

A Noise Resilient and Non-parametric Graph-based Classifier

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Abstract: In this paper, we propose a non-parametric and noise resilient graph-based classification algorithm. In designing the proposed method, we represent each class of dataset as a set of sub-graphs. The main part of the training phase is how to build the classification graph based on the non-parametric k-associated optimal graph algorithm which is an extension of the parametric k-associated graph algorithm. In this paper, we propose a new extension and modification of the training phase of the k-associated optimal graph algorithm. We compare the modified version of the k-associated optimal graph (MKAOG) algorithm with the original k-associated optimal graph algorithm (KAOG). The experimental results demonstrate superior performance of our proposed method in the presence of different levels of noise on various datasets from the UCI repository.

1 INTRODUCTION

Graph structure has been used in machine learning to deal with different tasks such as clustering, classification and feature reduction (Belkin 2003, Vathy 2009, Dhanjala 2014). Graph representation has some specific characteristics. It can present the topological structure of the data. It can also propose a hierarchical structure by presenting a graph as a set of sub-graphs and it presents an arbitrary shape for a class or a cluster. This is why the graph structure has attracted a lot of attention in machine learning. Perhaps, graph clustering is the most important application of graph structure as it is able to extract the arbitrary and unknown shape of clusters (Dhanjala 2014, Jun 2014). Semi supervised learning is another application of graph structure (Zhu 2008, Chen 2009) in which only a small portion of the data is labeled. Based on the labeled data, a graph-based classifier is trained and then it predicts the label of unlabeled part of the data. The newly labeled data now can be added to the former labeled data to retrain the classifier and improve the model accuracy. Classification is another application of graph structure which has not been receiving much attention in comparison with graph clustering and semi supervised learning. The graph classification problem can be discussed in two different ways. The first one is about learning to

classify separate, individual graphs in a graph database into two or more categories (Ketkar 2009).

The other application is how to represent a vector dataset as a set of sub-graphs, each of which illustrates a class of training dataset. In other words, a graph-based classifier consists of some sub-graphs representing the training dataset.

A number of algorithms have been introduced for building the set of sub-graphs for an input training dataset (Bertini Jr. et al 2011, Chatterjee 2012, Marios 2011). In this paper, we focus on the later application of graph classification for machine learning.

In (Bertini Jr. et al 2011), the authors propose a classifier based on K-associated graph which presents each class of data as a set of sub-graphs (components). Their proposed method is a non-parametric algorithm contrasting to K-nearest neighbor classifier, no need for model selection, does not consider relational data and neither makes use of graph kernel nor Laplacian. They also introduced a new concept, called purity, which measures the connectivity level of samples in each component. The output of the proposed method is K-Associated Optimal Graph (KAOG), based on K-associated graph algorithm (KAG). The detail of the K-Associated Optimal Graph and K-associated graph algorithms is presented in sections 2.1 and 2.2. In this research, we modify the training phase of the

KAOG algorithm, and propose a modified graph-based classifier which is more noise resilient than the KAOG.

The rest of the paper is organized as follows: in the next section, we present an overview on graph-based classifiers and relational data followed by explaining the KAOG algorithm. In section 3, we introduce the proposed algorithm. Section 4, is the experimental results and finally the last section is the conclusion.

2 GRAPH CLASSIFICATION AND RELATIONAL DATA

The problem of graph classification was first studied by (Gonzalez 2002) as a greedy method for finding a sub-graph. In (Deshpande 2005), the authors resented a graph classification algorithm that uses frequent sub-graph discovery algorithms to find all topological substructures in the dataset. By using highly efficient frequent sub-graph discovery algorithms, they reduced the computational complexity of the proposed algorithm based on which they were able to select the most discriminative sub-graph candidate to improve the accuracy of the classifier. Chatterjee and Raghavan in (Chatterjee 2012) proposed a data transformation algorithm to improve the accuracy of two classifiers (LD and SVM). First, they employed a similarity graph neighbourhoods (SGN) in the training feature subspace and mapped the input dataset by determining displacements for each entity and then trained a classifier on the transferred data. On the other hand, there are some applications which use the graph structure directly as a classifier (Bertini Jr. et al 2011).

