On the Impact of the Clipping Techniques on the Performance of Optical OFDM

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Abstract: Recently, OFDM modulations are being considered for both optical fiber and wireless optical communications, specially due to their efficiency to combat the inter-symbol interference. Between the modifications of the standard OFDM that meet the requirements of the incoherent OFDM optical communications such as the ones that consider intensity modulation/direct detection, the asymmetric clipping optical OFDM and the DC biased optical OFDM techniques are the most popular but both involve an asymmetric clipping operation. Therefore, due to the high sensitivity of OFDM signals to nonlinearities, the nonlinear distortion effects introduced by the asymmetric clipping in the form of in and out-band-distortion should not be neglected. In fact, in order to address the performance of such systems, these distortion effects must be accurately characterized. In this work, by making use of a Gaussian approximation, we study analytically the impact of the asymmetric clipping in the performance of optical OFDM techniques by deriving theoretical expressions for the power spectral density and for the distortion at the subcarrier level, which is a key step to understand the potential performance of these systems.

1 INTRODUCTION

Employed in many wired and wireless communications standards specially due to their facility to combat the inter-symbol interference (ISI), their simple equalization processes and their ease of implementation, OFDM modulations (Cimini, 1985) have been also recently considered to support optical communications (Armstrong, 2009). However, in optical wireless communications (OWC) that consider incoherent OFDM systems (with intensity modulation/direct detection IM/DD) some aspects of the typical OFDM techniques must be changed, since the OFDM signal is used to modulate the transmitted light and, for this reason, must be real and unipolar. There are two well-established techniques to transform a conventional OFDM signal into a real and positive signal: the DC biased optical OFDM (DCO-OFDM) (Carruthers and Kahn, 1996) where a DC-bias is added to the original OFDM signal and the residual negative part of the signal is clipped and the asymmetric clipping optical OFDM (ACO-OFDM) (Armstrong and Lowery, 2006) where the original OFDM signal is deliberately clipped at zero. While the former doesn't present a good power efficiency, the latter is shown to be power efficient and has been target of recent research (Armstrong and Schmidt, 2008)(Dimitrov and Haas, 2010). However, both techniques involve the use of clipping operations.

One of the major OFDM weaknesses is the large envelope fluctuations of their signals that lead to the existence of a high peak-to-average power ratio (PAPR) and, consequently, high sensitivity to nonlinear devices. Therefore, a clipping operation in the transmission chain will lead to existence of nonlinear distortion effects in the transmitted signals and, for this reason, it is important to evaluate its impact on the performance.

Under the assumption that the OFDM signal presents a large number of subcarriers and using the central limit theorem the OFDM signal can be seen as a Gaussian random process. Making use of this approximation, a nonlinearly distorted OFDM signal can be divided in two uncorrelated terms: one that is proportional to the input signal and another that concentrates the nonlinear distortion effects (Rowe,

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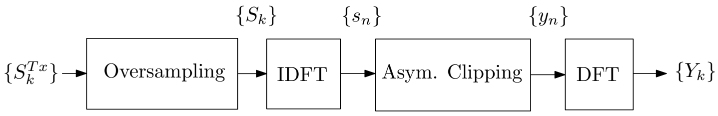

Figure 1: Considered OFDM transmission chain.

1982). In the literature, there are several works that employ statistical methods to derive closed-forms solutions for the output PSD of the nonlinearly distorted signal but they are mainly focused in odd nonlinear characteristics (Dinis and Gusmao, 2004)(Araújo and Dinis, 2010). The asymmetrical clipping employed in DCO-OFDM and in the ACO-OFDM is not an odd function and, for this reason, its analysis is different.

In this work we study the influence of an asymmetric clipping in optical OFDM transmissions by using a statistical approach that makes use of a Gaussian approximation. In order to access the performance of such systems, we derive theoretical expressions for the PSD at the nonlinearity output and we analyze both the in-band and out-band distortion introduced by this nonlinearity.

2 SYSTEM CHARACTERIZATION

In Fig. 1 it is represented the considered OFDM transmission chain.

