# Adaptive Filtering in Electricity Spot Price Models

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Keywords: Electricity Spot, Risk premium, Hyperbolic system, Kalman filter, Jump process, Parameter identification, Parallel filter.

Abstract: We study the adaptive filtering for risk premium and system parameters in electricity futures modes. Introducing the jump augmented Vasicek model as the spot price mode, the factor model of the electricity futures is constructed as the stochastic hyperbolic systems with jumps. Representing the main spike phenomena of the electricity spot price from one observed futures data by proxy, the filtering of the stochastic risk premium and its system parameters are developed in a Gaussian framework. By using the parallel filtering algorithm, the online system parameter estimation procedure is proposed.

## 1 INTRODUCTION

It is well known that the electricity is quoted same as any other commodity, e.g. crude oil, gold, copper and others. As shown in Fig. 1, the electricity spot prices present a higher volatility than equity prices and its mathematical model is required for pricing of electricity-related options, risk management and others.

The peculiar characteristic of electricity is that one can not store electricity, but there are many other characteristics which distinguish electricity from other commodities.

From Fig. 1, in which the spot price (a *day-ahead* market) is shown, we observe the special behaviors of electricity spot, i.e., many spikes frequently and seasonal effect.

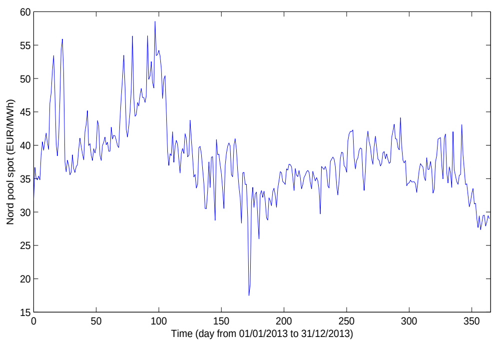


Figure 1: Nord Pool electricity spot price (day ahead implicit auction market).

In this paper, instead of modeling this process from the basic principle of supply and demand, the simple mathematical model for this spot price is proposed and leads to calibrate the model parameters and price the options by using the stochastic system approach. Along this line Schwartz and Smith (Schwartz and Smith, 2000) proposed a two-factor diffusion model and the system parameters are estimated from M.L.E. (Maximum likelihood estimate) by using Kalman filter. To apply this method one need to add *ad hoc* observation noise in order to derive the Kalman filter. This assumption has been made by numerous authors, either in the commodity or interest rate markets. The additional noise in the observation has been interpreted to bring into account bid-ask spread, price limits or errors in the information. The argument is clearly forced and unconvincing. By using the idea proposed by (Aihara and Bagchi, 2010), we approach the modeling differently. In our setup, on the one hand, the added measurement noise is built in the model. On the other hand, the modeling of the correlation structure between the futures (observation) is a natural component of our formulation. Hence the model parameters can be calibrated through the derived likelihood functional without any *ad hoc* observation noise.

All the same, in these works, the important spikes in the electricity prices could not be admitted, because including jumps[1] means giving up on the closed-form estimator like Kalman filter in (van Schuppen, 1977).

[1]The closed-form formulae for forwards and options are


Aihara S. and BAGCHI A..
Adaptive Filtering in Electricity Spot Price Models.
DOI: 10.5220/0005014206200628
In *Proceedings of the 11th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2014), pages 620-628
ISBN: 978-989-758-039-0

Fortunately, for the term structure model in the electricity problem, we can represent the jump process for the spike phenomena by using one observation component and transform the Non-Gaussian estimation problem into the Gaussian framework with correlated noises.

In this paper, from the real data (Nord Pool electricity spot data ), the linear trend is firstly identified by using a least squares method. After subtracting this linear trend from the spot data and taking the FFT, the prominent frequency of the seasonality effect is investigated in Sec.2. We choose the spot price dynamics as the jump-diffusion model proposed in (Duffie et al., 2000). According to the idea in (Aihara and Bagchi, 2010), we construct the arbitrary free model of the term structure, including jump-diffusion processes in Sec.3. In the electricity market, the averaged-type forward and futures contracts are observed and are used as the observation data for calibrating system parameters. After presenting this observation dynamics in Sec.4, we derive the closed form filter for estimating the whole term structure. To derive this filter, we choose one component of observation as the proxy for the spike process of the spot price, say $y_o(t)$. Reconstructing the spikes from $y_o(t)$ and plugging this into observation and system equations, the filtering problem with jumps is converted to the Gaussian framework in Sec.5. For figuring out the filtering problem, we need to work under the real world measure. Hence a suggested in (Carmona and Ludkovski, 2004), the stochastic market price of risk is introduced as the linear Ito equation. This implies that our extended state, including the stochastic market price of risk is still Gaussian. Instead of the use of MLE, we suggest the parallel filtering procedure in (Anderson and Moore, 1979) for obtaining the online state and parameter estimation. In the final section, some numerical examples are demonstrated.

# 2 IDENTIFICATION OF LINEAR TREND AND SEASONALITY

## 2.1 Linear Trend

From Fig.1, we can observe a slight downward trend in the spot data in 2013. A least squares fit gives the trend line for the log price,

$$-6\times10^{-4}t+3.74.$$

Now we subtract off this linear trend from the data shown in Fig.1 and obtain the data shown in Fig.2.

possible, even for the jump-diffusion and Levy processes in (Benth et al., 2008).