In this paper, we combine the idea of relational data and graph classification to improve the accuracy of the KAOG classifier in the presence of noise. Since we compare the proposed method with KAOG algorithm, in the next sub-section, we explain the main concept and functionality of the KAOG algorithm. The main core of the KOAG algorithm is K-associated graph (KAG) which builds a graph for an input parameter K. KAG is explained in the following section.

2.1 Constructing the K-associated Graph (KAG)

The following (Algorithm 1) illustrates the K-associated graph construction phase in which a

graph is built based on a fix value of K.

Algorithm 1. Constructing the K-associated graph from a data set (Bertini Jr. et al 2011)

Input: A constant K and a data set $X = \{(x_1, c_1), \dots, (x_i, c_i), \dots, (x_N, c_N)\}$

Symbols: $D_{v_i, K}$ is the label-dependent K-neighborhood set of vertex v_i

$findComponents()$ is a function that returns the components of a giving graph;

$purity()$ is a function that calculates the purity measure;

1: $C \leftarrow \emptyset$

2: $G^{(K)} \leftarrow \emptyset$

3: **for all** $v_i \in V$ **do**

4: $\Delta_{v_i, k} \leftarrow \{v_j | v_j \in \Lambda_{v_j, k} \text{ and } c_j = c_i\}$

5: $E \leftarrow E \cup \{e_{ij} | v_j \in \Delta_{v_i, k}\}$

6: **end for**

7: $C \leftarrow findComponents(V, E)$

8: **for all** $C_\alpha \in C$ **do**

9: $\phi_\alpha \leftarrow purity(C_\alpha)$

10: $G^{(k)} \leftarrow G^{(k)} \cup \{(C_\alpha(V, E); \phi_\alpha)\}$

11: **end for**

12: **Output:** The K-associated graph $G^{(k)} = \{C_1, \dots, C_\alpha, \dots, C_R\}$ where component $C_\alpha = (G(V, E); \phi_\alpha)$ and ϕ_α represents the purity of C_α

The input of the KAG training phase is a constant value K and the training dataset X, in which x_i shows the i^{th} sample of the training set and c_i shows the corresponding label.

Basically, the KAOG algorithm consists of three main parts. In the first part, for each vertex v_i , based on the input K, the k nearest neighbours of v_i is calculated ($\Lambda_{v_j, k}$). In the next step, from $\Lambda_{v_j, k}$, the samples which have the same label as v_i are selected as $\Delta_{v_i, k}$

Based on the $\Delta_{v_i, k}$, some edges are built, starting at v_i and ending at $\Delta_{v_i, k}$ members.

In the next step, the $findcomponent(V, E)$ function is responsible for finding the components (sub-graphs) which are built in previous step. Each component consists of some samples from the same class which form a component. Each class may have some components which are not connected to each other. In this function, V is the vertices of the graph and E is the edges that are generated in previous step.

In last step, based on Equations (1) and (2) the purity measure for each component is calculated. The purity measure illustrates how members of a component are well connected to each other.

$$D_\alpha = \frac{1}{N_\alpha} \sum_{v_i \in C_\alpha} (d_i^{in} + d_i^{out}) \quad (1)$$

In this equation C_α shows a component, v_i is a vertex which belongs to C_α , d_i^{in} shows the number of edges in C_α which start from v_i (in degree), d_i^{out} is the number of edges in C_α which end to v_i (out degree) and N_α is the number of vertices in C_α . D_α shows the average degree for C_α and more value shows the more number of connections and edges in the component. Based on D_α , the purity measure is calculated. Equation (2) illustrates the calculation of the purity measure for the current component.

$$\phi_\alpha = \frac{D_\alpha}{2K} \quad (2)$$

The maximum value for ϕ_α is one and the minimum value is zero. The value one shows that the component is fully connected and all the vertices are connected to each other. On the other side, the value zero shows a distinct vertex which is not connected to any other vertices. The more value of ϕ_α , the better and more well-connected component.

