Sliding Mode Control of Linear Time-varying Systems
Application to Trajectory Tracking Control of Nonlinear Systems

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Keywords: Sliding Mode Control, Linear Time-varying System, Non-linear System, Tracking Control.

Abstract: This paper concerns with the sliding mode controller design method for linear time-varying systems. For this purpose, using the time-varying pole placement technique, the state feedback is designed first so that the time-varying closed loop system is equivalent to the standard linear time invariant system. Then, conventional sliding mode controller design method is applied to this time invariant system to obtain the control input. Finally, using the time-varying transformation matrix, this sliding mode control input is put back to the control input for the original system. In this paper, this controller is applied to the trajectory tracking control problem for nonlinear systems.

1 INTRODUCTION

This paper concerns with the sliding mode controller design for linear time varying systems, and then, we apply this control technique to a trajectory tracking control of non-linear systems.

The author proposed the simple design method of the pole placement controller for linear time varying systems using the concept of the relative degree of the system (Mutoh,2011) (Mutoh and Kimura,2011). This pole placement design purpose is to make the time varying closed loop system equivalent to some linear time invariant system that has desired eigenvalues, by the state feedback. In this paper, we make use of this technique for designing the sliding mode controller for linear time varying systems. The first step is to find the state feedback for the linear time varying system so that the closed loop system is equivalent to some linear time-invariant standard system. Then, by using the conventional sliding mode controller design method, the sliding mode control input for this linear time invariant system can be obtained (Utkin,1992). After that, using an equivalent time varying transformation matrix, this control input can be transformed into the sliding mode control for the original linear time varying system. Since, the sliding mode controller is designed for the equivalent time invariant system, any type of conventional sliding mode controller design method can be applied.

If we need to control nonlinear systems to follow some particular desired trajectory in wide range, the most simple idea might be to approximate the nonlinear system along this trajectory using a linear time-varying system. However, since, controller design method for linear time-varying system is not necessarily simple, it seems that this approach is not commonly used. In this paper, the above time varying sliding mode control technique is applied to the trajectory tracking control problem of non-linear systems. Some simulation results will be also shown.

2 PRELIMINARIES

In this section, the basic properties of linear time-varying systems which we will use later are presented.

Consider the following linear time-varying multi-input system.

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{1}$$

Here, $x(t) \in R^n$ and $u(t) \in R^m$ are the state variable and the input signal, respectively. $A(t) \in R^{n\times n}$ and $B(t) \in R^{n\times m}$ are time varying coefficient matrices, which are bounded and smooth functions of t.

The matrix $B(t)$ is written as follows, using column vectors $b_i(t) \in R^n$ $(i = 1, \cdots, m)$.

$$B(t) = \begin{bmatrix} b_1(t) & b_2(t) & \cdots & b_m(t) \end{bmatrix} \tag{2}$$

Let $b_k^i(t) \in R^n$ be defined by the following recur-

Mutoh Y. and Kogure N..
Sliding Mode Control of Linear Time-varying Systems - Application to Trajectory Tracking Control of Nonlinear Systems.
DOI: 10.5220/0005019704920498
In *Proceedings of the 11th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2014), pages 492-498
ISBN: 978-989-758-039-0

sive equations.

$$\begin{cases} b_k^0(t) = b_k(t) \\ b_k^i(t) = A(t)b_k^{i-1}(t) - \dot{b}_k^{i-1}(t) \end{cases} \tag{3}$$
$$k = 1,2\cdots m,\ i = 1,2\cdots$$

Then, the controllability matrix of the system (1) can be written as follows.

$$U_c = [b_1^0(t)\cdots b_m^0(t)|\cdots|b_1^{n-1}(t)\cdots b_m^{n-1}(t)] \tag{4}$$

Theorem 1. *The system* (1) is completely controllable if and only if

$$rankU_c(t) = n \quad {}^{\forall}t \tag{5}$$

If the system (1) is completely controllable, we can define the controllability indices, μ_1, μ_2,$\cdots$, μ_m, which satisfy the following equations,

$$\begin{array}{rcl} R(t) & : & nonsingular \\ \sum_{i=1}^{m}\mu_i & = & n \end{array} \tag{6}$$

where

$$R(t) = \left[\ b_1^0(t)\cdots b_1^{\mu_1-1}(t)|\cdots|b_m^0(t)\cdots b_m^{\mu_m-1}(t)\ \right] \tag{7}$$

which is called the truncated controllability matrix. In this paper, it is assumed that if the system is completely controllable, its controllability indices satisfy the inequality, $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_m$, without loss of generality.