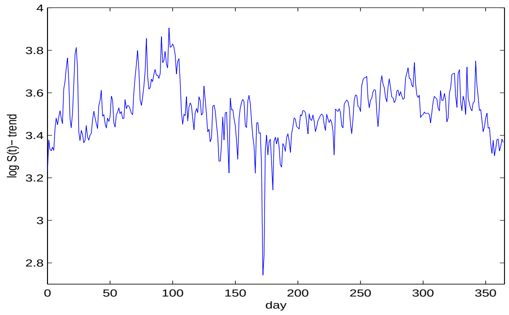


Figure 2: Log price (day ahead implicit auction market) minus the linear trend.

## 2.2 Seasonality

We take the FFT to the data shown in Fig.2 and obtain the periodogram shown in Fig.3. From Fig.3, we can



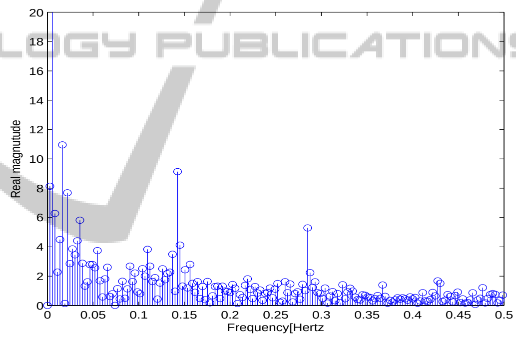


Figure 3: Periodogram of the log spot.

find big power points and zoom in on the plot and use the reciprocal of frequency to label the x-axis in Fig.4.

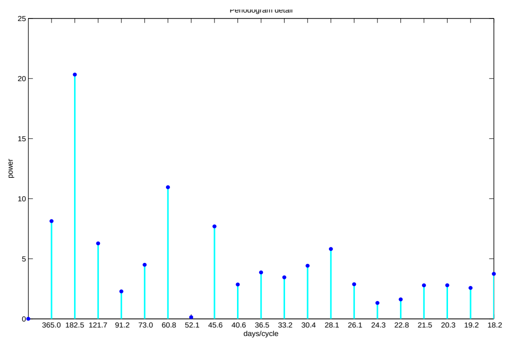


Figure 4: Detail of periodogram.

The 365 days/cycle is not important, but we find still three outstanding points; 182.5, 60.8 and 45.6 days/cycle. We also need to find phases at these points (Red marks) in Fig.5.

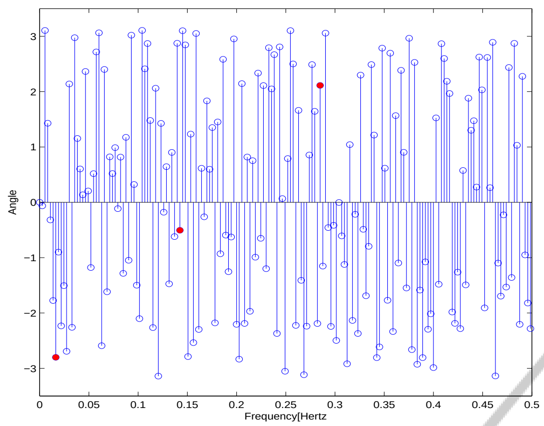


Figure 5: Angles at each frequencies.

Hence we find that

$$
\begin{aligned}
Se(t) &= \text{Linear trend + Seasonality} \\
&= -6\times 10^{-4}t + 3.74 \\
&\quad + a_1\cos(2\pi\times 16.4\times 10^{-3}t - 2.8) \\
&\quad + a_2\cos(2\pi\times 142.5\times 10^{-3}t - 0.05) \\
&\quad + a_3\cos(2\pi\times 284.9\times 10^{-3}t + 2.11) \quad (1)
\end{aligned}
$$

The coefficients $a_1, a_2$ and $a_3$ are identified to minimize the $|\log S(t) - \text{Seasonality - linear trend}|^2$ , i.e.,

$$a_1 = 0.114, a_2 = 0.006, a_3 = 0.042.$$

In Fig.6, we summarize the above results.

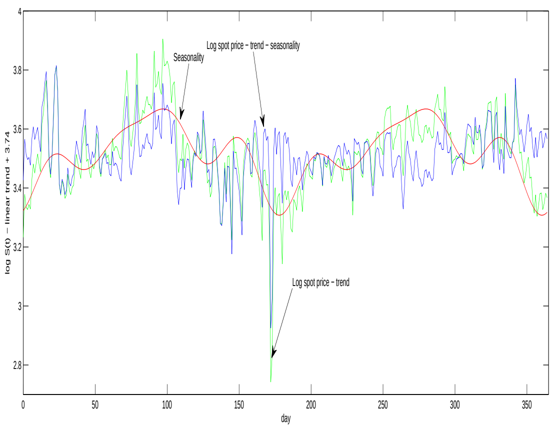


Figure 6: Log price (day ahead implicit auction market) with linear trend and seasonality.