As K is the input parameter of KAG algorithm, the KAG is a parametric algorithm. In the next section, the K -associated optimal graph (KAOG) algorithm is described. KAOG is a non-parametric algorithm.

2.2 Constructing the K -associated Optimal Graph -KAOG

KAG algorithm constructs a K -associated graph, based on a given input K consisting of some sub-graphs called *components*. The components constructed using a specific value of K can be different from the ones built by another value of K . As the components play a key role in classification task, the accuracy and the performance of the classification task may differ.

KAOG starts building the components based on the value of $K=1$ and increase the value of K for the components as long as the increment causes a better value for purity. Algorithm (2) shows the process in details.

Algorithm 2. Constructing the K -associated optimal graph from a data set (Bertini Jr. et al 2011)

Input: Data set $X = \{(x_1, c_1), \dots, (x_i, c_i), \dots, (x_N, c_N)\}$

Symbols: $Kac()$ function that creates the K -associated graph (Algorithm 1)

- 1: $K \leftarrow 1$
- 2: $G^{opt} \leftarrow Kac(X, K)$
- 3: **repeat**
- 4: $lastAvgDegree \leftarrow D^{(K)}$
- 5: $k \leftarrow k + 1$
- 6: $G^{opt(K)} \leftarrow Kac(X, K)$

- 7: **for all** $C_\beta^{(K)} \subset G^{(K)}$ **do**
- 8: **if** $(\Phi_\beta^{(K)} \geq \Phi_\alpha^{(opt)})$ **for all** $C_\alpha^{(opt)} \subseteq \Phi_\alpha^{(opt)}$ **then**
- 9: $G^{(opt)} \leftarrow G^{(opt)} - \bigcup_{C_\alpha^{(opt)} \subseteq C_\beta^{(K)}} C_\alpha^{(opt)}$
- 10: $G^{(opt)} \leftarrow G^{(opt)} \cup \{C_\beta^{(K)}\}$
- 11: **end if**
- 12: **end for**
- 11: **until** $D^{(k)} - lastAvgDegree < D^{(k)}/K$
- 12: **Output:** The K -associated optimal graph $G^{(opt)} = \{C_1^{(opt)}, \dots, C_\alpha^{(opt)}, \dots, C_R^{(opt)}\}$ where component $C_\alpha^{(opt)} = (G \setminus (V, E)); \phi_\alpha, k_\alpha$

The only input for KAOG is the dataset and the output is the optimal graph consisting of a number of components ($C_\alpha^{(opt)}$). Each component is a sub-graph built based on a value of K . This means that different components may have different values of K . The KAOG algorithm is described in (Bertini Jr. et al 2011) with more details. In our work, we modified the training phase and proposed Modified K -Association Optimal Graph (MKAOG). The next section presents the modified training phase in details.

3 THE PROPOSED GRAPH-BASED CLASSIFIER

The basis of the proposed method lies on representing the training set as a graph which consists of a number of components (sub-graphs). Each component is formed based on a subset of samples which are in the same class. In this section, we describe the modification applied on the original KAOG method to propose a new version of KAOG algorithm in the training phase, followed by a description of how the proposed classifier copes with the labelling process. Like any graph classification methods, MKAOG has two phases, building the graph based on the training samples (lies on KAOG) and assigning a label to each test sample.

3.1 The Modified Training Phase of KAOG (MKAOG)

Algorithm 2 starts with $K=1$ for all components. By increasing the value of K , some components are merged together generating a new component with a higher value of K . There is a condition based on which the new component is built by combining the old ones. The condition is explained in Algorithm 2, line 8, indicating that by adding the value of K ,

some components from the same class tend to merge into a larger component. The newly built component has a new purity measure. If the purity of the new component is greater or equal to the purity of each component combined (which are now subset of the new component), then the old components can merge together. Otherwise the merging does not proceed. This condition cannot be satisfied in many situations. Figure (1) shows a syntactic but very close to a real situation in many datasets.