Definition 1. *Consider the following output equation for the system (1),*

$$y(t) = C(t)x(t) \tag{8}$$

Here, $y(t) \in R^m$ is some output signal and $C(t) \in R^{m\times n}$ is a time varying coefficient matrices. Let p be a differential operator. System (1)(8) has the vector relative degree, r_1, r_2, $\cdots$, r_m from u to y, if there exist some matrix $D(t) \in R^{m\times n}$ and some nonsingular matrix $\Lambda(t) \in R^{m\times m}$, such that

$$\begin{bmatrix} p^{r_1} & & \\ & \ddots & \\ & & p^{r_m} \end{bmatrix} y(t) = D(t)x(t) + \Lambda(t)u(t). \tag{9}$$

It should be noted that p^{r_i} can be replaced by arbitrary monic polynomial of p of degree r_i. ∇

3 STANDARD TIME INVARIANT SYSTEM

To design the sliding mode controller for the system (1), we first design the state feedback with a new input vector $v(t) \in R^m$, so that the closed loop system is equivalent to the linear time invariant standard form.

Suppose that the system (1) is completely controllable. Then, if $\tilde{C}(t) \in R^{m\times n}$ is defined by

$$\tilde{C}(t) = W(t)R^{-1}(t) \tag{10}$$

where

$$\begin{cases} W(t) = diag(w_1(t), w_2(t), \cdots, w_m(t)) \\ w_i(t) = [0,\cdots,0,\lambda_i(t)] \in R^{1\times\mu_i}(i = 1,\cdots,m) \end{cases} \tag{11}$$
$$\lambda_i(t) \neq 0$$

and also if a new output signal $\tilde{y}(t) \in R^m$ is defined by

$$\tilde{y}(t) = \tilde{C}(t)x(t) \tag{12}$$

then, the vector relative degree from $u(t)$ to $\tilde{y}(t)$ is $\mu_1, \mu_2, \cdots, \mu_m$ (Mutoh and Kimura,2011).

Let $\tilde{y}(t)$ and $\tilde{C}(t)$ be

$$\tilde{y}(t) = \begin{bmatrix} \tilde{y}_1(t) \\ \vdots \\ \tilde{y}_m(t) \end{bmatrix}, \tilde{C}(t) = \begin{bmatrix} \tilde{c}_1(t) \\ \vdots \\ \tilde{c}_m(t) \end{bmatrix}. \tag{13}$$

By differentiating $\tilde{y}(t)$ successively, we have

$$\begin{array}{rcl} \tilde{y}_i(t) & = & \tilde{c}_i^0(t)x(t) \\ \dot{\tilde{y}}_i(t) & = & \tilde{c}_i^1(t)x(t) \\ \ddot{\tilde{y}}_i(t) & = & \tilde{c}_i^2(t)x(t) \\ & \vdots & \\ \tilde{y}_i^{(\mu_i)}(t) & = & \tilde{c}_i^{\mu_i}(t)x(t) + \tilde{c}_i^{\mu_i-1}(t)B(t)u(t) \\ & = & \tilde{c}_i^{\mu_i}(t)x(t) + \lambda_i(t)u_i(t) \\ & + & \gamma_{i(i+1)}u_{i+1}\cdots + \gamma_{im}(t)u_m(t) \\ & & i = 1,\cdots,m \end{array} \tag{14}$$

Here, $\tilde{c}_i^j(t)$ and $\gamma_{ij}(t)$ are obtained by the following recursive equation from $\tilde{C}(t)$.

