

Automated Segmentation of Folk Songs Using Artificial Neural Networks

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Abstract: Two different systems are introduced, that perform automated audio annotation and segmentation of Cypriot folk songs into meaningful musical information. The first system consists of three artificial neural networks (ANNs) using timbre low-level features. The output of the three networks is classifying an unknown song as “monophonic” or “polyphonic”. The second system employs one ANN using the same feature set. This system takes as input a polyphonic song and it identifies the boundaries of the instrumental and vocal parts. For the classification of the “monophonic – polyphonic”, a precision of 0.88 and a recall of 0.78 has been achieved. For the classification of the “vocal – instrumental” a precision of 0.85 and recall of 0.83 has been achieved. From the obtained results we concluded that the timbre low-level features were able to capture the characteristics of the audio signals. Also, that the specific ANN structures were suitable for the specific classification problem and outperformed classical statistical methods.

1 INTRODUCTION

The automatic annotation of a musical piece is an important subject in the field of computational musicology. The annotation of a musical piece indicates interesting and important musical events. Such events include the start and the end positions of a note, the start and the end positions of a part in which a singing voice is present, the repetitions of a melody and others. This procedure is often called audio thumb-nailing.

The main melody of a song is usually located where a singing voice is present. The knowledge of the position of a song that contains the main melody can give insights in the structure of the song and it is a starting point for further analysis and study. It is also desirable to detect the part in a song where only instruments are performing and no vocal singing is present. This can be considered a classification task of two classes. One class is the “vocal” where a singing voice is performing and the other is the “instrumental” where only instruments are performing. Several methods that tend to solve similar classification problems have been proposed in the past by Lu et al (Lu, Zhang, Li, 2003), Scheirer and Slaney (Scheirer and Slaney, 1997),

Fuhrmann et al (Fuhrmann, Herrera and Serra, 2009) and Vembu and Baumann (Vembu and Baumann, 2005). Panagiotakis and Tziritas (Panagiotakis and Tziritas 2004) propose a speech/music discriminator based on the Root Mean Square (RMS) and the zero crossing rates (ZCR). For the classification they employ a set of rules such as void interval frequencies between consecutive frames, information gathered by the product between RMS and ZCR, the probability of no zero crossings etc. Another common approach is the extraction of features from a training set that was previously annotated with the desired classes and the application of standard machine learning techniques. In the work of Pfeiffer et al, (Pfeiffer, Fischer and Effelsberg, 1996) perceptual features such as loudness or pitch were taken into account in their experiments. They claim that these features play a semantic role for the performance of their classifications and the audio content analysis. Experiments with additional features rather than using only the RMS and the ZCR were also introduced by Scheirer et al and Slaney (Scheirer and Slaney, 1997).

The latest publication and the most relevant to our work, is found in literature by Bonjyotsna and Bhuyan (Bonjyotsna and Bhuyan, 2014). They

suggest as main feature the Mel-Frequency Cepstral Coefficients (MFCC) for the classification of vocal/instrumental parts applied in the MUSCONTENT database. Three machine learning techniques were used for the classification: Gaussian Mixture Model (GMM), Artificial Neural Network (ANN) with Feed Forward Backpropagation algorithm and Learning Vector Quantization (LVQ). From their results, they claim that LVQ yields the higher accuracy in the classification. More precisely, they report 77% classification accuracy for the ANNs, 77.6% for the LVQ and 60.24% for the GMM. In our work, we included additional low-level features and we achieved higher accuracy by modeling our data with ANN.

In this paper, we introduce a two-stage approach for (a) the classification of an unknown song into “monophonic” or “polyphonic” and (b) the segmentation of a polyphonic song into positions of interest. Such positions include the boundaries of a part in a song that only instruments are performing and no vocal singing is present. We used low-level timbre features and we built trained artificial neural networks that are able to discriminate and predict with high accuracy the unknown songs as “monophonic” or “polyphonic” and polyphonic music as “vocal” or “instrumental”.

The main contribution of our work is the use of ANNs and their application in audio thumb-nailing. This use has numerous advantages in wide range of applications (Benediktsson, J., et. al., 1990). The ability of the adaptation of complex nonlinear relationships between variables arises from the imitation of the biological function of the human brain. Disadvantages include the greater time of training, and the empirical nature of model development. The authors have demonstrated how the disadvantages can be minimized in a wide area of applications including medicine (Neocleous C.C., et. al. 2011).