## 3 SPOT RATE MODEL WITH JUMPS

The spot price $S(t)$ is set as

$$S(t) = \exp(r(t) + Se(t))$$

where $Se(t)$ is identified by (1) and the short rate $r(t)$ is given by the jump augmented Vasicek model;

$$dr(t) = \kappa(\overline{r} - r(t))dt + \sigma_r dw_r(t) + \int_R \nu p(d\nu, dt), \quad (2)$$

where $w_r$ is a standard Brownian motion process which is independent of the Poisson random measure $p$ and the compensated Poisson measure $q_c$ is is given by

$$q_c(d\nu, dt) = p(d\nu, dt) - (\lambda^+\psi^P(d\nu) + \lambda^-\psi^M(d\nu))dt$$

and where $\lambda^+$ ($\lambda^-$) denotes the positive jump ( negative jump) time intensity and $\psi^P$ ($\psi^M$) is a distribution of the positive (negative) jump size. In this paper we more specify this jump process as the compound Poisson processes:

$$
\begin{aligned}
\int_R \nu p(d\nu, dt) &= J^P(t;\psi^P)dN(t;\lambda^+) \\
&\quad + J^M(t;\psi^M)dN(t;\lambda^-),
\end{aligned}
$$

where $J^{\cdot}(t;\psi^{\cdot})$ denotes the jump size with identically distributed law $\psi^{\cdot}$ and $N(t,\lambda^{\cdot})$ is a counting process with parameter $\lambda^{\cdot}$. Here we shall present the simulation results for these compounded Poisson processes:

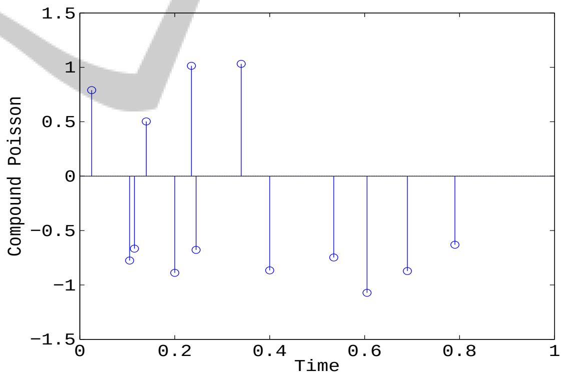


Figure 7: Compounded Poisson process.

## 4 ELECTRICITY MODEL

By a basic no-arbitrage argument it follows that the price of a futures contract $F(t, T-t)$ which has payoff $S(T)$ at future time $T$ equals to

$$F(t, T-t) = E\{S(T)|\mathcal{F}_t\},$$

with respect to the risk neutral measure. Hence we can write the futures price as

$$F(t, T-t) = \exp\{A(t, T-t) + B(t, T-t)r(t)\}, \quad (3)$$

where $A$ and $B$ satisfy deterministic equations. (See in (Duffie et al., 2000) for details.) Although this model is mathematically elegant, it is not consistent with the *forward curve* as stated in (Carmona and Ludkovski, 2004). From the systems identification view points, the observation futures data are summed to the artificial observation noises. In order to avoid this ambiguity, we append the extra noise in (3) as used in (Aihara and Bagchi, 2010). This noise represents the model errors from the basic property of $r(t)$. This will mean that the corresponding futures price should be given by a slight perturbation of (3),i.e.,

$$F(t,T-t) = \exp\{A(t,T-t)+B(t,T-t)r(t) + \int_0^t \sigma dw(s,T-s)\}, \quad (4)$$

where we use the same symbols in (3) and

$$\int_0^t \sigma dw(s,T-s) = \sum_{k=1}^{\infty}\int_0^t \sigma\frac{1}{\lambda_k}e_k(T-s)d\beta_k(s), \quad (5)$$

and where $e_k(\cdot)$ is a sequence of differentiable functions forming an orthonormal basis in $L^2(0,T^*)$[2] and $\{\beta_k(t)\}$ are mutually independent Brownian motion processes with $\sum\frac{1}{\lambda_k^2}<\infty$,i.e.,

$$\sigma^2 E\{w(t,x_1)w(t,x_2)\} = tq(x_1,x_2)$$

and

$$\int_0^{T^*} q(x,x)dx = \sum_{k=1}^{\infty}\frac{\sigma^2}{\lambda_k^2} < \infty.$$

Setting

$$f(t,x) = A(t,x)+B(x)r(t)+\int_0^t \sigma dw(s,x+t-s), \quad (6)$$

the futures contracts $F(t,T-t)$ becomes

$$F(t,T-t) = \exp(f(t,T-t)) \quad (7)$$

with $F(T,0)=\exp(f(T,0))=S(T)$. Now we derive the explicit forms of $A$ and $B$ so that $F(t,T-t)$ is a $\mathcal{F}_t$ martingale in the risk neutral measure. Applying the results by (Aihara and Bagchi, 2010), we get The explicit form of (6) is a solution of

$$\begin{aligned} df(t,x) &= \frac{\partial f(t,x)}{\partial x}dt - \tilde{q}_J(x)dt + e^{-\kappa x}\{\sigma_r dw_r(t) \\ &\quad + \int_R \nu q_c(d\nu,dt)\} + \sigma dw(t,x) \quad (8) \\ f(0,x) &= \bar{r}(1-e^{-\kappa x}) + \frac{\sigma_r^2}{2\kappa}(1-e^{-2\kappa x}) \\ &\quad + \frac{1}{2}\int_0^x q(z,z)dz + Se(x) + e^{-\kappa x}r(0) \\ &\quad + \int_0^x(\lambda^+C^P(z)+\lambda^-C^M(z))dz, \quad (9) \end{aligned}$$