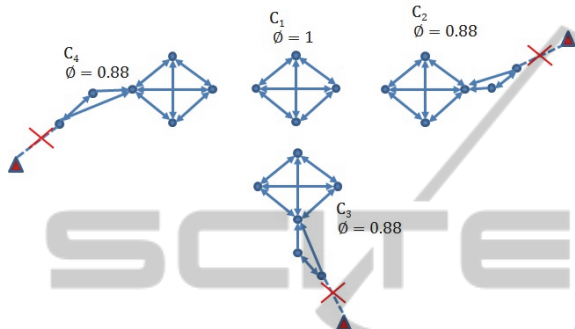


Figure 1: The components which are not allowed to be merged with each other in KAOG.

As shown in Figure (1), the samples indicated by “circle” come from the same class, based on which four components are built (C_1, C_2, C_3, C_4). The value of K for this figure is 3. The purity measure for C_1 is 1 as all the samples in this component are connected to each other with in and out degree of 3. The other components have the purity value of 0.88 because some samples connect to the samples from the other class. A purity value of 0.88 indicates that the majority of samples in the components are well-connected to each other. These components are potentially good candidates to connect to each other and create a larger component. But based on Algorithm 2 and its merging condition (line 8), it is impossible to merge these components because C_1 , with purity measure of 1, cannot be merged with any other components which have the purity measure lower than one (such as the one with purity measure of 0.88). This is why the algorithm does not permit the components to be merged.

Based on our experiments with real datasets, there are too many small components which are not allowed to be merged with each other. Even for small datasets like Iris, there are 16 components most of which are small and consist of only two or three samples. In our proposed modification, we change the merging condition and replace line 8 in algorithm 2 with the following condition:

$$\text{If } (\phi_{\beta}^{(K)} \geq \frac{1}{N} \sum \phi_{\alpha}^{(opt)}) \text{ for all } C_{\alpha}^{(opt)} \subseteq C_{\beta}^{(k)}$$

This means that if the purity measure of the newly merged component is higher than the average of the purity measure for all the components participating in the current merging process, then the component is allowed to be merged.

By applying the proposed new condition, the four components in figure (1) have the chance to be merged with each other as shown in Figure (2).

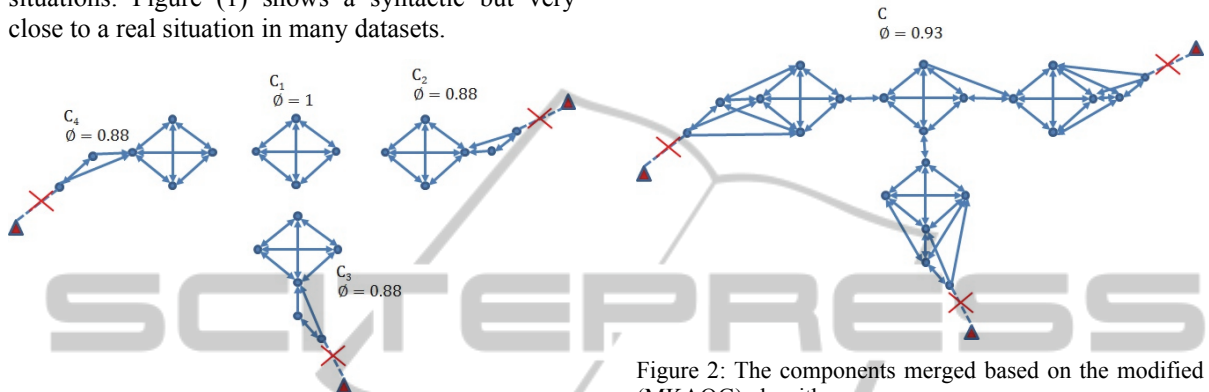


Figure 2: The components merged based on the modified (MKAOG) algorithm.