$$\begin{cases} \tilde{c}_i^0(t) = \tilde{c}_i(t) \\ \tilde{c}_i^{j+1}(t) = \tilde{c}_i^j(t)A(t) + \dot{\tilde{c}}_i^j(t) \end{cases} \tag{15}$$
$$i = 1,2\cdots m, j = 1,2\cdots$$

and

$$\gamma_{ij}(t) = c_i^{\mu_i-1}(t)b_j(t). \tag{16}$$

Hence, from (14), we have

$$\begin{bmatrix} p^{\mu_1} & & \\ & \ddots & \\ & & p^{\mu_m} \end{bmatrix} \tilde{y}(t) = D(t)x(t) + \Lambda(t)u(t) \tag{17}$$

where,

$$D(t) = \begin{bmatrix} \tilde{c}_1^{\mu_1}(t) \\ \tilde{c}_2^{\mu_2}(t) \\ \vdots \\ \tilde{c}_m^{\mu_m}(t) \end{bmatrix}, \Lambda(t) = \begin{bmatrix} \Lambda_1(t) \\ \Lambda_2(t) \\ \vdots \\ \Lambda_m(t) \end{bmatrix} \tag{18}$$

and

$$\Lambda_i(t) = [0,\cdots,0,\lambda_i(t),\gamma_{i(i+1)}(t),\cdots,\gamma_{ij}(t)] \quad (19)$$

Thus, by the state feedback

$$u(t) = \Lambda^{-1}(t)(-D(t)x(t)+v(t)) \quad (20)$$

with the new input signal $v(t) \in R^m$, the closed loop system becomes

$$\begin{bmatrix} p^{\mu_1} & & \\ & \ddots & \\ & & p^{\mu_m} \end{bmatrix} \tilde{y}(t) = v(t). \quad (21)$$

This system has the following state realization.

$$\begin{aligned} \dot{\omega}(t) &= A^*\omega(t)+B^*v(t) \\ &= \begin{bmatrix} A_1^* & & 0 \\ & \ddots & \\ 0 & & A_m^* \end{bmatrix}\omega(t) \\ &+ \left[\begin{array}{ccc} b_1^* & \cdots & 0 \\ \hline & \vdots & \\ \hline 0 & \cdots & b_m^* \end{array}\right] v(t) \quad (22) \end{aligned}$$

where $\omega(t) \in R^n$, $A^* \in R^{n\times n}$, $B^* \in R^{n\times m}$, and

$$\begin{aligned} A_i^* &= \begin{bmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ \vdots & & & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix} \in R^{\mu_i\times\mu_i} \\ & (i=1,\dots,m) \quad (23) \\ b_i^* &= \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{\mu_i}. \end{aligned}$$

The system (22)(23) is called the linear time invariant standard form. This new state variable $\omega(t) \in R^n$ is defined by

$$\omega(t) = \left[\begin{array}{c} \tilde{y}_1(t) \\ \vdots \\ \tilde{y}_1^{(\mu_1-1)}(t) \\ \hline \vdots \\ \hline \tilde{y}_m(t) \\ \vdots \\ \tilde{y}_m^{(\mu_m-1)}(t) \end{array}\right]. \quad (24)$$

From (14), the original state variable $x(t)$ and $\omega(t)$ satisfy the relation

$$\omega(t) = T(t)x(t) \quad (25)$$

where the transformation matrix, $T(t)$, is defined by

$$T(t) = \left[\begin{array}{c} \tilde{c}_1^0(t) \\ \vdots \\ \tilde{c}_1^{\mu_1-1}(t) \\ \hline \vdots \\ \hline \tilde{c}_m^0(t) \\ \vdots \\ \tilde{c}_m^{\mu_m-1}(t) \end{array}\right]. \quad (26)$$

4 SLIDING MODE CONTROLLER DESIGN

4.1 Controller for Linear Time-varying Systems

In this section, the sliding mode controller design for the linear time varying system (1) is presented. For this purpose, we first design the sliding mode control input $v(t)$ for the linear time invariant system (22)(23), and then, transform $v(t)$ into the sliding mode control input for the original system (1), using the relation (25)(26).