[2] $T^*$ denotes the longest future time in mind

where

$$\tilde{q}_J(x) = \sigma_r^2 e^{-2\kappa x} + \frac{1}{2}q(x,x) + (\lambda^+C^P(x)+\lambda^-C^M(x))$$

and

$$C^\bullet(x) = \int_R \exp(e^{-\kappa x}\nu)\psi^\bullet(d\nu) - 1. \quad (10)$$

## 5 REAL WORLD DYNAMICS

On the identification problem, we work in the real world measure. For example, we add a simple risk premium term to (8) . In this paper, we simplify the position that the market price of risk $\Lambda_w(t)$ comes mainly from $w_r(t)$ but this moves stochastically. We set this term as

$$d\Lambda_w(t) = \kappa_\lambda(\bar{\Lambda}-\Lambda_\omega(t))dt + \sigma_\Lambda dw_2(t), \quad (11)$$

where the BMP $w_2$ is independent of $w_r$ . Now under the real world measure the BMP $\tilde{w}_r(t)$ is represented by

$$w_r(t) = \tilde{w}_r(t) - \int_0^t \Lambda_w(s)ds.$$

Hence our system state $[f(t,x)\ \Lambda_w(t)]$ under the physical measure becomes

$$\begin{cases} df(t,x) = \frac{\partial f(t,x)}{\partial x}dt - \tilde{q}_J(x)dt \\ \quad +e^{-\kappa x}\{\sigma_r(-\Lambda_w(t)dt \\ \quad +d\tilde{w}_r(t)) + \int_R \nu q_c(d\nu,dt)\} + \sigma dw(t,x) \\ d\Lambda_w(t) = \kappa_\lambda(\Lambda-\Lambda_w(t))dt + \sigma_\Lambda dw_2(t). \end{cases} \quad (12)$$

## 6 OBSERVATION

Noting that electricity is essentially not storable, the futures contracts are based on the arithmetic averages of the spot prices over a delivery period $[T_0,T]$, given by

$$\frac{1}{T-T_0}\int_{T_0}^T S(\tau)d\tau.$$

Now, for $t<T_0$ we can calculate the futures prices by

$$\begin{aligned} F(t,T_0,T) &= E\{\frac{1}{T-T_0}\int_{T_0}^T S(\tau)d\tau|S(t)\} \\ &= \frac{1}{T-T_0}\int_{T_0}^T F(\tau,t)d\tau \\ &= \frac{1}{T-T_0}\int_{T_0-t}^{T-t}\exp[f(t,x)]dx. \quad (13) \end{aligned}$$

In practice we adopt the geometric average as an approximation;

$$F(t,T_0,T) \sim \exp[\frac{1}{T-T_0}\int_{T_0-t}^{T-t} f(t,x)dx]. \quad (14)$$

By using this geometric approximation, the observation data for the futures price is set as

$$y_i(t) = \frac{1}{T-T_0}\int_{\tau_i}^{\tau_i+(T-T_0)} f(t,x)dx, \quad (15)$$

where $\tau_1 < \tau_2 < \cdots < \tau_m$ .

Denoting

$$\vec{Y}(t) = [y_i(t)]_{m\times 1},$$

we have

$$\begin{aligned} d\vec{Y}(t) &= H_\delta f(t,\cdot)dt - H(\tilde{q}_J + B(x)\sigma_r\Lambda_w(t))dt \\ &\quad + H[dw_M(t,\cdot)] + H[B\int_R \nu q_c(d\nu,dt)], \quad (16) \end{aligned}$$

where $B(x) = e^{-\kappa x}$,

$$w_M(t,x) = B(x)\sigma_r w_r(t) + \sigma w(t,x), \quad (17)$$

$$H(\cdot) = \frac{1}{T-T_0}[\int_0^{\tau_1+(T-T_0)}(\cdot)dx,\cdots,\int_0^{\tau_m+(T-T_0)}(\cdot)dx]'$$

and

$$\begin{aligned} H_\delta(\cdot) = [\frac{1}{T-T_0}\int_G(\delta(\eta-(T-T_0+\tau_i)) \\ -\delta(\eta-\tau_i))(\cdot)d\eta]_{m\times 1}. \end{aligned}$$

## 6.1 Reconstruction of Jump Process

Choosing one yield data for $\tau_0 < \tau_1$,

$$y_0(t) = \frac{1}{T-T_0}\int_{\tau_0}^{\tau_0+(T-T_0)} f(t,x)dx, \quad (18)$$

we have

$$\begin{aligned} dy_0(t) &= H_\delta^0 f(t,\cdot)dt - H^0(\tilde{q}_J + B(x)\sigma_r\Lambda_w(t))dt \\ &\quad + H^0[dw_M(t,\cdot)] + H^0[B\int_R \nu q_c(d\nu,dt)], \end{aligned}$$

where $H^0(\cdot) = \frac{1}{T-T_0}\int_0^{\tau_0+(T-T_0)}(\cdot)dx$, and

$$\begin{aligned} H_\delta^0(\cdot) = \frac{1}{T-T_0}\int_G(\delta(\eta-(T-T_0+\tau_0)) \\ -\delta(\eta-\tau_0))(\cdot)d\eta. \end{aligned}$$

