## Price Responses of Grain Market under Climate Change in Pre-industrial Western Europe by ARX Modelling

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Keywords: ARX Modelling, Climate Change, Grain Market, Pre-Industrial Era, Western Europe.

Abstract: In academia, there are few studies adopted ARX modelling on historical datasets. Recently, the studies on notable effects of climatic changes upon past agrarian economy are paid by more attention. Here, this study first time seriously explores the relationship between climatic change and the grain market at a macro-scale in pre-industrial Western Europe by ARX modelling. The results show that a cold phase would raise grain price through lowering the supply in the grain market. Furthermore, according to the simulations on short-and long-term climate change, the long lasting climate change could be more disastrous to society than short-term change. Last, the application in the study proves ARX modelling is also a feasible choice in the field of historical research.

## **1 INTRODUCTION**

ARX, an autoregressive model with exogenous elements, can capture and reflect the variations in the temporally changing systems (Qin et al, 2010). This method is also useful in simulating the influence of past conditions and external systems on changing temporal factors (Hamilton, 1994). Moreover, the ARX model is regarded as extremely suitable for control theories with a simpler estimation in the field of signal studies and engineering (Huusom et al, 2010). However, there is few studies of application so far to adopt this statistical method to simulate the historical dataset (Pei et al, 2013).

In recent years, the studies on the notable effects of climatic changes upon past agrarian economy have attracted lots of attention in academia. Among different social-economic sectors, the grain market is the most sensitive to climate change because agricultural production is highly dependent on climatic conditions (IPCC, 2013). This was especially true in the past agrarian era (Beveridge, 1921). Surprisingly, these historical climatic impacts in relation to human agrarian society at a large spatial and long-term scale have been academically neglected from a quantitative perspective (Zhang et al, 2013), though important attempts recently have been made to use high-resolution palaeo-climatic records to explain several pre-historical socialeconomic changes in certain time periods of past society, which is studied at the level of cases (An et al, 2005; deMenocal, 2001; Polyak, 2001).

Under this background, the proposed study will focus on the impact of climate change on the grain market in pre-industrial Western Europe from AD 1500 to 1800. This study first time seriously explores the relationship between climatic change and the grain market at a macro-scale in preindustrial Western Europe by ARX. In the meantime, ARX could also be evaluated with its application to historical research. Furthermore, ARX is significantly useful to examine the temporal patterns of changes at the both short- and long-term scale. Through the check on short- and long-term of climatic impact, the vulnerability of the grain market under climate change could be further uncovered as well.

The quantitative analyses justify that the reduction of thermal energy input during a cold phase would raise the grain price and lead to price crisis through decreasing the agricultural supply in the grain market. According to the examinations on both long-term and short-term climate change, the study finds that the long lasting climate change could be more disastrous to society than short-term variations, particularly at the large spatial scale.

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## 2 DATA AND METHODOLOGY

#### 2.1 Study Region

The Western Europe in the study includes Albania, Belgium, Britain, Denmark, France, Germany, Greece, Portugal, Ireland, Italy, Netherland, Scotland, Spain, and Switzerland. This area is also overlapping the Temperate Region based on Koppen Climate Classification (Gerstengarbe and Werner, 2008).

## 2.2 Temperature

In pre-industrial society, agricultural production closely depended on climate. A cooling and variable climate can bring serious problems for food production, especially in the high and middle latitudes (Galloway, 1986). Temperature is a better indicator at a large scale (Jones and Bradley, 1992) and is most essential to plant growth (Mathias, 1990). Besides, cooler period is associated with greater variability in the short-term weather (Bryson and Murray, 1977; Gribbin and Lamb, 1978). Therefore, based on historical survey, the price of grain market has been pointed to highly correlate with temperature (Lamb, 1995).

#### 2.3 Population

Population is an interesting and crucial research topic in academia, because population always is assumed to play dual roles: labour and consumer. The population changes relate with social ability and contribute to the food productions (Robinson, 1959). This discussion leads to two possible relations between price and population. If the population acts as the labour, then relation will be negative. Because the more the available labour, the more supply will be realized. The affluent supply in the market will push the price decline. However, if the population plays as consumer, the relation will be positive. The price in the market certainly can be driven higher by more demand.

## 2.4 Real Price

In economics, real price or sometime is also used as the name of relative price, is a fundamental concept for study of economics, especially in microeconomics. The inflation in the business cycle could keep raising the price level, which changes the money value in the real world (Spencer and Orley, 1993). Hence, the real price must be adopted to correct the inflation rate, particularly when studying the prices over time in the long run (Pindyck and Rubinfeld, 1995). The nominal price or so-called money price could not reflect how costly it is in reality (Browning and Zupan, 1996). Through adopting the real price (relative price) into the analysis, it could avoid the influences from other commodities in the market and keep the consistently to reflect the commodities value (Landsburg, 1999). Therefore, in this study, the real price is adopted for the analysis. In this study, the study period is from AD 1500 to 1800.