The purity of the final component is 0.93 which is a high level of purity. Also, all the samples are presented as one component. By merging the small components, the proposed method is less sensitive to the input noise. In the section on experimental results, we demonstrate that the proposed method is less sensitive to noise in comparison with the original KAOG algorithm.

4 EXPERIMENTAL RESULTS

In this section, we evaluate the proposed MKAOG method on various datasets including some well-known datasets from the UCI repository. Table (1) shows the details of the datasets.

Table 1: Summary of the datasets.

Dataset	Number of features	Number of Samples
Iris	4	150
Wine	13	178
Glass	9	223
Ecoli	7	327
Ionosphere	34	351
Diabet	8	768

For evaluating the proposed method, we consider the number of components, correct detection rate, standard deviation (SD) and t-Test criteria to examine whether the improvements are significant

or not. Also, to demonstrate that our proposed method is less sensitive to noise compared to the original KAOG algorithm, we evaluate the algorithm efficiency in the presence of different levels of noise.

4.1 The Number of Components

In this section we compare the proposed MKAOG method with the KAOG in terms of the number of components. Table (2) shows the result of this experiment.

Table 2: Number of components for KAOG and MKAOG.

Dataset name	No. Component – KAOG	No. Components – MKAOG
Iris	16	7
Wine	29	25
Glass	38	29
Ecoli	50	15
Ionosphere	34	17
Diabet	126	65

As shown in Table (2), the number of components for MKAOG is always less than that of KAOG. In the case of Ecoli and Diabet datasets, the difference is much more significant than the other cases. In the case of Wine, they are almost the same. For the Iris dataset, the number of components for MKAOG is less than half of the number of components for KAOG. The values reported in this table are the average of 30 different independent runs which are rounded up to the nearest integer value. The lower number of components indicates existence of larger components, which means less sensitivity to noise. Next, we evaluate the proposed method based on the correct detection rate and standard deviation (SD).

4.2 The Correct Detection Rate and t-Test

In this section, we evaluate the correct recognition rate of the MKAOG algorithm, and compare the results with the ones for the KAOG. Table (3) shows the results of the experiment on different datasets. Each number in the tables is the average of 30 different independent runs. We also used t-Test to illustrate whether the improvement is significant or not. Table (3) shows the results on Iris, Wine and Glass datasets. For each level of noise, there is an associated t-Test result. A t-Test value of “one” indicates a significant difference between the accuracy and SD of KAOG and MKAOG. In such cases, the method with higher accuracy outperforms

the other one. A t-Test value of “zero” indicates that the difference between the two methods is not significant, and they are almost the same.

In addition to the original datasets, we also generated three modified datasets with adding three different levels of noise, 5%, 10% and 20%. The modified (i.e. noisy) datasets were generated by randomly changing the class of samples in the training dataset.

Table 3: The Accuracy and associated t-Test for the KAOG and MKAOG methods in presence of different levels of noise in Iris, Wine and Glass datasets.

Dataset	Noise level		Accuracy	SD	t-Test
Iris	0%	KAOG	94.8444	3.0111	0
		MKAOG	94.8224	3.007	
	5%	KAOG	89.4	4.8917	0
		MKAOG	92.5176	3.927	
	10%	KAOG	83.9778	5.775	1
		MKAOG	90.3474	4.5117	
20%	KAOG	74.3111	7.6219	1	
	MKAOG	83.5119	6.974		
Wine	0%	KAOG	85.5472	4.0341	0
		MKAOG	86.5471	3.7573	
	5%	KAOG	83.717	5.6052	1
		MKAOG	85.3047	4.0434	
	10%	KAOG	72.0189	5.9485	1
		MKAOG	77.7547	5.8232	
20%	KAOG	68.0566	7.2052	1	
	MKAOG	71.7547	6.8851		
Glass	0%	KAOG	69.4478	4.3607	0
		MKAOG	69.434	4.3587	
	5%	KAOG	66.8955	5.7762	0
		MKAOG	68.0215	5.2685	
	10%	KAOG	63.2463	6.1238	1
		MKAOG	64.4925	5.1054	
20%	KAOG	58.6269	6.5044	1	
	MKAOG	61.5794	6.0127		