Hence it is possible to reconstruct the jump process from $y_0(t)$ such that

$$\begin{aligned} &\int_0^t\int_R \nu q_c(d\nu,ds) \\ &= \frac{1}{B^0}(y_0(t) - \int_0^t H_\delta^0 f ds - H^0 w_M(t,x)) \\ &+ \int_0^t \frac{1}{B^0}H^0(\tilde{q}_J + B(x)\sigma_r\Lambda_w(s))ds, \quad (19) \end{aligned}$$

where $B^0 = H^0 B$. Plugging (19) into (12), we have

$$\begin{aligned} df(t,x) &= \frac{\partial f(t,x)}{\partial x}dt - (\tilde{q}_J(x) + B(x)\sigma_r\Lambda_w(t))dt \\ &+ dw_M(t,x) + \frac{B(x)}{B^0}\{dy_0(t) - H_\delta^0 f dt \\ &+ H^0(\tilde{q}_J + B(x)\sigma_r\Lambda_w(t))dt - H^0 dw_M(t,x)\} \quad (20) \end{aligned}$$

We transform the above equation as the robust form for jump term. Define [3]

$$\tilde{f}(t,x) = f(t,x) - \frac{B(x)}{B^0}y_0(t). \quad (21)$$

Hence we get

$$\begin{aligned} d\tilde{f}(t,x) &= (\frac{\partial}{\partial x} - C_\delta)(\tilde{f}(t,x) + \frac{B(x)}{B^0}y_0(t))dt \\ &- (1-C_0)(\tilde{q}_J(x) + B(x)\sigma_r\Lambda_w(t))dt \\ &+ (1-C_0)\sigma dw(t,x), \quad (22) \end{aligned}$$

where

$$C_\delta = \frac{B(x)}{B^0}H_\delta^0, \quad C_0 = \frac{B(x)}{B^0}H^0. \quad (23)$$

## 6.2 Reconstruction of Observed Yields

The original yield $y_j(t)$ becomes [4]

$$\begin{aligned} y_j(t) &= H^j f(t,\cdot) \\ &= H^j\tilde{f}(t,\cdot) + \frac{H^j B}{B^0}y_0(t). \quad (24) \end{aligned}$$

Now we construct the new observation such that

$$\begin{aligned} \tilde{y}_j(t) &= y_j(t) - \frac{H^j B}{B^0}y_0(t), \\ &= H^j\tilde{f}(t,\cdot). \quad (25) \end{aligned}$$

Denoting

$$\vec{\tilde{Y}}(t) = [\tilde{y}_j(t)]_{m\times 1},$$

and from $(H - HC_0)B\sigma_r\Lambda_w = HB\sigma_r\Lambda_w - HB\sigma_r\Lambda_w = 0$, we get

$$\begin{aligned} d\vec{\tilde{Y}}(t) &= (H_\delta - HC_\delta)\tilde{f}(t,\cdot)dt + (H_\delta - HC_\delta)\frac{B}{B^0} \\ & y_0(t)dt - (H - HC_0)\tilde{q}_J dt + (H - HC_0)\sigma dw(t,x). \\ & \quad (26) \end{aligned}$$

[3] We ued $w_M - \frac{B}{B^0}H^0 w_M = (1 - \frac{B}{B^0}H^0)\sigma w$.

[4] We used $H^j(\cdot) = \frac{1}{(T-T_0)}\int_{\tau_i}^{\tau_i+(T-T_0)}(\cdot)dx$.

# 7 THE KALMAN FILTER

In (8), Poisson jump processes are included and this is not a usual Kalman filter problem. There are many articles for Non-Gaussian filtering problem by using a martingale approach, e.g. (van Schuppen, 1977) and however it is still difficult to derive the closed form filtering algorithm. In our position, the transformed system (22) with the observation (26) do not include jump processes explicitly. Hence our estimation problem is in the Gaussian framework;

$$d\begin{pmatrix} \tilde{f}(t,x) \\ \Lambda_w(t) \end{pmatrix} = \begin{pmatrix} (\frac{\partial}{\partial x} - \mathcal{C}_\delta) - (1-\mathcal{C}_0) \\ 0 - \kappa_\lambda \end{pmatrix} \begin{pmatrix} \tilde{f}(t,x) \\ \Lambda_w(t) \end{pmatrix} dt + \begin{pmatrix} (\frac{\partial}{\partial x} - \mathcal{C}_\delta)\frac{B}{B^0}y_0 - (1-\mathcal{C}_0)\tilde{q}_J \\ \kappa_\lambda \bar{\Lambda} \end{pmatrix} dt + d\begin{pmatrix} (1-\mathcal{C}_0)w(t,x) \\ w_\Lambda(t) \end{pmatrix}. \tag{27}$$

with

$$d\vec{\tilde{Y}}(t) = (H_\delta - H\mathcal{C}_\delta, 0)\begin{pmatrix} \tilde{f}(t,x) \\ \Lambda_w(t) \end{pmatrix} dt + ((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) - (H - H\mathcal{C}_0)\tilde{q}_J)dt + (H - H\mathcal{C}_0)dw(t,x). \tag{28}$$

