Design of a Filter-bank by the Wave Digital Filter Technique
An approach for the Chebishev Bank-Filter by the Wave Digital Filter Technique

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Abstract: This paper presents a simple procedure for design of the filter-bank with Wave digital filter (WDF). The filter-bank is constructed using Chebyshev and Inverse Chebyshev filters. Wave digital filters are derived from LC filters and consist of cascade connections of serial and parallel adapters. These adapters contains the necessary adders, multipliers and inverters. A great advantage of this procedure is that the filters in the wave digital filter-bank synthesis are obtained from the wave digital filter-bank analysis only by changing some signs in the end of delay elements.

1 INTRODUCTION

In our procedure we use adapters with three-ports. The block of the serial and parallel reflection-free adapters and they signal-flow diagram are shown in figure 3, (Fettweis, 1972),(Sedlmeyer and Fettweis, 1973)

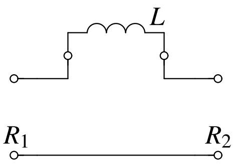

Figure 1: Three-port circuit with inductor.

The coefficient B of the three-port reflection-free serial adapter in figure 3A) is calculated from the port resistance R_i, $i = 1,2$ by equation (1), (Keiser, 1985), (Fettweis and Meerkotter, 1975). The coefficient A of the three-port reflection-free parallel adapter in figure 3B is calculated from the port conductance G_i, $i = 1,2$ by equation (2).

$$B = \frac{R1}{R1+R2} \tag{1}$$

$$A = \frac{G1}{G1+G2} \tag{2}$$

The inductor in Fig. 1 can be realized in the discrete form by serial three-port adapter Fig. 3 A) terminated at the port a2-b2 with the delay element in series circuit with the multiplier -1. Coefficient of the multiplier B we get by equation (3) (Fettweis and Meerkotter, 1975).

$$B = \frac{R_1}{R_1+L} \tag{3}$$

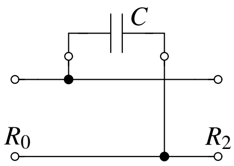

Figure 2: Three-port circuit with capacitor.

Capacitor in the Fig. 2 is realized in the discrete form by parallel adapter Fig. 3 B) terminated at the port a2-b2 with the delay element. Coefficient of the multiplier A we get by equation (4)

$$A = \frac{G_0}{G_0+C} \tag{4}$$

The coefficients of the dependent parallel adapter in the figure 4 B) can be get from port conductances G_i, $i = 1,2,3$ by equation (5), (Fettweis and Meerkotter, 1975)

$$A_1 = \frac{2G_1}{G_1+G_2+G_3} \qquad A_2 = \frac{2G_2}{G_1+G_2+G_3} \tag{5}$$

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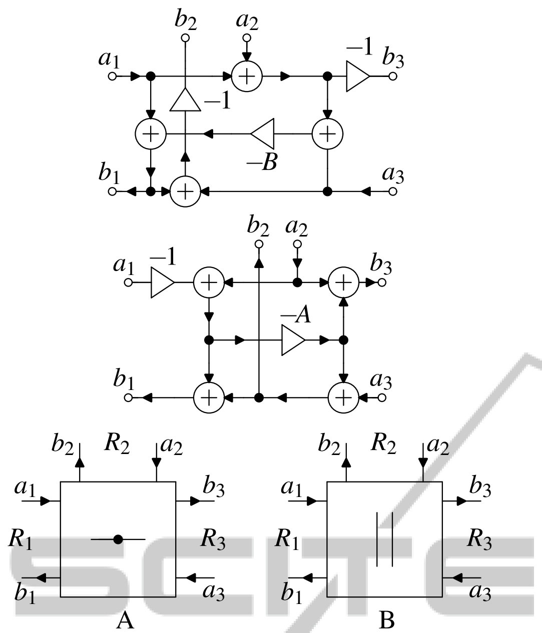


Figure 3: A) Three-port serial adapter whose port 3 is reflection-free and its signal flow-graph. B) Three-port parallel adapter.

The coefficients of the dependent serial adapter in the Fig. 4 A) can be get from port resistances R_i, $i = 1,2,3$ by equation (6)

$$B_1 = \frac{2R_1}{R_1+R_2+R_3} \qquad B_2 = \frac{2R_2}{R_1+R_2+R_3} \tag{6}$$

Three-port serial and parallel dependent adapters will be used only in the end of the structure in order to connect the filter to the load R_n.