While the interest of the MIR community on the audio thumb-nailing focused in popular music, little work has been done for folk music. Main differences between popular music and folk music include the western/non-western instrumentation as well as fundamental rules in music theory. For instance, the use of traditional instruments in folk music, create a significantly different sound in comparison to the popular music. One common feature of the folk music is the monophonic performances. These can be either using a musical instrument or only with singing voice. Our contribution with a classification system of a song into “monophonic” or “polyphonic” is reported in this paper.

We compare our results with two other methods, named Support Vector Machines and the statistical Bayesian Classification.

2 METHODS

2.1 Overview

The database we used contains audio signals of 98 Cypriot folk songs. Each audio signal has been extracted from original cd's and it has been encoded with a sampling frequency of 44100 Hz and 16 bit amplitude. The sampling frequency of 44100 Hz and the amplitude of 16 bit is the quality that is typically used in the audio cd's.

From this database, we isolated 24 songs for creating a training set while the remaining 74 songs were used for validation of our system. In the training set, 17 monophonic songs and 7 polyphonic songs were chosen. The monophonic songs were 6 vocal songs sung by male performers, 6 vocal songs sung by female performers and 5 songs performed with the traditional Cypriot instrument called “pithkiavli”.

The main idea of our method is illustrated in Figure 1. The first system takes as input an unknown song and predicts if it is monophonic or polyphonic. The second system takes a polyphonic song and predicts the boundaries of parts of the song that only instruments are performing (instrumental parts) and parts in which a singing voice is present (vocal parts).

Each audio signal was segmented into a sequence of overlapping audio frames of length 2048 samples (46 ms) overlapping by 512 samples (12 ms). For each of these audio frames we extracted the following audio features: Zero Crossing Rate, Spectral Centroid, MFCC (13 coefficients). For the first system the mean and the standard deviation values of each feature are calculated and are used to build three feed-forward ANNs. Each of them has 20 neurons in the hidden layer and was trained for 200 epochs. The ANNs were built using monophonic songs for the first class and polyphonic songs for the second class. The difference between the three ANNs is that the instrument that is performing in the monophonic songs is different for each network. This system classifies an unknown song into the class “monophonic” or “polyphonic”. Both systems 1 and 2 require audio frame segmentation and feature extraction.

For the second system, the entire feature vectors were used to build one ANN that predicts a value in

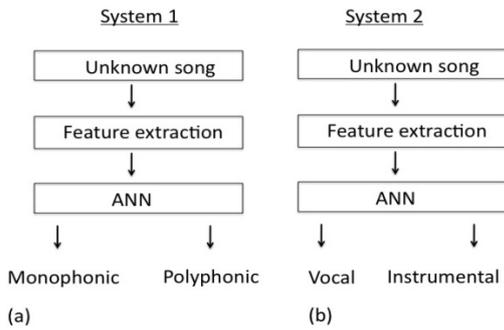


Figure 1: (a) System 1 takes as an input an unknown song and classifies the song as “monophonic” or “polyphonic”. (b) System 2 takes as an input a polyphonic song and segments it into “vocal” and “instrumental” parts.

the range between 0 and 1 for every audio frame. The output is then quantified with a threshold to the binary values 0 or 1. The value 0 corresponds to a frame from a purely instrumental part. The output is 1 if vocal singing is present in the frame. We use this system to annotate the instrumental and the vocal parts in a song.

2.2 Neural Network

Many different ANN structures had been proposed and used by researchers in different fields. The most common and widely used for classification, generalization, and prediction is the commonly known fully connected multilayer feed forward structure (FCMLFF). Mathematically this is represented by equation 1 as:

$$y_{iL}^{[L]} = f_{iL}^{[L]} \left(\sum_{j_{L-1}=1}^{n_{L-1}} y_{j_{L-1},L}^{[L]} w_{L-1,L}^{[L]} \right) \quad (1)$$

where,

$y_{iL}^{[L]}$ is the output value of each neuron iL of layer L that has a total of nL neurons. Typically, this function has a squashing function form such as the logistic or the hyperbolic tangent.

$W_{L-1,L}$ is the set of weights associating neurons in layer $L-1$ to neurons in layer L .