Denoting $\hat{\cdot} = E\{\cdot|\mathcal{Y}_t\}$, for $\mathcal{Y}_t = \sigma\{\vec{\tilde{Y}}(s), y_0(s); 0 \le s \le t\}$, we have

$$\begin{aligned} d\hat{\tilde{f}}(t,x) &= (\frac{\partial}{\partial x} - \mathcal{C}_\delta)(\hat{\tilde{f}}(t,x) + \frac{B(x)}{B_0}y_0(t))dt \\ &-(1-\mathcal{C}_0)(B(x)\hat{\Lambda}_w(t) + \tilde{q}_J(x))dt \\ &+\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - H\mathcal{C}_\delta)^* \\ &+(1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\}\vec{\Phi}^{-1} \\ &\times\left\{ d\vec{\tilde{Y}}(t) - (H_\delta - H\mathcal{C}_\delta, 0)\begin{pmatrix} \hat{\tilde{f}}(t,x) \\ \hat{\Lambda}_w(t) \end{pmatrix} dt \right. \\ &\left. -((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) - (H - H\mathcal{C}_0)\tilde{q}_J dt \right\}, \end{aligned} \tag{29}$$

and

$$\begin{aligned} d\hat{\Lambda}_w(t) &= \kappa_\lambda(\bar{\Lambda} - \hat{\Lambda}_w(t)dt \\ &+\tilde{\mathbf{P}}_{\Lambda f}(t)(H_\delta - H\mathcal{C}_\delta)^*\vec{\Phi}^{-1} \\ &\times\left\{ d\vec{\tilde{Y}}(t) - (H_\delta - H\mathcal{C}_\delta, 0)\begin{pmatrix} \hat{\tilde{f}}(t,x) \\ \hat{\Lambda}_w(t) \end{pmatrix} dt \right. \\ &\left. -((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) + (H - H\mathcal{C}_0)\tilde{q}_J dt \right\}, \end{aligned} \tag{30}$$

where $Q = \int q(x,z)(\cdot)dz$,

$$\vec{\Phi} = (H - H\mathcal{C}_0)((H - H\mathcal{C}_0)Q)^*, \tag{31}$$

and

$$\begin{aligned} \frac{\partial \tilde{\mathbf{P}}_{ff}(t)}{\partial t} &= (\frac{\partial}{\partial x} - \mathcal{C}_\delta)\tilde{\mathbf{P}}_{ff}(t) + \tilde{\mathbf{P}}_{ff}(t)(\frac{\partial}{\partial x} - \mathcal{C}_\delta)^* \\ &-\tilde{\mathbf{P}}_{f\Lambda}(1-\mathcal{C}_0)^* - (1-\mathcal{C}_0)\tilde{\mathbf{P}}_{\Lambda f} + (1-\mathcal{C}_0)Q(1-\mathcal{C}_0)^* \\ &-\left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^* + (1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\right\}\vec{\Phi}^{-1} \\ &\left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^* + (1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\right\}^*, \end{aligned} \tag{32}$$

$$\begin{aligned} \frac{\partial \tilde{\mathbf{P}}_{f\Lambda}(t)}{\partial t} &= (\frac{\partial}{\partial x} - \mathcal{C}_\delta)\tilde{\mathbf{P}}_{f\Lambda}(t) - \tilde{\mathbf{P}}_{f\Lambda}(t)\kappa_\lambda \\ &-\left\{\tilde{\mathbf{P}}_{ff}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^* + (1-\mathcal{C}_0)Q(H - H\mathcal{C}_0)^*\right\}\vec{\Phi}^{-1} \\ &\left\{\tilde{\mathbf{P}}_{\Lambda f}(t)(H_\delta - \frac{HB}{B^0}H_\delta^0)^*\right\}^*. \end{aligned} \tag{33}$$

# 8 ADAPTIVE FILTERING PROBLEM

## 8.1 Parallel Filtering

To construct the parameter estimation algorithm, we confine the general jump process as the compound Poisson process as stated in Remark, i.e., $J^P(t;\phi^P) =$ Gaussian with mean $m_J^P$ and variance $\sigma_J^P$ and i.i.d for each jump time and same as the negative jump. Hence we have

$$C^P(x) = \exp[\int_0^x B(y)dy(m^P + \frac{1}{2}(\sigma_J^P)^2)].$$

As used in (Aihara and Bagchi, 2010), we set the function form $q(x,x)$. Hence the unknown system parameters [5]

$$\theta = [\kappa\ \bar{r}\ \sigma\ \sigma_r\ m^P\ m^M\ \sigma_J^P\ \sigma_J^M\ \lambda^+\ \lambda^-\ \kappa_\lambda\ \bar{\Lambda}\ \sigma_\lambda] \in \Theta \subset R^{13},$$

where $\Theta$ is a known bounded set. Following from (Anderson and Moore, 1979), the following parallel filtering algorithm is made:

- Set candidates of unknown parameter $\theta$ such that

  $$\theta^{(j)} \in \text{uniform random vectors in } \Theta, j = 1, \cdot, \cdot, m_p.$$

- For each $\theta^{(j)}$, we solve the Kalman filter (29, 30) for $t_i \le t \le t_{i+1}$.