2 FILTER-BANK WITH WAVE DIGITAL FILTERS

A filter-bank with wave digital filters is designed on following example. Two channel filter-bank includes a connection of low-pass and high-pass filter Fig. 5. In our proposal $H_0(z)$ is a transfer function of Chebyshev wave digital low-pass filter and $H_1(z)$ is a transfer function of Inverse Chebyshev high-pass filter. To avoid an aliasing must be fulfilled the conditions (Mitra, 1998).

$$G_1(z) = -H_0(-z) \qquad G_0(z) = H_1(-z) \tag{7}$$

Chebyshev approximation with order $N = 3$, and ripple $A_{max} = 1$ dB was used. The transfer function of the low-pass filter is given by relation (8).

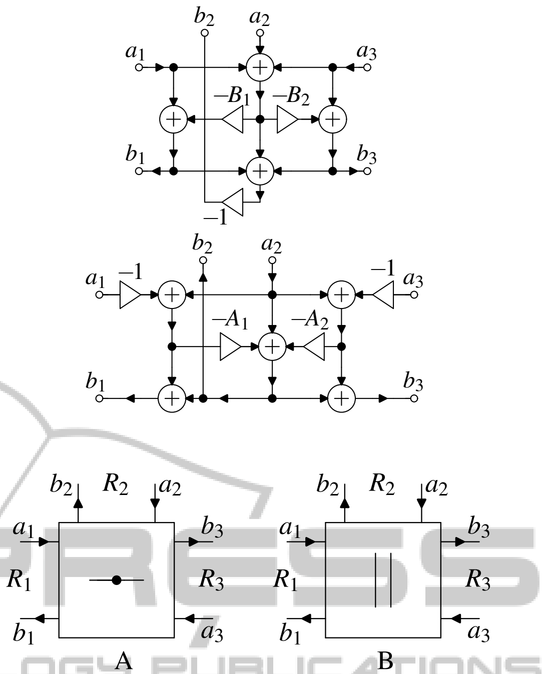


Figure 4: A) Three-port serial dependent adapter and its signal flow-graph. B) Three-port parallel dependent adapter.

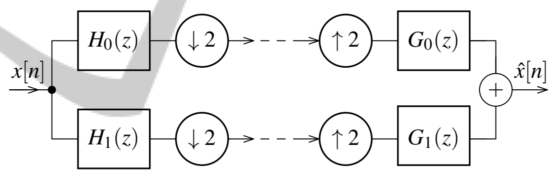


Figure 5: Two channel filter bank.

$$H(s) = \frac{1}{2.035s^3+2.0116s^2+2.5206s+1.000} = \frac{1}{h(s)} \tag{8}$$

Relevant characteristic function of the transfer function (8) is

$$F(s) = \frac{1}{2.5206s^3+1.5265s} = \frac{1}{f(s)} \tag{9}$$

From these relations input impedance $Z_e(s)$ of the circuit can be obtained in form:

$$Z_e(s) = \frac{h(s)+f(s)}{h(s)-f(s)} = \frac{4.0707s^3+2.0116s^2+4.072s+1}{2.0116s^2+0.9941s+1} \tag{10}$$

We expand $Z_e(s)$ in continued fraction about zero. The impedance $Z_e(s)$ can be constructed using the Foster preamble techniques (Weinberg, 1962).

$$Z_e(s) = \frac{1}{2.0235s+\frac{1}{0.9941s+\frac{1}{2.0235s+1}}} \tag{11}$$

The corresponding LC circuit is shown in figure 6. Discrete realization of the wave digital filter is obtained from this LC prototype of the Chebyshev filter, see Fig. 8. High-pass filter in the filter-bank synthesis has the same structure and values as high-pass filter in the filter-bank analysis, only the multiplier -1 at the end of each delay elements must be added, see Fig. 9, (B. Psenicka and Rodriguez, 2006).

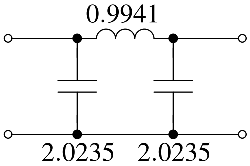


Figure 6: LC prototype of Chebyshev low-pass filter.

LC ladder network in Fig.6 can be redrawn network in Fig. 7. The elements are connected with the parallel and serial adapters together. Parallel dependent adapter have to be used in the end of the structure.

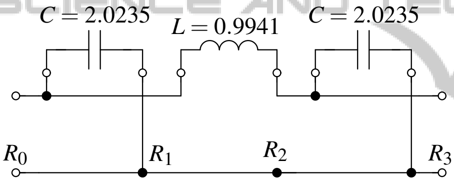


Figure 7: LC prototype of Chebyshev low-pass filter – redrawn diagram.

