Multi-loop Control Using Gershgorin and Ostrowski Bands

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Abstract: The goal of this paper is to develop a new method of decentralized control tuning. This method is based on Nyquist-Arrays and independently designs monovariable controllers for each loop of the plant while ensuring the robust stability of the multivariable system. It works on the optimization of a frequency criterion using the controller's design parameters. PID controllers have been chosen in this study because of their good performances for most applications. Finally, the proposed method allows to achieve good performances and the stability is ensured thanks to the analysis of Gershgorin and Ostrowski bands.

1 INTRODUCTION

The design of the control of a multivariable process can be achieved with two strategies. The centralized strategy consists in designing one full MIMO (Multiple Inputs Multiple Outputs) controller for the whole system. The different techniques of this strategy (Skogestad and Postlethwaite, 1996), including state-feedback, model predictive control, H-infinity loop-shaping… are usually efficient and achieve good performances. However, these methods need a precise enough model, and the obtained controllers are usually of high-order. The decentralized strategy consists in dividing the MIMO process into a combination of several SISO (Single Input Single Output) processes and to design mono-loop controllers in order to control the MIMO process (Albertos and Sala, 2004).

Compared to the centralized strategy, the decentralized one provides flexibility and needs fewer parameters to tune, while it is easier to implement and increases the loop failure tolerance of closed loop systems. Because of these benefits, decentralized controllers have been widely used and different types of methods have been developed as described in (Huang et al., 2003).

Independent design method (Skogestad and Morari, 1989) is chosen in this paper, which means that each loop is designed independently from the others. The Nyquist array techniques have shown themselves to be well-suited for practical design of controllers for multivariable interacting processes (Garcia, Karimi and Longchamp, 2005), (Chen and Seborg, 2002).

This paper proposes a new method based on Nyquist-Arrays. The goal is to design SISO controllers for any multivariable process (unstable poles, unstable zeros, dead time) with medium interactions. Two alternatives are under discussion, the first one uses Gershgorin bands whereas the second one uses Ostrowski bands. The study is limited to PID controllers but it can easily be generalized.

This paper is organized as follows: Section 2 surveys theoretical preliminaries about the Nyquist-array methods. Section 3 presents the design of the control laws and Section 4 exposes simulation results that demonstrate the efficiency of this method. Conclusions are presented in Section 5.

2 NYQUIST-ARRAY METHODS

The proposed method is based on the Nyquist-array methods (Leigh, 1982), (Rosenbrock, 1969), in which the design of the controller is divided into two steps. The first one consists in reducing the interactions in the system so that each control loop

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can be closed separately and independently from the remaining loops. In the second step, controllers of the different loops are designed.

This paper focuses on the design of the controllers but a short summary of the general methods is recalled.

2.1 Diagonal Dominance

The design of the control laws often requires the diagonal dominance of the system. A $p \times p$ matrix Z is called row (respectively column) diagonally dominant if it satisfies (1) (respectively (2)):

$$\left|Z_{ii}\right| > Rr_i(Z) = \sum_{j=1, j \neq i}^{p} \left|Z_{ij}\right|, i = 1,..., p \quad (1)$$

$$\left|Z_{ii}\right| > Rc_i(Z) = \sum_{j=1, j \neq i}^{p} \left|Z_{ji}\right|, i = 1,..., p \quad (2)$$

If the frequency response matrix of a MIMO system is row (respectively column) diagonal dominant for the whole frequency domain, it means that each output is mainly determined by its corresponding input (respectively each input determines mainly its corresponding output). Furthermore, it is clear that a higher degree of diagonal dominance yields a smaller difference between the MIMO performance and the performance of the SISO designs.

2.2 Principe of Nyquist-Array Methods

Nyquist-array methods are divided into two classes: the Direct Nyquist-Array (DNA) and the Inverse Nyquist-Array (INA). Both methods have identical design objectives and the method proposed here can be applied both with DNA and INA.