If we write $\omega(t)$ and $v(t)$ as

$$\begin{aligned} \omega(t) &= \begin{bmatrix} \omega_1(t) \\ \vdots \\ \omega_m(t) \end{bmatrix}, \quad \omega_i(t) \in R^{\mu_i} \\ v(t) &= \begin{bmatrix} v_1(t) \\ \vdots \\ v_m(t) \end{bmatrix} \quad (27) \\ & (i=1,\cdots,m) \end{aligned}$$

the system (22)(23) is presented as following m subsystems.

$$\begin{aligned} \dot{\omega}_i(t) &= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \omega_i(t) + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v_i(t) \\ & (i=1,\cdots,m) \quad (28) \end{aligned}$$

Since the system (28) is the standard form, the design procedure of the ordinary sliding mode controller is very simple stated as follows. First, divide $\omega_i(t)$ into two part.

$$\omega_i(t) = \begin{bmatrix} \overline{\omega}^i(t) \\ \omega_i^{\mu_i}(t) \end{bmatrix} \quad (29)$$

where $\overline{\omega}^i(t) \in R^{\mu_i-1}$ and $\omega_i^{\mu_i}(t) \in R$. Then, the sliding surface is defined by

$$S_i\omega_i(t) = \overline{s}_i\overline{\omega}_i(t) + \omega_i^{\mu_i}(t) = 0 \tag{30}$$

where

$$S_i = [\overline{s}_i, 1] \in R^{1\times\mu_i}, \quad \overline{s}_i \in R^{1\times(\mu_i-1)} \quad (i = 1,\cdots,m). \tag{31}$$

From (28),(29) and (30), the dynamics on the i-th sliding surface becomes

$$\dot{\overline{\omega}}_i(t) = \left[\begin{array}{ccccc} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \dots & \dots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ \hline \multicolumn{5}{c}{-\overline{s}_i} \end{array}\right] \overline{\omega}_i(t). \tag{32}$$

From the above, if the desired stable characteristic polynomial of the i-th sliding dynamics is chosen as

$$\alpha^i(p) = p^{\mu_i-1} + \alpha^i_{\mu_i-2}p^{\mu_i-2} + \cdots + \alpha^i_0 \tag{33}$$

then, the i-th sliding surface is

$$S_i\omega_i(t) = [\ \alpha^i_0,\ \cdots, \alpha^i_{\mu_i-2},\ 1\ ]\omega_i(t) = 0. \tag{34}$$

Since, the i-th subsystem is

$$\dot{\omega}_i(t) = A_i^*\omega_i(t) + b_i^*v_i(t) \tag{35}$$

it is well known that the i-th sliding control input $v_i(t)$ can be defined by

$$v_i(t) = -(S_ib_i^*)^{-1}\{S_iA_i^*\omega_i(t) + q_i\ sgn(\sigma_i) + k_if_i(\sigma_i)\} \tag{36}$$

where

$$\sigma_i = S_i\omega_i(t) \tag{37}$$

and $q_i > 0$ and $k_i > 0$ are constant parameters and $f_i(\sigma_i)$ is a function such that $\sigma_if_i(\sigma_i) > 0$. In fact, it is readily shown that, using (36), we have the following Lyapunov function.

$$V = \frac{1}{2}\sum_{i=1}^m \sigma_i^2 > 0, \ \ \dot{V} = \sum_{i=1}^m \sigma_i\dot{\sigma}_i < 0 \tag{38}$$

Using (23), $v_i(t)$ in (36) becomes

$$\begin{aligned} v_i(t) \ = \ & -\{[0,\ \alpha^i_0,\ \cdots, \alpha^i_{\mu_i-2}]\omega_i(t) \\ & + q_i\ sgn(\sigma_i) + k_if_i(\sigma_i)\}. \end{aligned} \tag{39}$$

Hence, from (25)(26), the sliding mode control input $u(t)$ for the original system becomes as follows.

$$u(t) = \Lambda^{-1}(t)(-D(t)x(t) + v(t)) \tag{40}$$

here

$$\begin{aligned} v(t) &= \begin{bmatrix} v_1(t) \\ \vdots \\ v_m(t) \end{bmatrix} \\ v_i(t) &= -\{[0\ \ \overline{s}]T_i(t)x(t) + q_i\ sgn(\sigma_i) + k_if_i(\sigma_i)\} \\ \sigma_i &= S_iT_i(t)x(t) \\ & \qquad (i = 1,\cdots,m) \end{aligned} \tag{41}$$

where

$$T_i(t) = \begin{bmatrix} \tilde{c}_i^0(t) \\ \vdots \\ \tilde{c}_i^{\mu_i-1}(t) \end{bmatrix}. \tag{42}$$

From the above, the design procedure of the sliding mode controller for the system (1) is summarized as the following steps.