In each OFDM frame, a data sequence $\{S_k^{Tx}; k = 0, 1, \ldots, N-1\}$ composed by N complex symbols from a given constellation (as for instance a quaternary phase shift keying (QPSK) constellation) is transmitted. The transmitted symbols are equiprobable, i.e., $\mathbb{E}[S_k^{Tx}] = 0$, and uncorrelated, i.e., $\mathbb{E}[S_k^{Tx} S_k^{Tx^*}] = 2\mathbb{E}[|S_k^{Tx}|^2]\delta_{k,k'}$, where $\delta_{k,k'}$ is the Kronecker delta. Moreover, as the OFDM signal modulates the transmitted light, its time-domain samples must be real and positive. To avoid the existence of complex samples at the inverse discrete Fourier transform (IDFT) output, the data vector $\{S_k^{Tx}; k = 0, 1, \ldots, N-1\}$ is constrained to have Hermitian symmetry, i.e.,

$$S_k^{Tx} = \begin{cases} 0, & k = 0, N/2 \\ S_{N-k}^{Tx} = S_k^{Tx^*}, & \text{otherwise.} \end{cases} \qquad (1)$$

We also considered an oversampling operation with oversampling factor M, performed through the addition of $M(N-1)$ idle subcarriers at the useful band edges. Thus, the final block that represents the signal to be transmitted is $\{S_k; k = 0, 1, \ldots, MN-1\}$. In this condition, the IDFT output is represented by $\{s_n; n = 0, 1, \ldots, MN-1\} = \text{IDFT}\{S_k; k = 0, 1, \ldots, MN-1\}$, with the n^{th} sample given by

$$s_n = \frac{1}{MN} \sum_{k=0}^{MN-1} S_k \exp\left(j2\pi \frac{kn}{MN}\right), \qquad (2)$$

and $\text{Im}(s_n) = 0 \quad \forall n$. The autocorrelation between the time-domain samples can be expressed as

$$R_{s,n-n'} = \mathbb{E}[s_n s_{n'}^*] = \frac{1}{(MN)^2} \sum_{k=0}^{MN-1} \mathbb{E}[|S_k|^2] \exp\left(-j2\pi \frac{k(n-n')}{MN}\right). \qquad (3)$$

Moreover, the autocorrelation and the power spectral density (PSD) form a Fourier pair, i.e., $\{R_{s,n-n'}; n, n' = 0, 1, \ldots, MN-1\} = \frac{1}{MN}\text{IDFT}\{G_{S,k}; k = 0, 1, \ldots, MN-1\}$, and $\mathbb{E}[S_k S_{k'}^*] = MNG_{S,k}$. To assure the positivity of the time-domain samples $\{s_n; n = 0, 1, \ldots, MN-1\}$, we consider an asymmetric clipping operation. This operation is represented by the following nonlinear function

$$f(x) = \begin{cases} x, & x > -s_M \\ -s_M, & x \leq -s_M. \end{cases} \qquad (4)$$

where s_M is the clipping level. In Fig. 2 it is depicted the nonlinear function that models the asymmetric clipping considering several clipping levels.

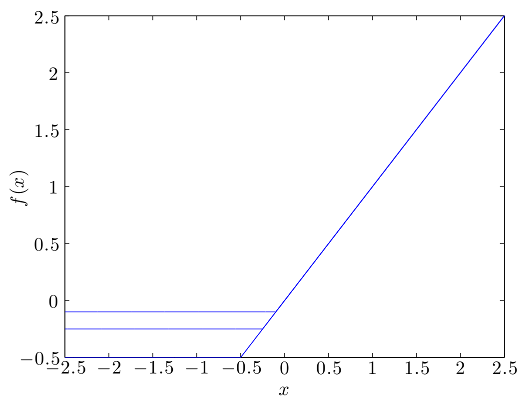

Figure 2: Asymmetric clipping considering different values of s_M.