Consider a MIMO plant G. The open-loop transfer matrix Q described in (3) is used in DNA whereas the inverse of the open-loop is considered in INA. The structure of the control laws is described by (4).

$$Q(s) = G(s)K(s) \quad (3)$$

$$K(s) = K_a K_b(s) K_c(s) \quad (4)$$

K_a is a constant matrix that permutes rows or columns to reorder the outputs or inputs. It can be used to avoid unstable off-diagonal elements. K_b is used to achieve diagonal dominance. An overview of the methods to find these matrices is found in (Vaes, 2005) and (Maciejowski, 1989). K_c is a diagonal matrix composed of separate SISO controllers for each loop.

In DNA, the diagonal matrix K_c post-multiplies the plant G_d as in (5). The effect of each element is to multiply each column of G_d by the same transfer function. Hence, column dominance of G_d is preserved.

However, in the INA, K_c^{-1} pre-multiplies the inverse of the plant and row dominance of the inverse is conserved.

$$Q(s) = G_d(s) K_c(s) \quad (5)$$

$$G_d(s) = G(s) K_a K_b(s) \quad (6)$$

This paper focuses on the design of the diagonal controller K_c only. Therefore, G_d is considered as being column diagonally dominant when working with DNA and G_d^{-1} is considered as being row diagonally dominant when working with INA in the following.

2.2.1 Direct Nyquist-Array Method

Closed-loop stability of a SISO system is obviously analyzed with the Nyquist stability theorem. The Generalized Nyquist theorem extends it to MIMO systems (Macfarlane and Postlethwaite, 1977): Considering an open loop transfer matrix Q presenting n_{pol} unstable poles, defining the characteristic loci as the images of the Nyquist contour by the eigenvalues of Q, the Generalized Nyquist theorem states that the closed loop is stable if and only if the sum of the anticlockwise encirclements around the critical point of the characteristic loci of the open-loop transfer equals n_{pol}.

The characteristic loci can be approached by the diagonal elements of Q thanks to Gershgorin's theorem:

The eigenvalues of a complex $p \times p$ matrix Z lie in the union of the p circles, each with center Z_{ii} and radius Rr or Rc defined in (1) and (2). When this theorem is applied to the gain matrix $Q(j\omega)$, a circle is obtained around each diagonal element of the loop gain at each frequency ω. The bands obtained by taking these circles together over the frequency domain are called Gershgorin bands.

Using Gershgorin's theorem, it can be claimed that the eigenvalues of a gain matrix Q over all frequencies are trapped into these Gershgorin's bands. Based on the generalized Nyquist theorem, it can be concluded that if all Gershgorin bands exclude the critical point, then closed-loop stability can be assessed by counting the number of encirclements of the critical point by the Gershgorin bands.

The open-loop matrix in (5) can be written:

$$Q = \begin{bmatrix} G_{11}K_1 & \cdots & G_{1p}K_p \\ \vdots & \ddots & \vdots \\ G_{p1}K_1 & \cdots & G_{pp}K_p \end{bmatrix} \quad (7)$$

The width of the i[th] column Gershgorin band is:

$$Rc_i(j\omega) = \sum_{k=1, k\neq i}^{p} \left| G_{ki}(j\omega) K_i(j\omega) \right| \quad (8)$$

It is clear that the width of this band only depends on the system and on the i[th] controller K_i. The stability of the i[th] loop can thus be ensured independently of the other controllers.

2.2.2 Inverse Nyquist-Array Method

The principle of the INA (Bell, Cook, and Munro, 1982) method is different. For notational convenience, the inverse of a matrix H is noted as $\hat{H}$.

Let us consider Q an open-loop transfer matrix composed of a plant and its SISO controllers as described in (5) and H the closed-loop transfer matrix. We denote l_i the open-loop transfer function between e_i and y_i with all the other loops are closed as shown in Figure 1 for a TITO (Two Inputs Two Outputs) process.