The accuracy of the MKAOG is higher than that of KAOG most of the time, and the standard deviation for 30 different runs is always lower than that of KAOG. Moreover, when the level of noise is increased, the difference between the two methods is more significant. Additionally, t-Test measurement shows that, while on the original iris dataset (without any noise) the difference is not significant, but by adding noise to the training set, the t-Test indicates a significant difference between the KAOG and MKAOG results. This confirms the proposed method is less sensitive to the input noise than the KAOG. For the original Glass dataset and the one with 5% noise, the improvement is not significant. However, with increasing the level of noise, the

difference between the correct detection rates is obvious, and the t-Test confirms this improvement. This illustrates that even if the performance of KAOG and MKAOG on the original dataset is the same, however, by adding noise to the dataset, the difference between the two methods becomes more significant, as the MKAOG is much more resilient to noise

5 CONCLUSIONS

In this paper, we proposed a non-parametric and noise resilient classification algorithm based on the graph structure. Our proposed algorithm consists of a modification of the training phase of the KAOG algorithm. The MKAOG method produces less number of components, which makes it a robust algorithm in the presence of different level of noise. We evaluated the proposed method on various datasets from the UCI repository. The experimental results demonstrate that the proposed algorithm has high correct detection rate (accuracy), while exhibiting robustness in the presence of noise. MKAOG outperforms KAOG in terms of accuracy. The t-Test evaluation confirms that the difference between accuracies of MAKOG and KAOG is significant.

REFERENCES

- Belkin, P. Niyogi, Laplacian eigenmaps for dimensionality reduction and data representation, *Neural Computation* 15 (2003) 1373–1396.
- Bertini Jr, Liang Zhao, Robson Motta, Alneu de Andrade Lopes, ‘A nonparametric classification method based on K-associated graphs’, *Information Sciences* 181 (2011) 5435–5456.
- Chatterjee A, Raghava P, ‘Similarity Graph Neighborhoods for Enhanced Supervised Classification’, *Procedia Computer Science*, Volume 9, 2012, Pages 577–586.
- Chen, L. Li, J. Peng, Error bounds of multi-graph regularized semi-supervised classification, *Information Sciences* 179 (2009) 1960–1969.
- Deshpande, M. Kuramochi, and G. Karypis, “Frequent Sub-Structure-Based Approaches for Classifying Chemical Compounds,” *IEEE Transactions on Knowledge and Data Engineering*, vol. 17, no. 8, pp. 1036–1050, 2005.
- Dhanjala C, Gaudelb R, Cl emen onc S, ‘Efficient eigen-updating for spectral graph clustering’, *Neurocomputing*, Volume 131, 5 May 2014, Pages 440–452.
- Gonzalez, L. Holder, and D. Cook, “Graph-based relational concept learning,” *Proceedings of the Nineteenth International Conference on Machine Learning*, 2002.
- Jun Ye, Zhong Jin, “Dual-graph regularized Concept Factorization for Clustering”, *Neurocomputing*, In Press, Accepted Manuscript, Available online 13 April 2014.
- Ketkar, N.S. ; Holder, L.B. ; Cook, D.J., ‘Empirical comparison of graph classification algorithms’, *Computational Intelligence and Data Mining*, 2009. CIDM '09. 259 – 266.
- Marios Iliofotou, Hyun-chul Kim, Michalis Faloutsos, Michael Mitzenmacher, Prashanth Pappu, George Varghese, ‘Graption: A graph-based P2P traffic classification framework for the internet backbone’, *Computer Networks* 55 (2011) 1909–1920.
- Vathy-Fogarassy, J. Abonyi, Local and global mappings of topology representing networks, *Information Sciences* 179 (2009) 3791–3803.
- Zhu X, *Semi-Supervised Learning Literature Survey*, Technical Report 1530, Computer-Science, University of Wisconsin-Madison, 2008.