Throughout this work we considered that $s_M < 0$ which is the typical scenario of the DCO-OFDM schemes. In these conditions a DC component must be added to the clipped signal in order to assure that the samples are positive. In the case of the ACO-OFDM schemes the clipping is made with $s_M = 0$ and there is no need for a DC-bias. For modeling purposes, when the number of subcarriers is large (let's say that $N \geq 32$), the time-domain samples of an OFDM signal are Gaussian distributed and can be modeled by s whose the probability density function (PDF) is

$$p(s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{s^2}{2\sigma^2}\right), \qquad (5)$$

where σ^2 is the variance of s. Thanks to the Gaussian nature of OFDM signals, the Bussgang theorem (Rowe, 1982) can be used. This theorem states that a nonlinearly distorted signal is given by the sum of two uncorrelated components: a scaled replica of the input signal and a term that concentrates the nonlinear distortion effects. Therefore, the n^{th} time-domain sample at the nonlinearity output can be described as

$$y_n = f(s_n) = \alpha s_n + d_n, \qquad (6)$$

where α is a relation between the cross-correlation between the input and the output signals of the nonlinearity and autocorrelation of the input signal

$$\alpha = \frac{\mathbb{E}[s_n y_n]}{\mathbb{E}[|s_n|]^2} = \frac{\mathbb{E}[s_n y_n]}{\sigma^2}, \qquad (7)$$

and $\{d_n; n = 0, 1, \ldots, MN-1\}$ represents the distortion term. In the frequency domain, the output is $\{Y_k; k = 0, 1, \ldots, MN-1\}$ and for the k^{th} subcarrier we have

$$Y_k = \alpha S_k + D_k, \qquad (8)$$

where $\{D_k; k = 0, 1, \ldots, MN-1\}$ is the distortion introduced in the k^{th} subcarrier. In Fig. 3 it is shown the evolution of α with the clipping level. In fact, throughout this work we consider a normalized clipping level s_M/σ since we are working with random signals and the clipping level must be related to their standard deviation σ.

As s_M/σ increases, it is less likely that the samples of the OFDM signal enter in the nonlinear region and the value of α tends to the unity (when $\alpha = 1$, no nonlinear distortion effects are introduced).

3 OUTPUT AUTOCORRELATION FOR MEMORYLESS NONLINEARITIES

In this section, in order to obtain the average PSD of a nonlinearly distorted random process we study its autocorrelation function, since it forms a Fourier pair

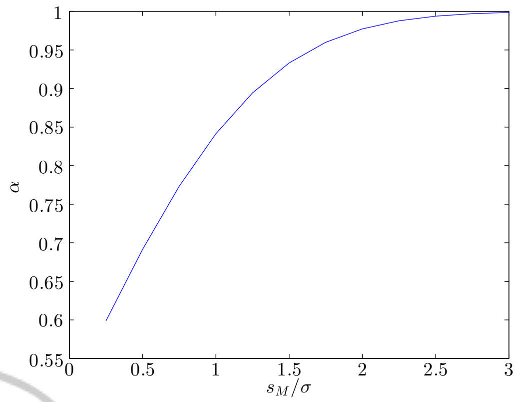

Figure 3: Evolution of α with the normalized clipping level s_M/σ.

with the PSD. Although the output autocorrelation for memoryless nonlinearities with Gaussian inputs were already studied in (Dinis and Gusmao, 2004)(Araújo and Dinis, 2010), these works only consider nonlinearities with odd characteristics, which is not the case of the asymmetrical clipping function characterized in (4). Our aim is to derive the autocorrelation of an OFDM signal that is submitted to this type of nonlinearities by making use of its Gaussian nature. Let us start by expressing the autocorrelation at the input of the nonlinearity. Due to the stationarity of the random process that models the OFDM signal, the autocorrelation only depends on the time lag between the observation moments, τ, and we can write

$$R_s(\tau) = \mathbb{E}[s_1 s_2] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s_1 s_2 p(s_1, s_2) ds_1 ds_2, \qquad (9)$$