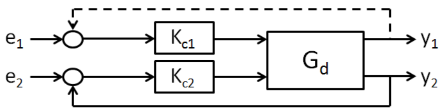


Figure 1: TITO process (the second loop is closed and the first one is being closed).

Considering this open-loop transfer function l_i takes into account the stability of the whole system. l_i is not a priori known but $\hat{Q}_{ii}$ gives a good approximation of the inverse of l_i thanks to Ostrowski's theorem: Considering a complex $p\times p$ matrix Z diagonally row dominant, then:

$$\left| Z_{ii} - 1/\hat{Z}_{ii} \right| \leq \sum_{k=1, k\neq i}^{p} \left| Z_{ik} \right| \times \max_{j\neq i} \left(\sum_{k=1, k\neq j}^{p} \left| Z_{jk} \right| \Big/ \left| Z_{jj} \right| \right) \quad (9)$$

Applying this theorem to the i[th] row of $\hat{H}$ that is supposed row diagonally dominant, we obtain after calculation:

$$\left| \hat{Q}_{ii}(j\omega) - 1/l_i(j\omega) \right| \leq Rr_i(\hat{Q}(j\omega)) \Phi_i(j\omega) \quad (10)$$

$$\Phi_i(j\omega) = \max_{j\neq i} \left(Rr_j(\hat{Q}(j\omega)) \Big/ \left| \hat{Q}_{jj}(j\omega) \right| \right) \quad (11)$$

Consequently, $1/l_i(j\omega)$ is contained within a circle centered in $\hat{Q}_{ii}(j\omega)$. We call this an Ostrowski circle and the union of all these circles an Ostrowski band.

The terms Φ_i physically represent the maximal relative couplings in the other loops. Since $\hat{Q}$ is assumed to be row diagonally dominant, Φ_i is smaller than 1. The i[th] Ostrowski band is thus contained within the i[th] Gershgorin band of the inverse of the open-loop transfer matrix. Moreover, although each term of Φ_i depends on the controllers of the other loops, Φ_i is independent of these. It can be concluded that the width of row Ostrowski bands only depends on the plant and on the i[th] controller. The stability of loop i can thus be ensured independently of the other loops' controllers.

As in DNA with Gershgorin bands, Ostrowski bands can be used to characterize the stability of the system in INA. The inverse Nyquist criterion (Bell, Cook, and Munro, 1982) is then used: A feedback loop with n_{zeros} unstable zeros in the loop gain Q is stable if and only if the sum of the anticlockwise encirclements around the critical point of the inverse Nyquist locus of Q equals n_{zeros}.

3 CONTROL LAWS DESIGN

The goal is to define a method to design a decentralized control for MIMO systems. There is no restriction about the structure of the controller. PID controllers have been chosen in this study because they remain the industry standard and reach good performances for most applications with an easy to understand structure. Nevertheless, it is easy to implement other controller structures in the algorithm.

The method consists in tuning SISO controllers independently for each SISO system thanks to the optimization of a cost function depending on the controllers parameters. Similar criteria are defined thereafter for each loop of the system for DNA and INA analysis.

3.1 Stability

As seen before, to ensure stability, Gershgorin (respectively Ostrowski) bands must not include the critical point. Moreover, the bands have to encircle anticlockwise the critical point a number of times corresponding to the number of open-loop unstable poles (respectively unstable zeros).

To take in consideration the number of encirclements, one idea is to force the Nyquist locus (respectively the inverse of the Nyquist locus) to travel through specific areas. After a brief study of the plant and the shape of its Nyquist locus, it is possible to define attractive areas depending on the number of the open-loop unstable poles (respectively unstable zeros).

For instance, let us consider an open-loop monovariable system Q with an unstable pole, the Nyquist locus of which presents infinite branches that do not encircle the critical point. Ensuring that locus crosses the real axis at the left of the critical point and travelling below it are sufficient conditions to have closed-loop stability as in Figure 2.