[5] $\bar{r}$ is estimated from the initial forward cure. See (Aihara and Bagchi, 2010)

- Calculate the posteriori probability,

$$p(\theta^{(j)}|\vec{\tilde{Y}}(t_i)) = \frac{p(\theta^{(j)}|\vec{\tilde{Y}}(t_i))p(\vec{\tilde{Y}}(t_{i+1})|\theta^{(j)},\vec{\tilde{Y}}(t_i))}{\sum_{k=1}^{m} p(\vec{\tilde{Y}}(t_{i+1})|\theta^{(k)},\vec{\tilde{Y}}(t_i))p(\theta^{(k)}|\vec{\tilde{Y}}(t_i))},$$

where

$$p(\vec{\tilde{Y}}(t_{i+1})|\theta^{(j)},\vec{\tilde{Y}}(t_i)) \propto \exp\left(\int_{t_i}^{t_{i+1}} \vec{\tilde{Y}}(s)'\tilde{\Phi}^{-1} \times \left\{d\vec{\tilde{Y}}(t) - (H_\delta - H\mathcal{C}_\delta, 0)\begin{pmatrix}\hat{\vec{f}}(t,x)\\ \hat{\Lambda}_w(t)\end{pmatrix}dt - ((H_\delta - H\mathcal{C}_\delta)\frac{B}{B^0}y_0(t) - (H - H\mathcal{C}_0)\tilde{q}_J dt\right\}$$

- The estimates of $f$ and $\theta$ are given by

$$\hat{f}(t_{i+1},x) = \sum_{i=1}^{m_p} \hat{f}(t_{i+1},x;\theta^{(j)})p(\theta^{(j)}|\vec{\tilde{Y}}(t_{i+1}))$$

$$\hat{\theta} = \sum_{i=1}^{m_p} \theta^{(j)}p(\theta^{(j)}|\vec{\tilde{Y}}(t_{i+1})).$$

## 8.2 Resample Procedure for Parameters

The parallel filter algorithm is not sensitive for identifying many unknown parameters. In this paper, we propose a new resampling procedure to increase the diversity of parameter estimates.

- Set the resampling time period $t_{resamp}$.
- At the time $t_r = (n-1)t_{resamp}$ for $n = 1,2,\cdots$, calculate

$$\hat{\sigma}^2_{\theta(i)} = \sum_{j=1}^{m_p}\{(\theta^{(j)})^2 p(\theta^{(j)}|\vec{\tilde{Y}}(t_r)) - \hat{\theta}^2(i)\}$$

$$\text{for } i = 1,2,\cdots,12.$$

- Construct the posteriori distribution by using the Gaussian approximation:

$$P(\theta(i)|\vec{\tilde{Y}}(t_r)) \sim \mathcal{N}(\theta(i);\hat{\theta}(i),\varepsilon_i\hat{\sigma}_{\theta(i)}),$$

where $\varepsilon$ denotes a user defined parameter.

- Generate new samples $\theta^{(j)}(i)$ in $\Theta$ from $P(\theta(i)|\vec{\tilde{Y}}(t_r))$. To get new samples in $\Theta$, we use the systematic resampling procedure.
- Reset $p(\theta^{(j)}|\vec{\tilde{Y}}(t_r)) = \frac{1}{m_p}$.

# 9 SIMULATION STUDIES

In this simulation study, we set the system parameters in Table-1

Table 1: Systems parameters.

| $\kappa$ | $\bar{r}$ | $\sigma_r$ | $\sigma$ | $m_J^P$ | $\lambda^+$, | $\sigma_J^P$ |
|---|---|---|---|---|---|---|
| 5.00 | 3.00 | 0.80 | 0.1 | 0.30 | 8.00 | 0.20 |
| $m_J^M$ | $\lambda^-$ | $\sigma_J^M$ | $\kappa_\lambda$ | $\bar{\Lambda}$ | $\sigma_\lambda$ | |
| -0.30 | 8.00 | 0.20 | 6.00 | 0.40 | 2.00 | |

Setting the seasonality and linear trend functions are set in Sec.2, we simulate (12) by using the finite difference method with $dx = 0.01, dt = 0.005$ in Fig.8.

We also generate the observation data $\vec{Y}(t) = [10y_i(t)]_i$ for $i = 1,2,\cdots,7$ with $\tau_1 = 0, \tau_2 = 0.1,\cdots,\tau_7 = 0.6$ shown in Fig.9.



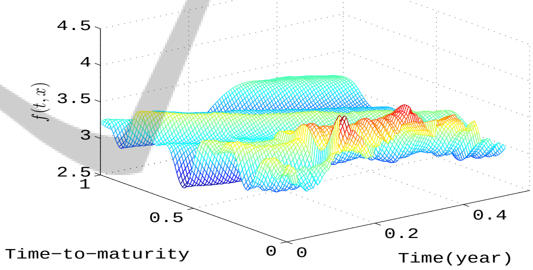


Figure 8: $f(t,x)$- process.