Once the ANN is decided, an effective training and tuning procedure needs to be implemented, so that the network will achieve a useful capability for doing the desired task, such as classification, generalization, recognition etc. Many training procedures had been proposed and are available for implementation. The most widely used for feed

forward networks is the backpropagation algorithm (Werbos, 1974). In this work, we implemented fully connected feed-forward neural networks with backpropagation learning.

2.3 Feature Selection

Twenty-four songs were selected to form a training set to be used in the artificial neural network classifier. The training set was chosen in such a way that all the musical instruments that were of our interest for classifying them were present. The positions of the vocal parts and the instrumental parts were manually annotated to the training data and a set of low-level timbre features were extracted for each class respectively. Specifically, the features extracted were the Zero Crossing Rate, Spectral Centroid, Spectral Spread and 13 coefficients of MFCC, thus creating a feature set of 16 features in total. We applied a statistical analysis to the features and from the results we assume that this set of features is considered to be suitable for solving the particular classification problem.

2.3.1 Zero Crossing Rate

The feature Zero Crossing Rate (Benjamin 1986) is a measure on how many times does the waveform crosses the value of zero within a frame:

$$ZCR = \frac{1}{2(N-1)} \sum_{n=1}^{N-1} \text{sgn}[x(n+1) - \text{sgn}[x(n)]] \quad (2)$$

Where:

$X(n)$: is the discrete audio signal, $n=1 \dots N$

$\text{sgn}[\cdot]$: is a sign function.

The ZCR is a powerful feature for identifying noisy signals. It is also used as a main feature for fundamental frequency detection algorithms (Roads 1996).

2.3.2 Spectral Centroid

The feature Spectral Centroid it is the geometric center of the distribution of the spectrum and is a measure of the spectral tendency of a random variable x . It is a useful feature for classification problems such as instrument identification or the separation of audio signals into speech/music. It is defined as:

$$\mu_1 = \int xf(x)dx \quad (3)$$

Where:

x : is a random variable

$f(x)$: is the probability distribution of the random variable x characterized by that distribution.

2.3.3 Spectral Spread

The feature Spectral Spread it is defined in eq. 4 and it is essentially the standard deviation of the spectrum. It describes how much energy is distributed by the frequencies across the spectrum.

$$\sigma = \sqrt{(x - \mu)^2 f(x) dx} \quad (4)$$

2.3.4 Mel Frequency Cepstral Coefficients (Mfcc)

The feature MFCC (Mermelstein 1976) describes the timbre characteristics of an audio file within a number of coefficients. Usually the number of the coefficients taken into account is 13. The computation of the MFCCs is as follow: first the spectrum of a framed windowed excerpt audio signal is computed using the Fast Fourier Transformation (FFT). The result from the FFT is then mapped into 13 Mel bands using triangular overlapping windows. The cosine transformation is applied to the logarithm of each one of those Mel bands. The results of each transformation for every band are considered to be the MFCC coefficients. The mapping of the spectrum from the linear scale to the Mel scale is done in order to approximate the functionality of the human auditory system where, in one of its processes it separates the perceived sound into non-linear frequency bands. The most popular formula for converting the frequencies from hertz to Mel is described below:

$$mel = 2595 \log_{10} \left(1 + \frac{f}{700} \right) \quad (5)$$

Where:

f : is the frequency in Hertz.

The mel scale has been proposed by Stevens et al in 1937 (Stevens, Volkman and Newman, 1937) and the name comes from the word melody. It is pointed that the MFCC features are widely used in speech processing and are considered to be a powerful feature for describing timbre characteristics. They carry most of the spectral information within 13 coefficients, in contrast with the row spectrum that has at least 5000 frequency values. In Figure 2 we present an example of the spectrogram of a polyphonic song for an excerpt of 50 seconds. In this case, there are three positions where only instruments are performing, and two positions where

the singing voice is also performing together with the instruments. The first parts of the instrumental and the singing voice are annotated in the same Figure, on the lower plot. It is rather obvious that the distribution of the energy across the spectrum for the two classes “vocal” and “instrumental” is different.