For each attractive area k, a measure of distance between the nearest point of the Nyquist locus with the segment $[P_{k1},P_{k2}]$ is constituted by (12). To force the Nyquist locus to travel through these attractive areas, the distances D_k will be minimized. In Figure 2, two attractive areas are defined with $[P_{11},P_{12}]$ and $[P_{21},P_{22}]$. To facilitate the readability, only the calculation of D_1 is presented.

$$D_k = \inf_{\omega}\left(\left|Q(j\omega)-P_{k1}\right| + \left|Q(j\omega)-P_{k2}\right|\right) \Big/ \left|P_{k1}-P_{k2}\right| \qquad (12)$$

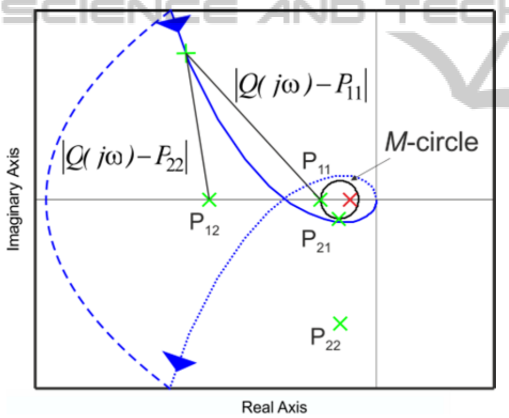


Figure 2: Calculation of D_1 with checkpoints P_{11}, and P_{12}.

This allows to find a controller that stabilizes the closed-loop system, even if the initial conditions of the optimization match with a controller configuration leading to an unstable closed-loop.

For SISO systems, robustness against model uncertainties is ensured if the direct or inverse Nyquist loci present sufficient phase margin. Note that the phase margin of the inverse of a SISO system is the opposite of the phase margin of the system.

The determination of the phase margin of a MIMO system is not obvious (Ye et al., 2008). In this paper, phase margin is assessed applying the previous considerations for SISO systems to Gershgorin bands (respectively Ostrowski bands). In (Ho, Lee, and Gan, 1997), the circle at the cutoff frequency is used to determine the phase margin. The problem is that in some configurations, circles at other frequencies can be closer from the critical point than the one at the cutoff frequency.

An example is presented in Figure 3 where the blue point represents the crossing of the circle at the cutoff frequency with the unit circle, and the green point corresponds to the point associated with true phase margin. Thus, it seems more relevant to consider the envelope of the circles.

It is then possible to define an objective with a specified phase margin M_φ* (when using INA, the opposite of the specified phase margin is used). Besides ensuring stability, the phase margin leads to an upper bound for the damping of the system. It is also possible to define the gain margin using the envelope of the bands in the same way. Thus, a minimum gain margin can be obtained considering a specified gain margin M_g* in the criterion to optimize (when using INA, the opposite of the specified gain margin is used).

In DNA, Gershgorin bands ensure the same stability margins for all loops. Indeed, when the bands are superimposed (which is often the case when approaching the critical point), the characteristic loci of each diagonal element are not necessarily contained in the Gershgorin bands of this element. The global stability margins finally match with the worst stability margins determined from the different bands.

By contrast, Ostrowski bands can ensure different stability margins for each loop.

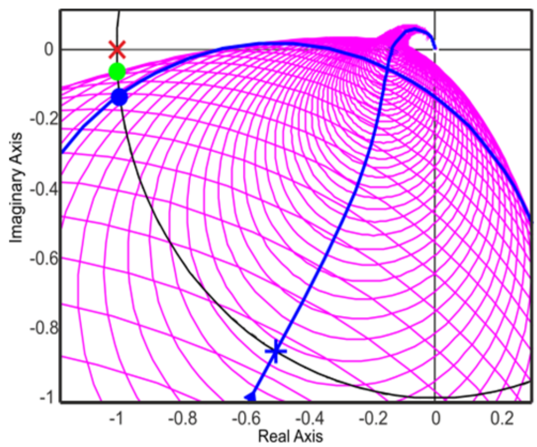


Figure 3: Phase margin for a MIMO process.