where s_1 and s_2 are two random variables resulting from the observation of the random process at $t = 0$ and $t = \tau$, respectively. The variance of these random variables is σ^2 and their joint PDF is given by

$$p(s_1, s_2) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp\left(-\frac{s_1^2 + s_2^2 - 2\rho s_1 s_2}{2\sigma^2(1-\rho^2)}\right), \qquad (10)$$

where ρ is their correlation that is defined as

$$\rho = \rho(\tau) = \frac{R_s(\tau)}{R_s(0)}. \qquad (11)$$

At the nonlinearity output the autocorrelation is given by

$$R_y(\tau) = \mathbb{E}[f(s_1) f(s_2)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(s_1) f(s_2) p(s_1, s_2) ds_1 ds_2. \qquad (12)$$

However, it can be shown that the joint PDF of s_1 and s_2 can be expressed as a function of their marginal densities and the Hermite polynomials as

$$p(s_1,s_2) = p(s_1)p(s_2)\sum_{m=0}^{+\infty}\frac{\rho^m}{2^m m!}H_m\left(\frac{s_1}{\sqrt{2}\sigma}\right)H_m\left(\frac{s_2}{\sqrt{2}\sigma}\right), \tag{13}$$

which allows us to rewrite (12) as

$$R_y(\tau) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(s_1)f(s_2)p(s_1)p(s_2)\sum_{m=0}^{+\infty}\frac{\rho^m}{2^m m!}\times H_m\left(\frac{s_1}{\sqrt{2}\sigma}\right)H_m\left(\frac{s_2}{\sqrt{2}\sigma}\right)\mathrm{d}s_1\mathrm{d}s_2. \tag{14}$$

In addition, as $f(s_1) = f(s_2) = f(s)$, $p(s_1) = p(s_2) = p(s)$ and $H_m\left(\frac{s_1}{\sqrt{2}\sigma}\right) = H_m\left(\frac{s_2}{\sqrt{2}\sigma}\right) = H_m\left(\frac{s}{\sqrt{2}\sigma}\right)$ we can rewrite the output autocorrelation as

$$R_y(\tau) = \sum_{m=0}^{+\infty}\frac{\rho^m}{2^m m!}\left(\int_{-\infty}^{+\infty} f(s)p(s)H_m\left(\frac{s}{\sqrt{2}\sigma}\right)\mathrm{d}s\right)^2. \tag{15}$$

By defining P_m as the power of the intermodulation product of order m

$$P_m = \frac{1}{2^m m!}\left(\int_{-\infty}^{+\infty} f(s)p(s)H_m\left(\frac{s}{\sqrt{2}\sigma}\right)\mathrm{d}s\right)^2, \tag{16}$$

we can rewrite (15) as

$$R_y(\tau) = \sum_{m=0}^{+\infty}\rho^m P_m = \sum_{m=0}^{+\infty}\left(\frac{R_s(\tau)}{\sigma^2}\right)^m P_m. \tag{17}$$

With the average PSD of the output given by $G_y(f) = \mathrm{DFT}(R_y(\tau))$.

4 PERFORMANCE RESULTS

In this section we present a set of results to demonstrate the accuracy of the proposed analytical expression for the autocorrelation of a nonlinearly distorted signal. In Fig. 4 it is shown the PSD of the nonlinearity output $\{G_{Y,k}; k = 0,1,\ldots,MN-1\}$ both theoretically and by simulation using the fact that $G_{Y,k} = \mathbb{E}[|Y_k|^2]/MN$. We considered two oversampling factors $M = 8$ and $M = 4$, $N = 512$ data subcarriers, $m = 40$ and $s_M/\sigma = 1.0$. From the figure, we note that regardless of the oversampling factor, the accuracy of the theoretical expression for the PSD is very high. As $P_0 \neq 0$ there is a peak in the subcarrier in the middle of the spectrum ($k = 0$), since a DC component is introduced by the nonlinearity.