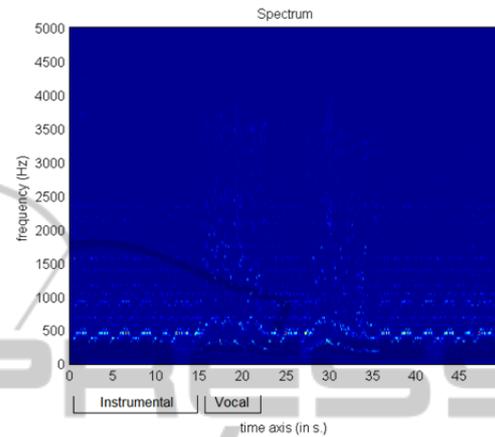


Figure 2: Spectrogram of a polyphonic song. The first 15 seconds in this figure are being performed with instruments. The position from the 15th to 20th second is performed with singing voice together with instruments. It is shown that the distribution of the energy across frequencies between the two positions “instrumental” and “vocal” significantly differ.

Figure 3 shows the 13 coefficients of the MFCC features for the same excerpt of the same song. It is also clear that the MFCC in the instrumental part have higher values with respect to the part of a singing voice.

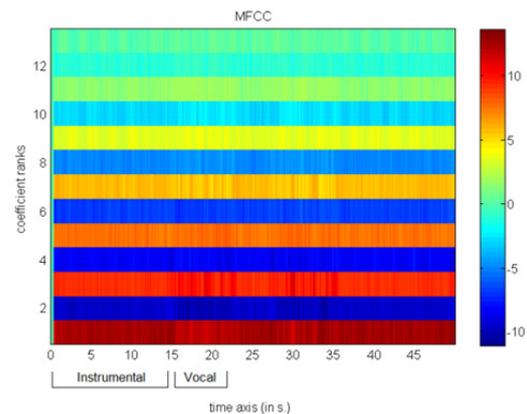


Figure 3: The 13 coefficients of MFCCs versus time. The “instrumental” and the “vocal” parts are annotated manually in the lower plot.

2.4 Statistical Analysis

In an attempt to visualize the data and to understand

better the contribution of each feature to the performance of the system, we applied statistical methods and we report our results in this section. In this analysis we are concerned to explore how significant is the difference between the values of a given feature for the two classes “instrumental” and “vocal”. Our null hypothesis is that the median of the data in class “instrumental” is equal with the median of the data in class “vocal”.

Several methods exist for testing a statistical hypothesis, such as z-test, t-test, Chi-squared test, Wilcoxon signed-rank test and others. The t-test is the most widely used method for testing significant differences between two populations whose size is less than 30 (Mankiewicz, R., 2004). It assumes that the distribution of the two populations being compared is normal. In our case, not all features we used are following a normal distribution. More precisely, the features Zero Crossing Rate, Spectral Centroid and Spectral Spread did not follow a normal distribution for any of the two classes. These features were tested with the Wilcoxon signed-rank test (Siegel, 1956). The 13 coefficients of the MFCC were following a normal distribution. The normality of each distribution was tested with a graphical method and with the Kolmogorov-Smirnov test (Stephens, 1974). For the graphical method we used a normal probability plot. In order to get such plot, first the histogram of the data is approximated with a normal distribution.

In the normal probability plot, the probability distribution follows a normal distribution and it is plotted against the unknown distribution of the data. If the data follow a normal distribution, the function of the normal probability plot will be a straight line. If the normal probability plot does not fit to a straight line, it is an indication that the distribution of the data does not follow a normal distribution. In Figure 4 we present an example of this method for the features (a) Zero Crossing Rate and (b) MFCC of the first coefficient.

In Figure 4a the upper plot shows with blue color the distribution of the feature ZCR and with red color the normal approximation. From this plot it is obvious that the normal distribution cannot model the distribution of the data. This is also observable from the normal probability plot in the lower plot. In Figure 4b we present an example where the distribution of the data of the feature MFCC can be modeled with a normal distribution.

The Wilcoxon signed-rank test is a non-parametric method for testing the significance of the difference between two populations. This method does not assume that the distribution of the

populations is normal. We used this method for testing the features that did not have normal distribution. For the 13 MFCC coefficients we used the t-test. Both the t-test and the Wilcoxon signed-rank test rejected the null hypothesis that the two populations are not different for all the features we used.