3.2 Performances

To give an upper bound of the peak modulus of the closed-loop frequency response of system, the complementary modulus margin is considered

(Bourlès, 2010). It represents physically the inverse of this maximum gain.

$$M_c = \inf_{\omega}\left|\frac{1+K(j\omega)Gd(j\omega)}{K(j\omega)Gd(j\omega)}\right| \quad (13)$$

A criterion is then defined with an objective M_c* to reach. This specification can be directly interpreted in the Nyquist diagram thanks to M-circles (Mirkin, 2011) which are the contours of the constant closed-loop magnitude. M-circles are described by the equation:

$$(X+\tfrac{M^2}{M^2-1})^2+Y^2=\tfrac{M^2}{(M^2-1)^2} \quad (14)$$

X and *Y* are the real and imaginary coordinates in the complex plane and *M* is the magnitude of the closed-loop transfer function. In order to satisfy simultaneously stability and closed loop maximum modulus conditions, the points P_{k1} can be chosen adequately on the specified M-circle as shown in Figure 2. There are no real rules to set the points P_{k2}, they only have to be far enough from the points P_{k1}.

The crossover frequency highly impacts the bandwidth of the closed-loop system. It is then useful to take it into account in a criterion, defined with a desired crossover frequency ω_c*.

In order to cancel the static error and reduce the tracking error, the integral action of the controller, whose structure is given in (15), is maximized:

$$K(s)=K_p\left(1+\tfrac{1}{T_i s}+\tfrac{T_d s}{\frac{T_d}{N}s+1}\right) \quad (15)$$

The criteria for previously considered concepts, are summarized here:

$$J_1=\sum_k D_k \quad (16)$$

$$J_2=(M_\varphi^*-M_\varphi)\big/M_\varphi^* \quad (17)$$

$$J_3=(M_g^*-M_g)\big/M_g^* \quad (18)$$

$$J_4=\left|M_c^*-M_c\right|\big/M_c^* \quad (19)$$

$$J_5=\left|\omega_c^*-\omega_c\right|\big/\omega_c^* \quad (20)$$

$$J_6=T_i \quad (21)$$

3.3 Optimization

For each SISO loop, the controller parameters are determined by solving a least-square optimization problem characterized by a criterion *J* taking into account the criteria previously described:

$$J=\sum_{k=1}^{6} q_k J_k^2 \quad (22)$$

This cost function presents weighting factors q_k that give more or less importance to each criterion. If the initial choice for the parameters lead to a stable closed-loop system, J_1 is not necessary. Often, it is sufficient to take into account J_2 for robustness stability so that J_3 may not be considered. To speed up the optimization, controllers found with classical SISO methods (Aström and Hägglund, 1995) can be used for initial conditions.

Each optimization gives the controller parameter settings for one SISO loop and the tuning of the other SISO loops do not affect the stability of the loop already tuned, which makes this method interesting.

3.4 DNA and INA

As seen in the previous part, the two approaches work with a similar algorithm. However, conditions for stability are not the same and the algorithm lead to different solutions. It is not obvious to guess a priori which one is the least conservative. The size of Gershgorin bands of the plant only depends on the magnitude of the coupling terms. The ratio between off-diagonal terms and diagonal terms gives the distance between the bands and the origin of the Nyquist diagram. However, we are interested in the distance between the bands and the critical point and there is no information about that. That is why it is difficult to know which method to prefer.

Even if the Gershgorin bands can be used to predict stability when the gains in all the loops change simultaneously, the DNA method deals with eigenvalues of the system that can be sensitive to model perturbations. It is thus less robust than the INA method where stability is ensured considering a monovariable system. Another advantage of INA is that it can be used to indicate whether the system would be stable if one loop failed.