## 2.5 Data Source

In recent years, scientists around the world have carried out intensive research on past climate change, increasingly using multi-proxy data networks to reconstruct past climate variations in terms of temperature anomaly. As suggested by Zhang et al. (2007), Osborn's (2006) temperature anomaly series and Luterbacher's palaeo-climate reconstructions (Luterbacher et al, 2004) over the AD 1500 to1800 were apt to be chosen together to carry out the quantitative analysis.

In this study, population size of Europe was extracted from McEvedy and Jones' (1978) *Atlas of World Population History*. This is a remarkably accurate work, which have been repeatedly used by other scholars.

The cited price data and CPI data in the study is all from the International Institute of Social History Database and Allen - Unger Database European Commodity Prices AD 1260-1914. The price data covers four types of grains (wheat, rye, barley, and oats). The price and CPI data are from major European regions: Amsterdam and Holland, Antwerp and Belgium, Augsburg, Leipzig, London and Southern England, Madrid and New Castile, Munich, Naples, Northern Italy, Paris, and Strasbourg. Figure 1 shows the curves of each data series in the study.

## **3 RESULTS**

In the study, the ARX modelling is adopted to simulate the price responses under climate change in pre-industrial Western Europe systematically. The final modelling is selected based on whole consideration of Residual Analysis, Parameter Analysis and  $R^2$ . Based on criteria of model selections, the following model is chosen as the

fitted model for European grain market from AD 1500 to 1800. The results are shown in Table 1.

Based on the modelling results, a cold phase would raise grain price through lowering the supply in the grain market, while the mild climate would be favourable to agrarian economy.

In the meantime, the population is acting as role of farmer in the study period. The simulation results do not imply that population in pre-industrial Western Europe did not act as consumer at all in the past. However, the role of producer exceeds the role of consumer in the long term.

#### 4 **DISUCUSSION**

#### 4.1 Annual Impact of Climate Change

First, the impact of climate on the grain market virtually exists. The statistical model results are consistent with the literature survey. The hypothesis of study is not only theoretically sound, but also quantitatively verified.

Second, the fluctuation rate in temperature will be enlarged when it impacts on the grain market, based on the modelling. The larger the temperature changed, the larger price change would be, according to the pattern of ARX modelling results.

Besides, based on the modelling results, the interaction between climate change and grain market is not a linear process, though it is due to the modelling design in the research. However, as pointed out, the process of climate change can be non-linear (Schneider, 2004), and its corresponding effect on the socioeconomic system can also reflect non-linear patterns (Adger et al, 2009). Therefore, following the current research on climate change issues, price and temperature can be considered exponential functions according to our statistical results as well as to our studies.

# 4.2 Long Term Impact of Climate Change

In order to examine the long term impact of temperature parameter, the time series theory should be reviewed. Generally, the ARX model can also be written by the formula transformation, which is listed in the Section of "Equations".

Based on the result, the impact of climate change from year t will be lasting long in the following year. Furthermore, attenuation speeds of obvious climatic impact last 10 years according to Figure 2. After 10 year, the impact is almost equal to zero. The attenuation speeds of temperature impact show the buffering capacity of human society to relieve the climatic impact gradually though still exists for 10 years. This result justified again that the preindustrial Western Europe could try to make the adaptation and relief to the climate change, while with limited effectiveness. However, compared to whole Europe of 25 years lasting effect (Pei et al, 2013), the higher population density makes Western Europe is more vulnerable to climate change.

Due to the low speed of attenuation, cooling impact could pile up, especially during the long term cooling period. In the short-term (several years), those social buffers are effective in stabilizing grain prices. However, institutional and social buffering mechanisms would be ultimately exhausted by the recurrent subsistence crises caused by long termcooling (Orlove, 2005). Worldwide empirical studies also have revealed that in the face of persistent agricultural shortages induced by long-term cooling, social buffers ultimately became ineffective and were unable to prevent social-economic crisis (Lee et al, 2008; Pei et al, 2014; Zhang et al, 2007). Therefore, the long lasting climate change could be more disastrous to society than short-term climate variations.

Lastly, in addition to above theoretical implications of a specific field, the simulation in the study proves that ARX modelling is a feasible choice in the field of historical research.