[Design procedure]

STEP 1. Using the controllability matrix, $U_c(t)$ in (4), check the controllability of the system (1). If the system is controllable, calculate the controllability indices $\mu_1,\cdots,\mu_m$ and the truncated controllability matrix $R(t)$ in (7).

STEP 2. From (10)(11), calculate $\tilde{C}(t)$.

STEP 3. Using the recursive equation (15), obtain $\tilde{c}_i^j(t)$ and, using (16), calculate $\gamma_{ij}(t)$ ($i = 1,\cdots,m \quad j = i+1,\cdots,m$). Then, using (18)(19), define $D(t)$ and $\Lambda(t)$.

STEP 4. Using (26) (or (42)), calculate $T(t)$ (or $T_i(t)$).

STEP 5. Define the desired stable characteristic polynomial (33). Then, define

$$\begin{aligned} S_i &= [\overline{s}_i, 1] \\ \overline{s}_i &= [\ \alpha^i_0,\ \cdots, \alpha^i_{\mu_i-2}\ ] \\ & \quad (i = 1,\cdots,m) \end{aligned} \tag{43}$$

STEP 6. The sliding mode control input is obtained by (40)(41) and (42).

4.2 Trajectory Tracking Controller for Nonlinear Systems

In this paper, sliding mode controller for linear time-varying systems is concerned so far. However, in practice, we can hardly find any system that can be modeled by a linear time-varying system. And, the most of practical systems are nonlinear systems. So, one of the most important application of the control problem for linear time-varying systems is a control design problem of linear time varying approximate model around some particular trajectory of nonlinear systems.

Consider the following non-linear system.

$$\dot{x}(t) = f(x(t), u(t)) \tag{44}$$

Here, $x(t) \in R^n$ and $u(t) \in R^m$ are the state variable and the input signal. Let $x^*(t)$ and $u^*(t)$ be some particular desired trajectory and the desired input for $x^*(t)$.

The problem is to design a controller to track this desired trajectory $x^*(t)$ stably around it. This can be done by stabilizing this trajectory in the neighborhood of $x^*(t)$ and $u^*(t)$. Let $\Delta x(t)$ and $\Delta u(t)$ be defined by

$$\begin{cases} \Delta x(t) = x(t) - x^*(t) \\ \Delta u(t) = u(t) - u^*(t). \end{cases}$$

Then, we have a linear time-varying approximation around $x^*(t)$ and $u^*(t)$ as follows.

$$\Delta \dot{x}(t) = A(t)\Delta x(t) + B(t)\Delta u(t) \tag{45}$$

$$\begin{cases} A(t) = \frac{\partial}{\partial x} f(x^*(t), u^*(t)) \\ B(t) = \frac{\partial}{\partial u} f(x^*(t), u^*(t)) \end{cases} \tag{46}$$

Then, using time-varying sliding mode control technique, error equation can be stabilized around the desired trajectory $x^*(t)$ and $u^*(t)$.

5 NUMERICAL EXAMPLE

Consider the following nonlinear system with two input.

$$\begin{aligned} x_1(t) &= x_2(t) \\ x_2(t) &= 0.5(1 - x_1^2(t))x_2(t) - x_1(t)x_3(t) + u_1(t) \\ x_3(t) &= x_1^2(t) - x_3(t) + u_1(t) + u_2(t) \end{aligned} \tag{47}$$

Let the desired trajectory $x^*(t)$ for this system be

$$\begin{aligned} \dot{x}_1^*(t) &= \cos t \\ \dot{x}_2^*(t) &= -\sin t \\ \dot{x}_3^*(t) &= 1. \end{aligned} \tag{48}$$

Then, the desired input $u^*(t)$ for $x^*(t)$ is obtained as follows.

$$\begin{aligned} u_1^*(t) &= 0.5\sin t(1 - \cos^2 t) \\ u_2^*(t) &= -\cos^2 t + 1 - u_1^*(t) \end{aligned} \tag{49}$$

A linear time-varying approximation of the system (47) around $x^*(t)$ and $u^*(t)$ becomes as follows.

$$\frac{d}{dt}\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \Delta x_3(t) \end{bmatrix} = A(t)\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \Delta x_3(t) \end{bmatrix} + B(t)\begin{bmatrix} \Delta u_1(t) \\ \Delta u_2(t) \end{bmatrix} \tag{50}$$

where

$$A(t) = \begin{bmatrix} 0 & 1 & 0 \\ \cos t \sin t - 1 & 0.5\sin^2 t & -\cos t \\ 2\cos t & 0 & -1 \end{bmatrix} \tag{51}$$

$$B(t) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}. \tag{52}$$

The sliding mode control input to stabilize the system (50) can be obtained according to the following steps.