Finally, the choice of the method is determined by the shape of the frequency response of the plant.

Ability to make the direct (inverse) open loop matrix transfer column (row) diagonal dominant can lead the choice for the method DNA (INA).

3.5 Case of TITO Plants

The case of TITO processes is specific because of the form of the inverse of the system:

$$G_d=\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{bmatrix} \quad \hat{G}_d=\frac{1}{G_{11}G_{22}-G_{12}G_{21}}\begin{bmatrix}G_{22} & -G_{12}\\ -G_{21} & G_{11}\end{bmatrix} \quad (23)$$

If G_d is column diagonally dominant, it implies that G_d^{-1} is row diagonally dominant. In addition, the ratio between magnitudes of diagonal terms with off-diagonal terms is conserved.

The consequence is that DNA and INA are not exclusive. Indeed, the first one requires the column diagonal dominance of the system and the second one requires the row diagonal dominance of the inverse of the system, and these properties are equivalent. These considerations are only true for TITO systems. Indeed, in the general case, the inverse of a column diagonally dominant system is not a priori row diagonally dominant.

4 SIMULATION EXEMPLES

Academic examples are now considered to demonstrate the efficiency of this method with a great variety of processes.

4.1 Optimization with INA

In this first example, G_d is a TITO plant described by:

$$G_d = \begin{bmatrix} \dfrac{(s+0.1)}{(s+0.4)(s+0.25)} & \dfrac{1}{(s+0.5)(s+0.3)} \\ \dfrac{1}{(s+2)(s+1)} & \dfrac{0.2(s-5)}{(s^2+0.6s+0.1)} \end{bmatrix} \quad (24)$$

Technical specifications are described in Table 1. The inverse of the plant is row diagonal dominant, the INA can thus be applied. Settings of the designed controller are presented in Table 2. Due to its negligible derivative action, the first controller has been simplified in a PI one.

Hereafter figures present Nyquist loci and inverse Nyquist loci respectively in blue and green, and the points defining phase margins represented by crosses. Gershgorin and Ostrowski bands are respectively drawn in magenta and yellow. Nyquist diagrams and Gershgorin bands are plotted in Figure 4 for the two diagonal terms. The plots on the left-hand side give an overview of the Nyquist diagrams.

Table 1: Technical specifications.

Controller	Complementary Modulus Margin	Crossover frequency (rad/s)	Phase Margin (°)
K_1	1/1.05	0.7	35
K_2	1/1.05	0.7	35

Table 2: Controllers parameters.

Controller	K_p	T_i	T_d	N
K_1	0.26	0.47	X	X
K_2	-0.62	4.2	1.0	33

In the first case, the inverse Nyquist locus and Ostrowski bands do not encircle the critical point. In the second loop, the inverse Nyquist locus encircles the critical point once that is logical because the second loop contains one unstable zero.

Stability can also be analyzed with Gershgorin bands. The open-loop transfer matrix is stable and it can be checked that Gershgorin bands do not encircle the critical point.

The right diagrams zoom on the critical point to check that specifications are satisfied. As it can be seen, phase margins are compliant. For the first loop, the phase margin obtained with Gershgorin bands is similar to the one obtained with Ostrowski ones. However, for the second loop, the phase margin obtained with Gershgorin bands is clearly smaller than the one obtained with Ostrowski bands. If DNA had been chosen, the settings of the controller would not have been found because the phase margin would not have been satisfied. The benefits of INA appear clearly in this case.

M-circles are also drawn to visualize the complementary modulus margins. For each loop, the Nyquist loci tangent the M-circles. That means the complementary modulus margins are fulfilled.