## 5 CONCLUSIONS

Climate change has played a very important role in Western European agrarian economy in the preindustrial era. The current study first time adopts ARX modelling to scrutinize climate-economy association in pre-industrial Western Europe AD 1500-1800. This study fills the gap in previous quantitative analyses about the short- and long-term effect of climate change on past agrarian economies.

Through the statistical analysis, temperature is important to economy of pre-industrial Western Europe at a large spatial scale. In the short term, cooling climate could cause high prices because of poor production and scarcity in the grain market. The larger the changes in temperature, the larger the price changes are, which shows the non-linear interaction between climate and economy in the past. In the long term, the impact from climate change could last around 10 year, which reflects the social buffering capacity. The long term climatic impact, especially 10 year or even longer could pile up and finally destroy the economic equilibrium. In consequence the long last climate change could be more disastrous to society in the past era.

The findings of this study do not refute other theories on climate change and economic mechanism in history. This study is different from its predecessors in terms of both temporal scale and hierarchies of reasoning (levels of quantitative association). The long-term economic mechanism is embedded in a complex system that includes both environmental and social components. Any complex system is determined by different factors at different spatial-temporal scales (O'Neill et al 1989; Norton, and Ulanowicz, 1992). At a given spatial-temporal scale, some processes are more fundamental than the rest in the system (Tilly, 1984; Pei and Zhang, 2014). Other economic theories generated from case and short-term studies have been limited by their spatial-temporal scales. The explanation and generalization to long term economic mechanism in this study may, of course, not be appropriate in other studies with different temporal scales.

We explored the long historical consequences of climate change by examining the high-resolution frequency and time domains of different time series. The characteristics of this large unit are not simple combinations of the attributes of small units but demonstrate the climatic impacts on economic fluctuations, which is a new theory of economic change. Hence, this study is an innovative way of identifying dominant causes in social and historical processes across a broad range of temporal scales. Research concerning scale in the social sciences has been criticized as being insufficiently explicit and precise due to its complexity (Gibson et al, 2000). Nevertheless, our accurate and comprehensive explanation of a complex system reveals that social science research is capable of attaining the standards applicable to physical scientific research by using novel quantitative methods and scientific thought.

## TABLE

Table 1: ARX Model in Western Europe at lag=2 (Significant level = 90%).

		-	Estimate	SE	t	Sig.
Constant			-1.012	0.469	-2.156	0.032
LnRP	AR	Lag 1	0.860	0.058	14.858	0.000
		Lag 2	-0.143	0.058	-2.477	0.014
Tem		Lag 0	-0.015	0.009	-1.712	0.088
LnPop		Lag 0	-0.195	0.110	-1.769	0.078

Stationary R<sup>2</sup>=0.616

FIGURES



Figure 1: Climatic changes and parameters of grain market in Europe, AD 1500-1800. (a) Normalized temperature change records in Europe. (b) Western European population size. (c) Real grain price of Western European.



Figure 2: Decline rate of temperature impact in the long term.

#### **EQUATIONS**

Then the base year is set as AD 1500. It is calculated as follows:

$$RP_{t} = \frac{CPI_{BaseYear}}{CPI_{t}} \times P_{t}$$
(1)

$$\phi(B)Y_t = X_t'\beta \tag{2}$$

Where:

 $\phi(B)$  is the backward equation for  $\{Y_t\}$ 

$$X'_{t}\beta$$
 is the regression part of ARX

$$Y_{t} = \frac{1}{\phi(B)} X_{t}^{\prime} \beta$$

$$Y_{t} = \varphi^{(0)}(B) X_{t}^{\prime} \beta$$
(3)

Where:

$$\varphi^{(0)}(B) = \frac{1}{\phi(B)} = \sum_{j=0}^{\infty} \varphi_j^{(0)} B^j \Longrightarrow Y_t = \sum_{j=0}^{\infty} \varphi_j^{(0)} X'_{t-j} \beta \tag{4}$$

Here the time point t-m is the time point when abnormal phenomena happened, then the impact on the following year  $Y_t$  is calculated as below:

$$\frac{1}{\phi(B)} = \sum_{j=0}^{\infty} \varphi_j^{(0)} B^j \Longrightarrow \phi(B) \times \sum_{j=0}^{\infty} \varphi_j^{(0)} B^j = 1$$
(5)