STEP 1. The controllability matrix, $U_c(t)$, of the system (50)-(52) is

$$U_c(t) = \left[\begin{array}{cc|c|c|c} 0 & 0 & 1 & 0 & \cdots \\ 1 & 0 & 0.5\sin^2 t - \cos t & -\cos t & \cdots \\ 1 & 1 & -1 & -1 & \cdots \end{array}\right]. \tag{53}$$

This implies that the system (50)-(52) is controllable, and the controllability indices are $\mu_1 = 2$ and $\mu_2 = 1$.

STEP 2. From STEP 1, the truncated controllability matrix, $R(t)$, becomes

$$R(t) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & \frac{1}{2}(\sin^2 t - 2\cos t) & 0 \\ 1 & -1 & 1 \end{bmatrix} \tag{54}$$

And from (10)(11), we have

$$\begin{aligned} \tilde{C}(t) &= W(t)R^{-1}(t) \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.5\sin^2 t + \cos t & 1 & 0 \\ 1 & 0 & 0 \\ 1 + 0.5\sin^2 t - \cos t & -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 1 + 0.5\sin^2 t - \cos t & -1 & 1 \end{bmatrix} \end{aligned} \tag{55}$$

where we choose $\lambda_1(t) = \lambda_2(t) = 1$.

STEP 3. From (15) and (16), we have following $\tilde{c}_i^k(t)$ and $\gamma_{ij}(t)$.

$$\begin{aligned} \tilde{c}_1^1(t) &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\ \tilde{c}_1^2(t) &= \begin{bmatrix} \cos t \sin t - 1 & 0.5\sin^2 t & -\cos t \end{bmatrix} \\ \tilde{c}_2^1(t) &= \begin{bmatrix} \sin t + 2\cos t + 1 & 1 - \cos t & \cos t - 1 \end{bmatrix} \\ \gamma_{12}(t) &= 0 \end{aligned}$$

And, from these equations, $D(t)$ and $\Lambda(t)$ are calculated as follows.

$$D(t) = \begin{bmatrix} \cos t \sin t - 1 & 0.5\sin^2 t & -\cos t \\ \sin t + 2\cos t + 1 & 1 - \cos t & \cos t - 1 \end{bmatrix} \tag{56}$$

$$\Lambda(t) = \begin{bmatrix} \lambda_1 & \gamma_{12} \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

STEP 4. Using (26), the state transformation matrix, $T(t)$, is

$$T(t) = \begin{bmatrix} T_1(t) \\ T_2(t) \end{bmatrix}. \tag{57}$$

where

$$T_1(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{58}$$

$$T_2(t) = \begin{bmatrix} 1+0.5\sin^2 t - \cos t & -1 & 1 \end{bmatrix}. \tag{59}$$

STEP 5. From (33) and the fact that $\mu_1 = 2$, $\mu_2 = 1$, we choose

$$\alpha^1(p) = p + 1. \tag{60}$$

In this case, there is not $\alpha^2(p)$, because $\omega_2(t)$ is a scalar. From this, we define

$$S_1 = [\,1 \;\; 1\,], \quad S_2 = [\,1\,]. \tag{61}$$

STEP 6. From the above and (40)(41), the sliding mode control input is obtained as follows.

$$\begin{aligned} u_1(t) &= (1-\sin t\cos t)\Delta x_1(t) - 0.5\sin^2 t\Delta x_2(t) \\ &\quad + \cos t\Delta x_3(t) - sgn(\sigma_1) \end{aligned} \tag{62}$$

$$\begin{aligned} u_2(t) &= -(\sin t + 2\cos t + 1)\Delta x_1(t) \\ &\quad + (\cos t - 1)\Delta x_2(t) + (1-\cos t)\Delta x_3(t) \\ &\quad - sgn(\sigma_2) \end{aligned} \tag{63}$$

where σ_1 and σ_2 are defined by

$$\begin{aligned} \sigma_1 &= \Delta x_1(t) + \Delta x_2(t) \\ \sigma_2 &= (1+0.5\sin^2 t - \cos t)\Delta x_1(t) \\ &\quad - \Delta x_2(t) + \Delta x_3(t). \end{aligned} \tag{64}$$

and $q_1 = q_2 = 1$, $k_1 = k_2 = 0$.