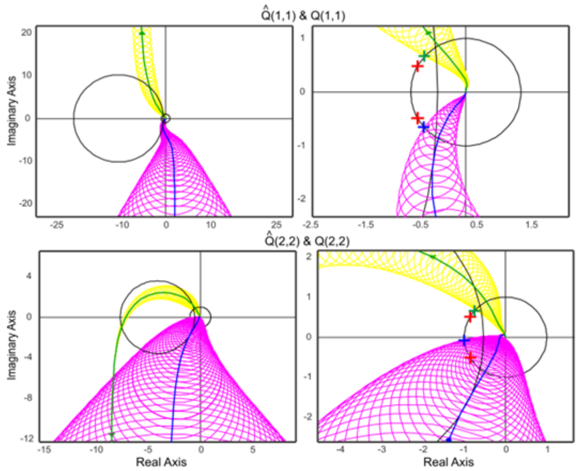


Figure 4: Nyquist-array of the designed loops.

4.2 Optimization with DNA

Consider the MIMO process described by the 3×3 transfer matrix G_d in (25). By analysing the plant, it can be seen that medium interactions are still present.

$$G_d=\begin{bmatrix}\frac{8\exp(-0.05s)}{4s^2+3s+2} & \frac{0.25}{s+10} & \frac{1}{(s+15)(s+2)}\\ \frac{0.5}{(s+2)(s+1)} & \frac{\exp(-0.05s)}{s-1} & \frac{0.2(s+10)}{(s+2)(s^2+6s+10)}\\ \frac{0.5}{(s+2)(s+1)} & \frac{0.5(s+20)}{(s+50)(s+15)} & \frac{3(s+10)}{s(s-20)(s-5)}\end{bmatrix} \quad (25)$$

To present several study cases, each diagonal element has a different structure. The first diagonal term is stable and includes a time delay. The second diagonal term has one unstable pole and includes a time delay as well. The third diagonal term includes an integrator and has two unstable poles.

G_d is column diagonally dominant, the algorithm with DNA can thus be applied and PID controllers have been chosen. Technical specifications are described in Table 3 and details of the controller settings K_1, K_2, and K_3 are presented in Table 4.

Due to its negligible derivation action, a PI controller has finally been designed for the second loop. Performances broadly match with technical specifications. Nyquist diagrams and Gershgorin bands are plotted in Figure 5 for each diagonal term. As in the first example, the plots on the left-hand side give an overview of the Nyquist diagrams to check that the number of anticlockwise encirclements matches with the number of unstable poles (respectively 0, 1 and 2 for the three loops). It can be seen on the diagrams on the right-hand side that Gershgorin bands do not include the critical point and fulfilled the specified phase margins. Moreover, complementary modulus margins are satisfied.

Table 3: Technical specifications.

Controller	Complementary Modulus Margin	Crossover frequency (rad/s)	Phase Margin (°)
K_1	1/1.05	10	30
K_2	1/1.4	10	30
K_3	1/1.15	300	30

Table 4: Controllers parameters.

Controller	K_p	T_i	T_d	N
K_1	4.6	1.1	1	9960
K_2	7.45	0.63	X	X
K_3	1610	0.38	0.065	990

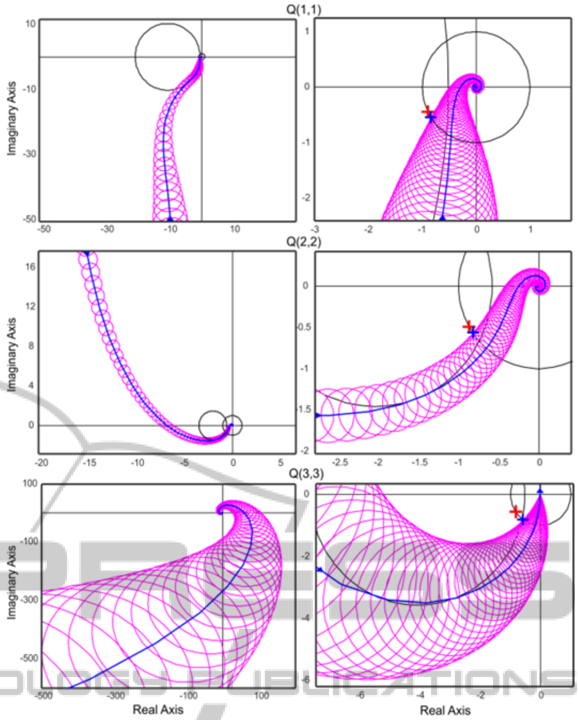


Figure 5: Nyquist-array of the designed loops.