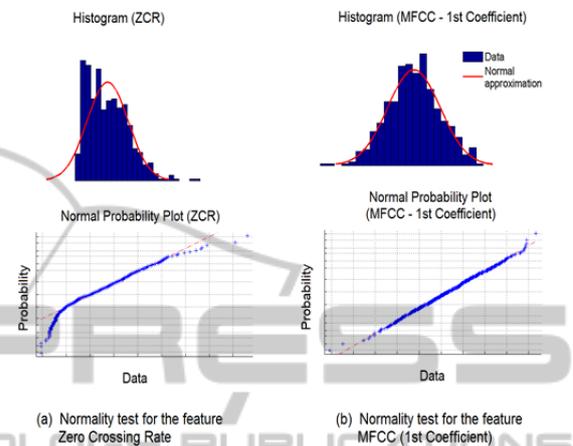


Figure 4: Normality test for the features (a) ZCR and (b) MFCC. In the upper plot, the histogram of every feature is plotted together with the approximation with a normal distribution. Lower plot shows the normal probability plots.

2.5 Classification into “Monophonic” or “Polyphonic”

For the classification into the two classes “monophonic” or “polyphonic” we built three ANN using the mean and the standard deviation of each feature. In total, 32 features were used to train the ANNs. The first ANN is called “male vocal – polyphonic” and is trained with 720 seconds of monophonic male singing performances which form the first class and 115 seconds of polyphonic music which form the second class. The second ANN is called “female vocal – polyphonic” and is trained with 720 seconds of monophonic female singing performances (1st class) and the same 115 seconds of polyphonic music (2nd class). The third ANN is called “pithkiavli – polyphonic” and is trained with 600 seconds of monophonic performances by the instrument “pithkiavli” (1st class) and 115 seconds of polyphonic music (2nd class). The output target for the polyphonic music was set to 1 and the output target for the class “female vocal”, “male vocal” and “pithkiavli” was set to 0.

The classification is done with the following procedure: an unknown song is represented numerically by a vector of 32 that is fed to the three

ANN “male vocal - polyphonic”, “female vocal - polyphonic” and “pithkiavli - polyphonic”. We quantize the outputs of the models to 0 or 1 by using a threshold of 0.5. We classified a song as “monophonic” if the binary output of at least two models is 0; otherwise the song is classified as “polyphonic”.

2.6 Classification into “Vocal” or “Instrumental”

The fourth ANN was built using all feature vectors that were extracted for all audio frames. The output target was added to the database after a manual segmentation of the training set into the “vocal” and “instrumental” positions. For every audio frame of a song the ANN gives a prediction value in the range 0 or 1. One example of the output of the ANN is shown in Figure 5 with continuous black line. The vocal parts and the instrumental parts are annotated manually. Even though in this example it is shown that most of the output values correspond to the correct class (if we set a threshold), some of the frames are misclassified.

In order to solve the misclassification problem, we introduced a set of rules and we used dynamic programming for correcting the possible misclassifications. In a first step we divide the frame sequence in groups of 100 frames each and we compute their vector mean as shown with red dots in Figure 5. These values are then converted into binary values by using an appropriate threshold. The threshold is calculated as the mean of the mean values and is shown with green line in the same figure. In this example the threshold is 0.6. The mean values that fall above the threshold are classified as “instrumental” while the values that fall below the threshold are classified as “vocal”.

Further processing was needed in order to correct additional misclassifications. One example of a misclassified sample is encircled in figure 5. In this example, the encircled output value exceeds the threshold and the system wrongly classifies that position as instrumental. For solving such misclassification problems, we introduced the following rule:

Each sample of the quantized vector is tested with the classes that belong to the frames around it. To consider a classification of a sample as true, the class of the previous frame has to be the same with the class of the following two frames. Regardless on what the classification of the testing frame is, after we apply this rule, the classification may change.

In order to illustrate an example, in Figure 5 we present a frame that was wrongly classified as instrumental while the annotation of this frame is to be vocal. This frame is encircled with yellow colour and in this example it is the testing frame. The previous frame and the two frames after the testing frame belong to the class “vocal” while the testing frame belong to the class “instrumental”. After we apply the rule described above, the class of the testing frame turns from “instrumental” to “vocal”.