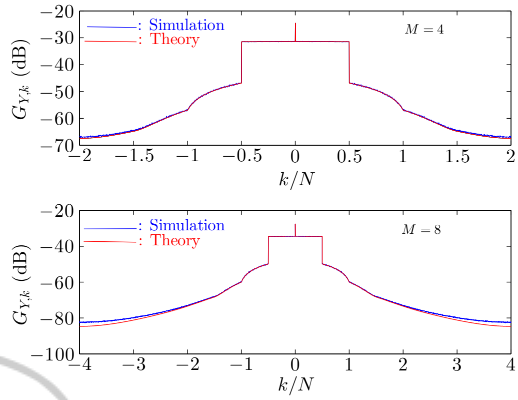

Figure 4: PSD of the nonlinearity output considering $M = 4$ (top figure) and $M = 4$ (bottom figure).

In Fig. 5 it is shown the PSD of the distortion term $\{G_{D,k}; k = 0,1,\ldots,MN-1\}$ obtained both theoretically (considering the contribution m intermodulation products except the one associated to the useful term, where $m = 1$) and by simulation considering that $G_{D,k} = \mathbb{E}[|D_k|^2]/MN$. We also considered that $M = 4$, $N = 512$ data subcarriers and $s_M/\sigma = 1.0$.

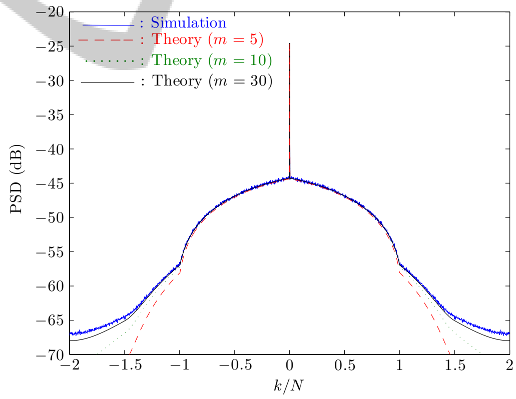

Figure 5: PSD of the distortion component obtained both theoretically and by simulation considering different values of m.

From the figure we can note that the results are very accurate with errors near 0 dB specially when we consider a large number of intermodulation products ($m = 30$). Regarding the performance evaluation, we can compute the signal-to-interference ratio (SIR). This ratio gives an indication of the performance losses at the subcarrier level that are associated with the in-band distortion introduced by the nonlinearity. The SIR for k^{th} subcarrier is defined as

$$\mathrm{SIR}_k = \alpha^2\frac{\mathbb{E}[|S_k|^2]}{\mathbb{E}[|D_k|^2]}. \tag{18}$$

In Fig. 6 it is shown the SIR computed both theoretically and by simulation considering $M = 4$, $N = 512$ data subcarriers, $s_M/\sigma = 1.0$ and $m = 40$. As the SIR is dependent on the average PSD of the distortion component, its accuracy is as high as the one of the Fig. 5 when m is high.

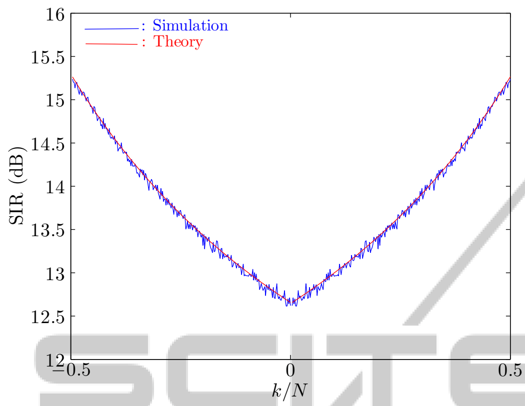

Figure 6: Evolution of the SIR obtained both theoretically and by simulation.

5 CONCLUSIONS

In this paper we considered incoherent optical OFDM systems that have a nonlinear operation in their transmission chain. It is presented an analytical method based on a statistical approach that can be used to characterize the distortion levels at the subcarrier level and, consequently, be used to access their performance. The analytical method is validated by a set of simulation results that demonstrate its high accuracy.

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