4.3 Analysis using Row and Column Dominance

Consider now the TITO process:

$$G_d=\begin{bmatrix}\frac{2000\exp(-0.1s)}{s^2+80s+1825} & \frac{30(s+0.1)}{(s+25)(s+2)^2}\\ \frac{100000(s+75)}{(s+100)^2(s+50)^2} & \frac{240}{(s+20)(s+5)}\end{bmatrix} \quad (26)$$

The system is column diagonal dominant and technical specifications are given in Table 5. The design is performed with PI controllers with the proposed algorithm for DNA and details of the controller settings are presented in Table 6.

Table 5: Technical specifications.

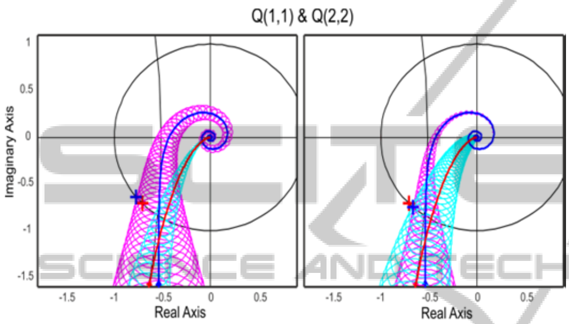
Controller	Complementary Modulus Margin	Crossover frequency (rad/s)	Phase Margin (°)
K_1	1/1.05	6	45
K_2	1/1.05	6	45

Table 6: Controllers parameters.

Controller	K_p	T_i
K_1	0.27	0.05
K_2	0.54	0.14

In figure 6, Nyquist diagrams of the two diagonal elements are plotted in blue and red and the associated bands are respectively in magenta and cyan.

Considering column Gershgorin bands in Figure 6a, the worst phase margin does not satisfy the specified one. It can be noted that Q_{12} is much smaller than Q_{21} close to the cutoff frequency (bands of the second loop are much thinner than those of the first one). By plotting row instead of column Gershgorin bands as in Figure 6b, the largest bands become further from the critical point. Thanks to this analysis, it is possible to ensure the specified phase margin for the MIMO system.



(a) (b)

Figure 6: Nyquist-array of the designed loops.

Row Gershgorin bands can not be considered for the design of the controllers as seen before but can be used to assess stability as well. It is similar for column Ostrowski bands.

Moreover, once design has been done whatever the chosen approach, Gershgorin and Ostrowski bands can be superimposed to determine stability margins.

5 CONCLUSIONS

This paper proposes a new method of tuning multi-loop controllers. SISO controllers can be designed independently using DNA or INA thanks to the optimization of similar cost functions. The described procedure aims to reach some performances while ensuring stability robustness of the closed-loop multivariable process thanks to Gershgorin bands in DNA and Ostrowski bands in INA. By superimposing both Gershgorin and Ostrowski bands, it is possible in some cases to reduce the conservatism of the chosen approach.

PID controllers have been chosen in this study but the method can be easily applied with other types of controllers.

To conclude, the proposed method offers a straightforward and systematic way of designing MIMO controllers, while still leaving freedom to the designer. Simulation results illustrate the good performances obtained by this method for a wide range of processes.

Future works will focus on the adaptation of the methodology to improve the multivariable performances, particularly concerning the dynamic couplings of the closed-loop system.

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