To obtain the multiplier coefficients of the discrete Chebyshev low-pass filter the conductances G_1 and G_2 must be calculated first.

$$R_0 = 1 \qquad R_3 = 1 \tag{12}$$

$$G_1 = G_0 + C = 3.0235 \qquad R_1 = 0.33074 \tag{13}$$

$$R_2 = R1 + L = 1.32484 \qquad G_2 = 0.754806 \tag{14}$$

Finally the multiplier coefficients of the discrete low-pass Chebychev filter can be calculated by following relations:

$$A_1 = \frac{G_0}{G_0+C} = 0.3307 \qquad B_2 = \frac{R_1}{R_1+L} = 0.2496$$

$$A_{31} = \frac{2G_2}{G_2+C+G_3} = 0.3995 \quad A_{32} = \frac{2G_3}{G_2+C+G_3} = 0.5293 \tag{15}$$

Attenuation of the low-pass Chebyshev filter can be obtained by following script. The script was written according the structure in Fig. 8. The attenuation of the low-pass and high-pass filter are presented in the Fig. 10. Attenuation of the high-pass filter can be obtained by changing the signs of variables in the script: $N2 = -N1$, $N4 = -N3$ and $N6 = -N5$.

```
A1=0.3307; B2=0.2496; A31=0.3995; A32=0.5293;
N2=0; N4=0; N6=0; XN=1;
for i=1:1:200
   XN1=N2-N2*A1+XN*A1;
   XN2=XN1+N4;
   BN2=XN2-A31*XN2+2*N6-N6*A31-N6*A32;
   BN1=XN1-B2*XN2-B2*BN2;
   N1=XN*A1-A1*N2+BN1;
   N3=BN1+BN2;
   N5=N6-XN2*A31-N6*A31-N6*A32;
   YN(i)=2*N6-N6*A31-N6*A32-XN2*A31;
   N2=N1; N4=N3; N6=N5; XN=0;
end
[h,w]=freqz(YN,1,50)
plot(w,20*log10(abs(h)))
axis([0.5184 2.623-40.3 0])
```

Minimum attenuation of the high-pass Inverse Chebyshev filter in the analysis part of filter-bank must be calculated from the ripple factor of the low-pass filter in the analysis part of the filter-bank by relation (16), (Weinberg, 1962).

$$A_{min} = 10log_{10}\left(\frac{1}{10^{\frac{A_{max}}{10}}-1}+1\right) \tag{16}$$

Transfer function of inverse Chebyshev filter in s domain is then given by:

$$H(s) = \frac{s^2+1.3333}{0.655s^3+1.6512s^2+1.3177s+1.3333} = \frac{k(s)}{h(s)} \tag{17}$$

Relevant characteristic function to the relation (17) is

$$F(s) = \frac{s^2+1.3333}{0.6550s^2} = \frac{k(s)}{f(s)} \tag{18}$$

Input impedance z_{11} and transfer impedance z_{12} can be calculated from equation (17) and (18). Inverse Chebyshev low-pass filter presented in Fig. 11 can be obtained from these impedances (z_{11}, z_{12}), (Storer, 1957).

Discrete realization of the parallel LC circuit from Fig. 11 is demonstrated in the Fig. 12, where K is given by equation (19), (Kammeyer and K, 1992).

$$K = \frac{1-LC}{1+LC} \tag{19}$$

The coefficients of the serial and parallel adapters of the discrete filter in figure 18 obtained by equation (5) are $A_1 = 0.6693, B_2 = 0.2496, A_{31} = 0.7032\, A_{32} = 0.5293$ and $K = 0.1428$. Discrete Chebychev high-pass filter is obtained from the discrete low-pass filter by changing the sings as is seen in Fig. 19.

Attenuation of the Inverse Chebyshev filter can be calculated by the following Matlab script. The script is written according the structure in Fig. 18.

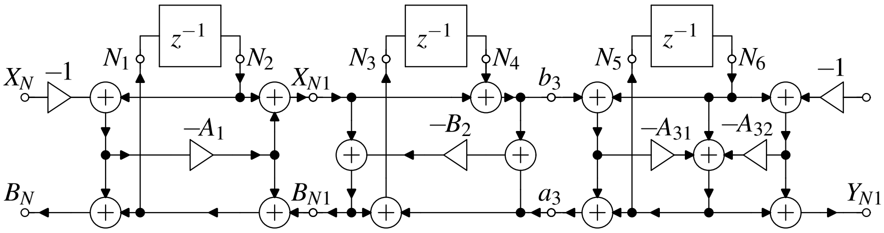


Figure 8: Discrete low-pass Chebyshev filter in analysis filter-bank.