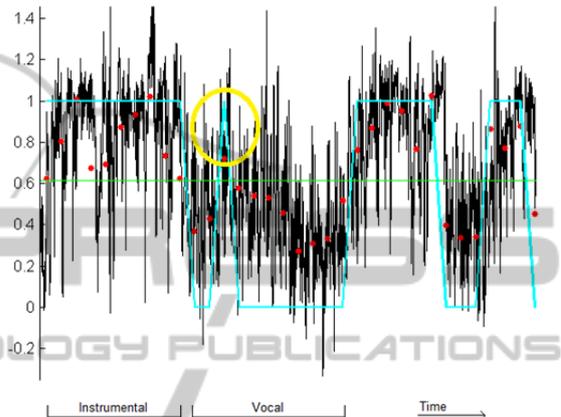


Figure 5: Black continuous line shows the output of the ANN for a chunk of 30 seconds of a polyphonic song. Red dots show the mean values for groups of 100 frames each. Blue continuous line shows the binary quantization of the mean values with respect to the threshold. Yellow circle shows an example of a misclassified value.

3 EVALUATION AND RESULTS

The validation set contained 74 songs of a total duration of 230 minutes. 46 songs were monophonic and the remaining 28 were polyphonic. For the classification of the “monophonic – polyphonic”, we call a “false positive” prediction when a song is annotated as “monophonic” and the prediction of the system is “polyphonic”. We present our results in terms of precision and recall. ANNs achieved a precision of 0.88 and recall of 0.78. SVMs gave precision of 0.85 and recall of 0.81 and Bayesian Statistics precision of 0.71 and recall of 0.69.

The precision is defined as:

$$precision = \frac{TP}{TP + FP} \quad (6)$$

The recall is defined as:

$$recall = \frac{TP}{TP + FN} \quad (7)$$

For the classification of the “vocal – instrumental” we call a false positive if a part of the signal was annotated as “vocal” but the prediction of the system was “instrumental”. Figure 6 shows an example on how we define the terms false positive, false negative and true positive for the specific classification problem. The audio signal is plotted with black continuous line. The red vertical lines indicate the limits in the audio signal where only instruments are performing, while the green vertical lines indicate an example of the limits where a prediction was done from our system. We call false positive the duration of the signal that the ground truth is annotated as “vocal” and the prediction was “instrumental”. ANNs achieved a precision of 0.85 and recall of 0.83. SVMs gave precision of 0.86 and recall of 0.82 and Bayesian Statistics precision of 0.76 and recall of 0.72.

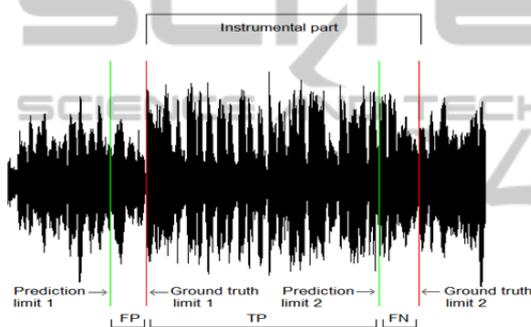


Figure 6: The interpretation of the terms “false positive”, “True positive” and “False negative”.

Where:

TP: True positive
 FP: False positive
 FN: False negative

4 CONCLUSIONS

We described a method for automatic annotation of Cypriot folk music into “polyphonic” or “monophonic” music. In the validation procedure, the audio signal is segmented into audio frames of 46 ms. A set of low level features are being extracted from each such audio frame and given to the input layer of the neural network. In the output layer, a value between 0 and 1 is extracted. This value is used for the classification using a threshold.

For polyphonic music we presented an automatic annotation into instrumental or singing parts. The system identifies these positions by classifying each audio frame into instrumental or vocal. This is done

automatically, while there is no need for any external assistance or guidance.

From our experiments we observed that timbre low-level features are suitable to capture the characteristics of each class. The advantage of our system is the use of ANNs and standard timbre low-level features. We consider ANNs a very powerful technique for classification problems. They have the ability to imitate the biological function of the human brain. Thus, they are able to efficiently identify patterns and correlations in the feature space. Our method does not need any perceptual features and it uses the row values of the features without any pre-processing such as feature normalization. The selected features are state of the art for audio analysis and classification.

The ANNs and the SVMs had similar results. In comparison with the statistical Bayesian classification the ANNs and the SVMs performed better. We present a precision of 0.78 and recall of 0.88 for the first system and a precision of 0.85 and recall of 0.83 for the second system. The results are not yet finalised but represent the basis on our future research will be based. Improvements of the results reported in this paper could be achieved by introducing additional features such as mid-level features. Principal component analysis could also be applied to the feature set for dimensionality reduction. These problems are currently under study and the results will be reported in the near future.

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