement. *Science*, 103:677–680.

Thurstone, L. L. (1927). A law of comparative judgment. *Psychological review*, 34(4):273–286.

Thurstone, L. L. (1928). Attitudes can be measured. *American journal of Sociology*, 33(4):529–554.

Van Leekwijck, W. and Kerre, E. E. (1999). Defuzzification: criteria and classification. *Fuzzy sets and systems*, 108(2):159–178.

Von Altrock, C. (1997). *Fuzzy logic and neuroFuzzy applications in business and finance*. Prentice Hall PTR, Upper Saddle River, NJ.

Weisberg, H. F. and Rusk, J. G. (1970). Dimensions of candidate evaluation. *The American Political Science Review*, 64(4):1167–1185.

White, M. (1926). Psychological technique and social problems. *Southwestern Political and Social Science Quarterly*, 7:58–73.

Yeh, C.-T. (2009). Weighted trapezoidal and triangular approximations of fuzzy numbers. *Fuzzy Sets and Systems*, 160(21):3059–3079.

Yu, J. H., Albaum, G., and Swenson, M. (2003). Is a central tendency error inherent in the use of semantic differential scales in different cultures? *International Journal of Market Research*, 45(2):213–228.