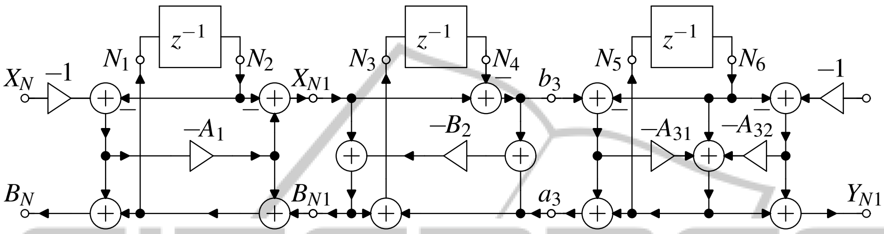


Figure 9: Discrete high-pass Chebyshev filter in synthesis filter bank.

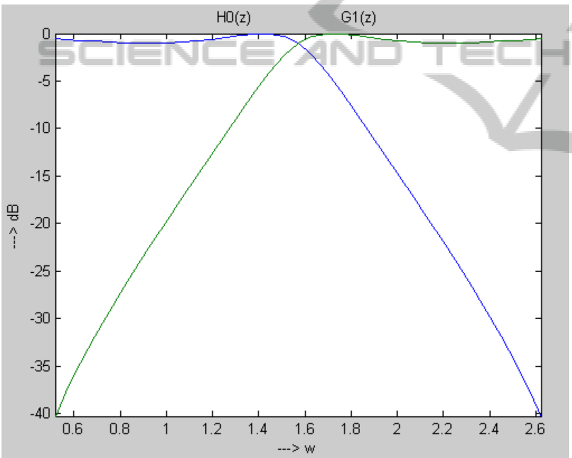


Figure 10: Attenuations of the low-pass and high-pass Chebyshev filter.

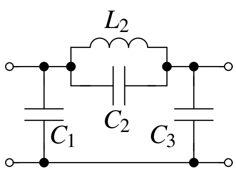


Figure 11: LC prototype of the inverse Chebyshev low-pass filter.

```
XN=1;A1=0.6693;B2=0.5423;A31=0.7032;A32=0.8678;
N2=0;N4=0;N6=0;N8=0;K1=0.1428;
for i=1:1:200
    XN1=N2-N2*A1+XN*A1;
    XN2=XN1+N6;
    BN2=XN2-XN2*A31-2*N8-N8*A31-N8*A32;
    BN1=XN1-BN2*B2-XN2*B2;
    N1=XN*A1-N2*A1+BN1;
    N3=BN1+BN2;
    N5=N3*K1+N4-N6*K1;
    N7=N8-N8*A31-N8*A32-XN2*A31;
    YN(i)=2*N8-N8*A31-N8*A32-XN2*A31;
    N2=N1;N4=N3;N6=N5;N8=N7;XN=0;
end
XN=1;K1=-0.1428;
for i=1:1:200
    XN1=N2-N2*A1+XN*A1;
    XN2=XN1+N6;
    BN2=XN2-XN2*A31-2*N8-N8*A31-N8*A32;
    BN1=XN1-BN2*B2-XN2*B2;
    N1=XN*A1-N2*A1+BN1;
    N3=BN1+BN2;
    N5=N3*K1+N4-N6*K1;
    N7=N8-N8*A31-N8*A32-XN2*A31;
    YN1(i)=2*N8-N8*A31-N8*A32-XN2*A31;
    N2=-N1;N4=N3;N6=N5;N8=-N7;XN=0;
end
[h,w]=freqz(YN,1,200);
[h1,w]=freqz(YN1,1,200);
plot(w,20*log10(abs(h)),w,20*log10(abs(h1)))
```

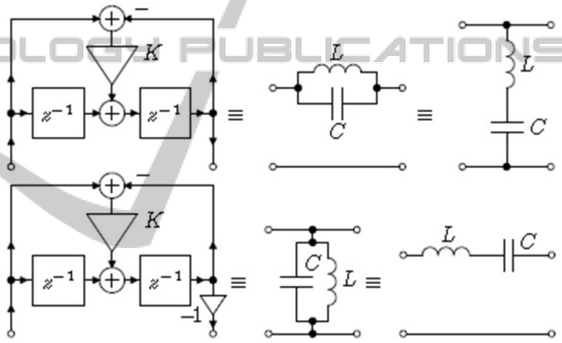


Figure 12: Realization of the dual LC structures by discrete structure.