The ARX in the article is an ARX (2) model. Then,

$$\begin{bmatrix} [1-\phi_{1}(B)-\phi_{2}(B^{2})] \times \sum_{j=0}^{\infty} \phi_{j}^{(0)} B^{j} = 1 \\ \sum_{j=0}^{\infty} \phi_{j}^{(0)} B^{j} - \phi_{1} \sum_{j=0}^{\infty} \phi_{j}^{(0)} B^{j+1} - \phi_{2} \sum_{j=0}^{\infty} \phi_{j}^{(0)} B^{j+2} = 1 \\ \phi_{0}^{(0)} = 1 \\ \phi_{1}^{(0)} - \phi_{1} \phi_{0}^{(0)} = 0 \\ \phi_{2}^{(0)} - \phi_{1} \phi_{1}^{(0)} - \phi_{2} \phi_{0}^{(0)} = 0 \\ \cdots \end{bmatrix} \Rightarrow \phi_{R}^{(0)} = \phi_{1} \phi_{R-1}^{(0)} + \phi_{2} \phi_{R-2}^{(0)} \quad R \ge 2$$

$$(6)$$

Where:

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$$\phi_1 = 0.860 
\phi_2 = -0.143$$
(7)

$$\varphi_{R}^{(0)} = 0.860\varphi_{R-1}^{(0)} - 0.143\varphi_{R-2}^{(0)} \quad R \ge 2$$

Based on the above calculation process, with the consideration of ARX model fitted, the final expression of temperature change impact in year t on the following year is as below.

$$\frac{\partial \ln RP_{i+j}}{\partial Tem_i} = [0.860\,\varphi_{R-1}^{(0)} - 0.143\,\varphi_{R-2}^{(0)}] \times (-0.015)$$

$$R \ge 2, \quad j = 1, 2, 3...$$
(8)

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#### APPENDIX

#### **Abbreviations and Acronyms**

RP represents real grain price.

CPI stands for Consumer Price Index.

P represents nominal grain price.

t is the time step and the base year is AD 1500.

LnRP stands for ln value of real grain price.

Tem stands for temperature.

LnPop stands for ln value of population size.

#### Units

Temperature:  $\delta$ , it is anomaly of past temperature reconstructions.

Real price: Ag Gram/liter. Population size: million.

## **Expression of Regression Modelling**

The Classic Linear Regression Model is expressed as below:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i$$
  $i = 1, 2, 3, \dots n$ 

There are p+1 parameters will be estimated. In matrix terms this becomes:

 $Y = X\beta + \varepsilon$ 

Where:	
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$Y = \begin{cases} y_1 \\ y_2 \\ y_3 \\ \dots \\ y_n \end{cases}$	$X_i = \begin{cases} 1\\1\\1\\\dots\\1 \end{cases}$	$x_{11}$ $x_{21}$ $x_{31}$  $x_{n1}$	$x_{12} \\ x_{32} \\ x_{32} \\ \dots \\ x_{n2}$	$x_{13}$ $x_{33}$ $x_{33}$  $x_{n3}$	   $ \left.\begin{array}{c}x_{1p}\\x_{2p}\\x_{3p}\\\dots\\x_{np}\end{array}\right\} $
$\boldsymbol{\beta} = \begin{cases} \boldsymbol{\beta}_0 \\ \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \boldsymbol{\beta}_3 \\ \cdots \\ \boldsymbol{\beta}_n \end{cases}$	$\mathcal{E} = \begin{cases} \mathcal{E}_1 \\ \mathcal{E}_2 \\ \mathcal{E}_3 \\ \dots \\ \mathcal{E}_n \end{cases}$				

The most commonly used criterion to estimate the parameters in the regression model is the principle of Least Squares, which involves minimizing the sum of Residual Square.

The regression model will be worked out as below:

 $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + ... + \hat{\beta}_p x_p$ 

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#### **Expression of AR Modelling**

If the time series is autoregressive process, then a general time series will be obtained.

 $Y_{t} = \phi_{1}Y_{t-1} + \phi_{2}Y_{t-2} + \dots + \phi_{p}Y_{t-p}$ 

 $\{Y_t\}$  is a mixed autoregressive process of orders p, that is AR (p) model.  $\{Y_t\}$  is the observed value at time *t*.

## **Expression of ARX Modelling**

The ARX modelling is realized by above two parts: regression and AR.

 $Y_{t} = \beta_{0} + \beta_{1}x_{i1} + \beta_{2}x_{i2} + \dots + \beta_{p}x_{ip} + \phi_{1}Y_{t-1} + \phi_{2}Y_{t-2} + \dots + \phi_{p}Y_{t-p} + \varepsilon_{t}$  $i = 1, 2, 3, \dots n$ 

Through model parameter estimation, the above ARX model will be expressed as below:

$$Y_{t} = \hat{\beta}_{0} + \hat{\beta}_{1}x_{1} + \hat{\beta}_{2}x_{2} + \ldots + \hat{\beta}_{p}x_{p} + \hat{\phi}_{1}Y_{t-1} + \hat{\phi}_{2}Y_{t-2} + \ldots + \hat{\phi}_{p}Y_{t-p}$$

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