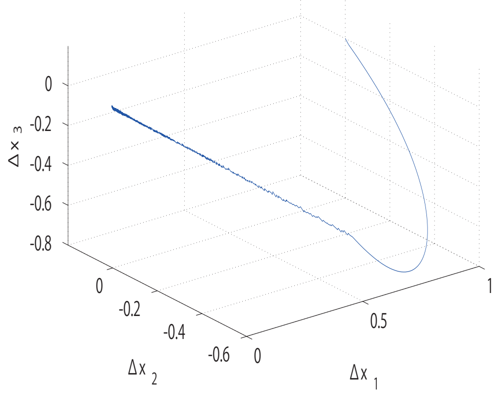

Figure 1: Response of $\Delta x(t)$.

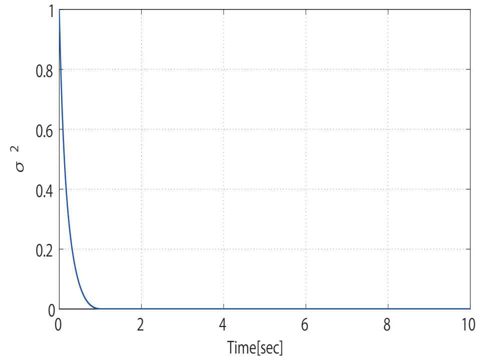

Figure 2: Response of $\sigma = \sigma_1^2 + \sigma_2^2$.

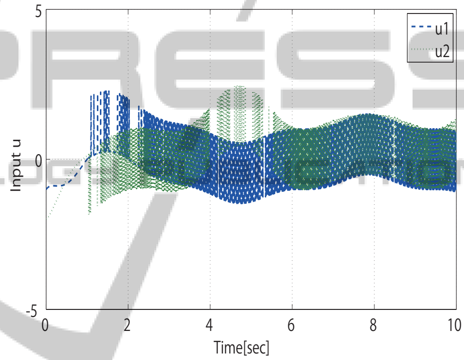

Figure 3: Response of $\Delta u(t)$.

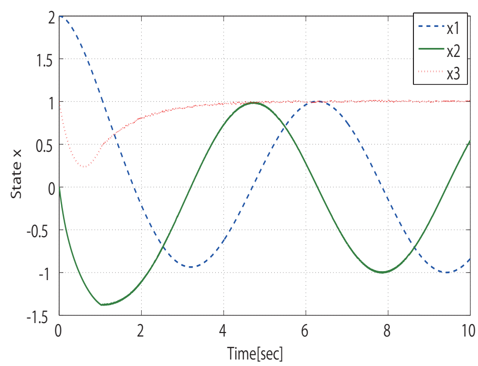

Figure 4: Response of $x(t)$.

Fig.1 shows that the response of $\Delta x(t)$ which converges to the origin. This implies that the state $x(t)$ converges to the desired trajectory $x^*(t)$.The value of $\sigma_1^2 + \sigma_2^2$ is plotted in Fig.2. It decreases monotonically to 0. Fig.3 and 4 show the control input $u(t)$ and state response $x(t)$. According to these graphs, the time varying sliding mode controller works well for the trajectory tracking control for non-linear systems.

6 CONCLUSIONS

In this paper, the design procedure of sliding mode controller for linear time-varying system is presented. For this purpose, the time-varying pole placement feedback is used so that the closed loop system is equivalent to some linear time invariant system. Then, the conventional design method of the sliding mode control can be applied to this time invariant system. And, finally by the time-varying transformation matrix, this control input is transformed into the sliding mode control input for the original system. It was shown that this controller has a good availability for the trajectory tracking control problem of nonlinear systems.

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