The attenuation of the Chebyshev inverse low-

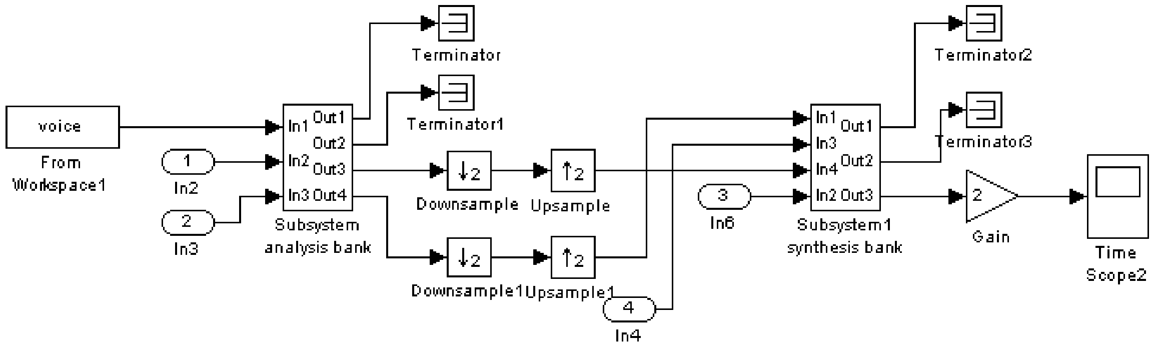
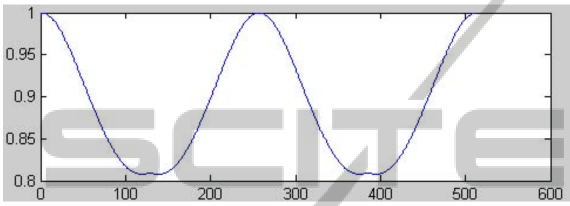
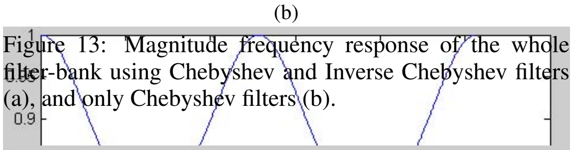


Figure 15: Simulink model of the wave digital filter-bank.



(a)



(b)

Figure 13: Magnitude frequency response of the whole filter-bank using Chebyshev and Inverse Chebyshev filters (a), and only Chebyshev filters (b).

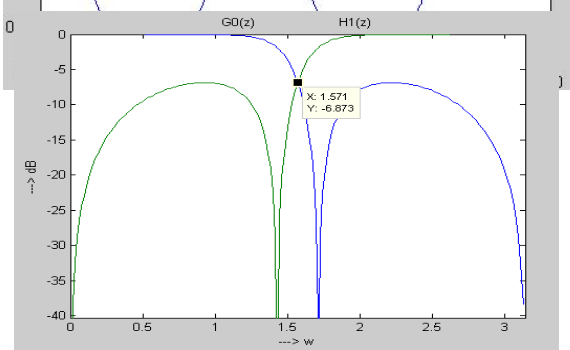


Figure 14: Attenuation of the low-pass and high-pass Inverse Chebyshev filter.

pass and high-pass wave digital filter is presented in figure 14.

A model of the wave digital filter-bank for Simulink is in Fig. 15. This model was constructed from filters in Fig. 8, 9, 18 and 19.

Transfer function $TF(z)$ of the whole filter-bank can be expressed as

$$TF(z) = H_0(z)G_0(z) + H_1(z)G_1(z)$$

The frequency response of the whole filter-bank are shown in Fig. 13. Our realization using Chebyshev and Inverse Chebyshev filters gives better results Fig. (a) in comparison with filter-bank using only Chebyshev filters, Fig. (b). The frequency response of the whole filter-bank cannot fully meet the condition for perfect reconstruction $TF(z) = 2z^k$. However the error is small enough for many applications.