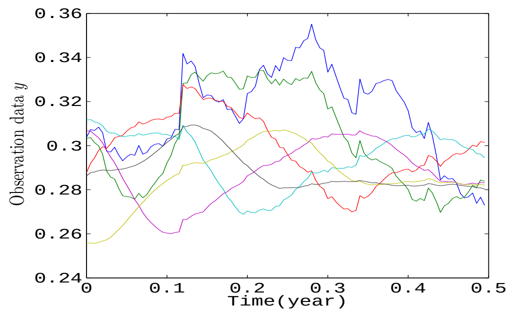


Figure 9: Observed data $\vec{Y}$.

If all systems parameters are known, the Kalman filter works good and such simulation results are found in (Aihara et al., 2014). Here we shall look into the feasibility of the proposed on-line algorithm. Hence we set the upper and lower bounds for the unknown parameters shown in Table-2 with the tuning parameters $\varepsilon_i$.

Table 2: Upper and lower bounds for systems parameters.

| | $\kappa$ | $\bar{r}$ | $\sigma_r$ | $\sigma$ | $m_J^P$ |
|---|---|---|---|---|---|
| Upper | 6.5 | 3.9 | 1.04 | 0.13 | 0.39 |
| Lower | 3.5 | 2.1 | 0.56 | 0.07 | 0.21 |
| $\varepsilon_i$ | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 |
| | $\lambda^+$ | $\sigma_J^P$ | $m_J^M$ | $\lambda^-$ | $\sigma_J^M$ |
| Upper | 10.4 | 0.26 | -0.21 | 10.4.00 | 0.26 |
| Lower | 5.6 | 0.14 | -0.39 | 5.6 | 0.14 |
| $\varepsilon_i$ | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 |
| | $\kappa_\lambda$ | $\bar{\Lambda}$ | $\sigma_\lambda$ | | |
| Upper | 7.8 | 0.52 | 2.6 | | |
| Lower | 4.2 | 0.28 | 1.4 | | |
| $\varepsilon_i$ | 1.05 | 1.05 | 1.05 | | |

The estimate $\hat{f}(t,x)$ is shown in Fig. 10.

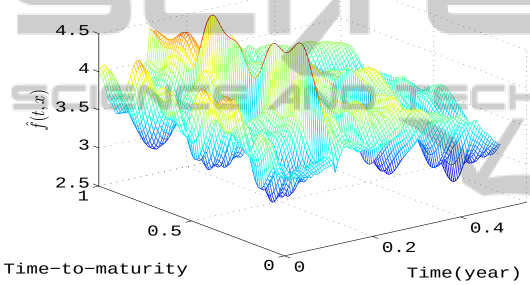


Figure 10: Estimated $\hat{f}(t,x)$.

The estimate of the market price of risk is demonstrated in Fig.11.

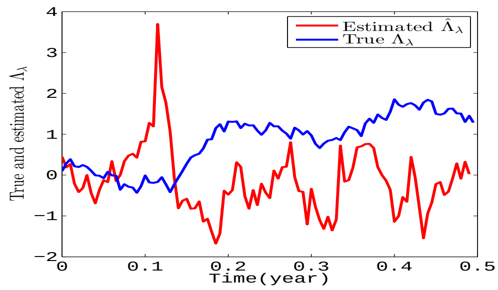


Figure 11: True and estimated $\Lambda_w(t)$.

Now we call demonstrate the estimates of unknown parameters where we selected for $\kappa$, and $\bar{r}$ in Figs.12 and 13. The estimates of other parameters have almost same beavers in these images.

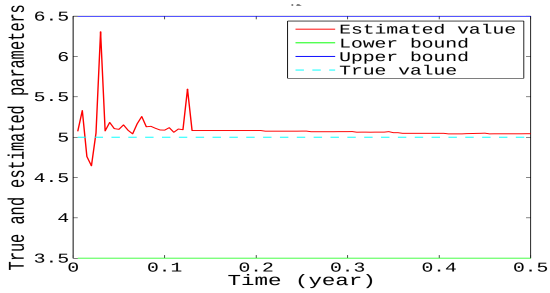


Figure 12: True and estimated $\kappa$.

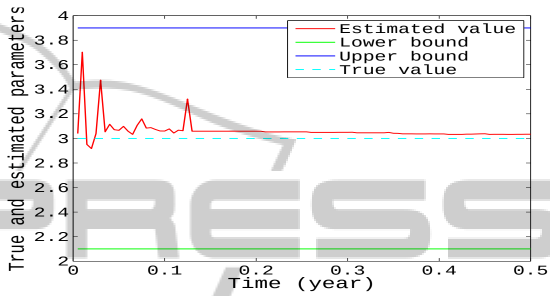


Figure 13: True and estimated $\bar{r}$.

## 10 CONCLUSIONS

Bringing in the compound Poisson jump process, the stochastic model for the electricity futures has been suggested. By using the idea proposed by (Aihara et al., 2014) the original filtering problem is changed to the Gaussian framework and its Kalman filter is derived. In the adaptive filtering algorithm, the parallel filtering method in (Anderson and Moore, 1979) is used to obtain the on-line parameter estimates with the new resampling procedure. From the proposed algorithm, it is not possible to estimate the noise correlation parameter, if it exits. The possible way to identify this parameter is to use the Rao-Blackwellized filter.

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