The properties of the designed filter-bank was tested by a speech signal. This signal was applied to the input of the filter-bank, while the output of the filter-bank was connected to Simulink oscilloscope. Input and output signal waveforms of the filter-bank excited by the speech signal is demonstrated in Fig. 16 and 17. Both input and output signals are near the same. It is confirmation of the previous result. Our realization of filter-bank is applicable for speech applications. Filter-bank composed from Chebyshev and Inverse Chebyshev filters has better properties in comparative with the filter-bank constructed only with low-pass and high-pass Chebyshev filters.

3 CONCLUSIONS

Though the structure of the wave digital filter is more complicated than other structures, the algorithm for implementation on the DSP is very simple and it is very easy to propose general algorithm for arbitrary order of wave digital filter. These structures are less sensitive to the quantization error as other types of filters. Tables of the values A_i and B_i of the wave digital filters can be easily created by small modification of the presented design.The parts of the presented programs can be utilized for implementation of the Wave Digital Fiter (WDF) in digital signal processors. Filter-bank from Chebyshev end inverse Chebyshev filters was designed in this article and simulated

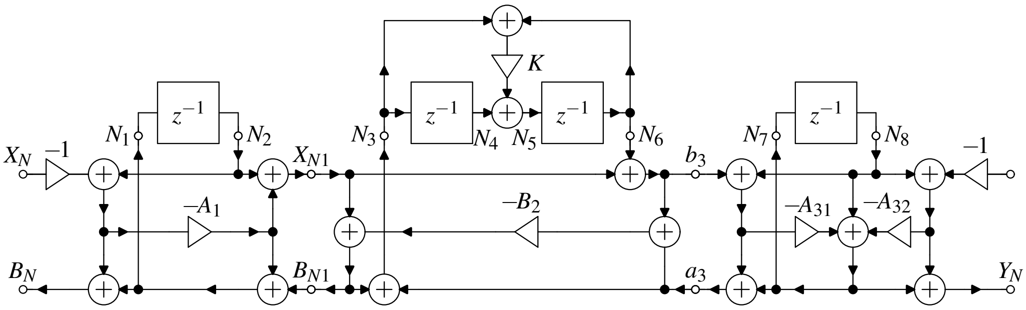


Figure 18: Discrete low-pass inverse Chebyshev filter in analysis filter-bank.

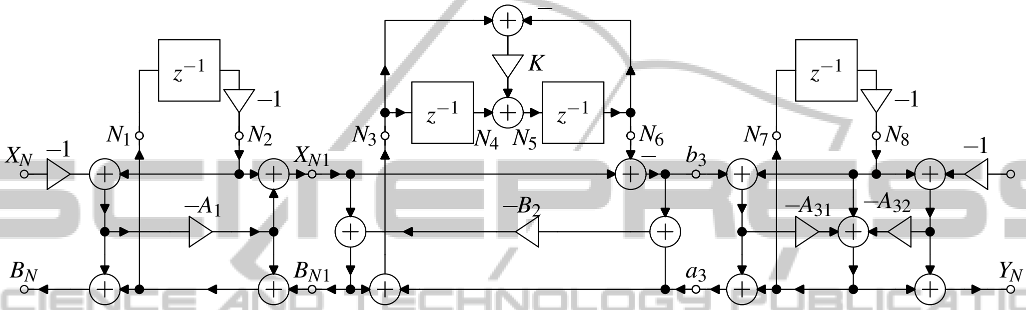


Figure 19: Discrete high-pass inverse Chebishev filter in synthesis filter-bank.

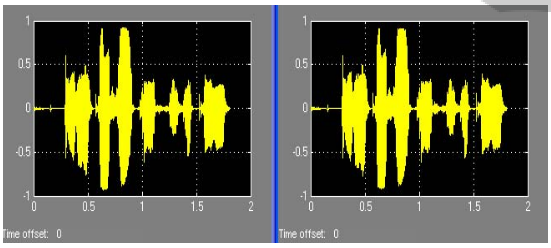


Figure 16: Input and output signal waveforms of the filter-bank – whole speech.

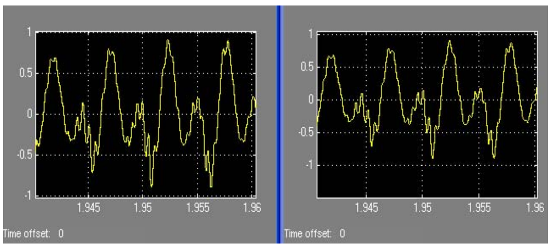


Figure 17: Input and output signal waveforms of the filter-bank – detail.

by signal processing Matlab toolbox containing the speech signal saved in Workspace.

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