Routing Strategy of a Prioritized Limited Multi-Server Processor-Sharing System

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Keywords: Processor-Sharing, Multi-Server, Routing Strategy, Performance Evaluation, Loss Probability, Sojourn Time, Traffic Simulation.

Abstract: In this work, routing strategies of an arriving request to a server in a prioritized limited multi-server processor-sharing (PS) system are studied in order to optimize a given performance criterion. In this system, an arriving request enters the dispatcher, which routes this request to each server according to a predetermined strategy. In the prioritized limited PS server, a high-priority request is allocated a service ratio that is m (called the priority ratio) times greater than that of a low-priority request. Moreover, the sum of the number of the requests receiving service is restricted to a fixed value. The arriving request which cannot receive service will be queued (waiting system) or rejected (loss system). In this server, at the arrival (or departure) of a request, the extension (or shortening) of the remaining sojourn time of each request that is receiving service can be calculated using the number of requests and priority ratio. Employing a simulation program to execute these events and calculations enables us to analyze the performance of this system, such as the loss probability, mean sojourn time, and mean waiting time. Based on the evaluation results, the most suitable routing strategy for the loss or waiting system is clarified.

1 INTRODUCTION

The processor-sharing (PS) discipline has gained an important role in evaluating the performance of a variety of resource allocation mechanisms. Under PS discipline, if there are n (> 0) requests in a single-server system, then each request receives 1 / n of the service facility capacity. No arriving request has to wait for the service because it will be served promptly, even if the service rate becomes slow. The PS paradigm emerged as an idealization of Round-Robin (RR) scheduling algorithms in time-shared computer system.

A PS discipline with a priority structure has been proposed as well, wherein a larger service ratio is allocated to requests that have high-priority. In such a prioritized PS paradigm, with an increase in the number of arriving requests, the service ratio for an individual request decreases; accordingly, the sojourn time of each request increases. In order to prevent an increase in the sojourn time of each request in such a prioritized single-server PS paradigm and to realize a realistic model of sharing, a method for limiting the number of requests receiving service has been proposed. In such a prioritized limited single-server PS system, a high-priority (class-1) request is allocated a service ratio that is m (≥ 1, called the priority ratio) times greater than that of a low-priority (class-2) request. Moreover, the sum of the number of the requests receiving service is restricted to a fixed value. The arriving requests that cannot receive service will be attached to the service-waiting queue (waiting system) or is rejected (loss system).

On the other hand, communication services such as web server-farms, database systems and grid computing clusters, routinely employ multi-server systems to provide a range of services to their customers. An important issue in such systems is to determine which server an arriving request should be routed to, in order to optimize a given performance criterion. Therefore, we investigate the routing strategies in a multi-server system where each server employs the prioritized limited PS rule. The following two routing strategies are proposed:

(a) RST (Remaining Sojourn Time)-based strategy – Routing to the server that has the smallest value of the sum of the remaining sojourn time of each request.
(b) NNR (Normalized Number of Requests)-based

Shikata Y. and Hanayama N..
Routing Strategy of a Prioritized Limited Multi-Server Processor-Sharing System.
DOI: 10.5220/0004998900050012
In *Proceedings of the 5th International Conference on Data Communication Networking* (DCNET-2014), pages 5-12
ISBN: 978-989-758-042-0

strategy – Routing to the server with the smallest value of ($m * n1_i + n2_i$), which is the normalized number of requests. Here, $n1_i$ and $n2_i$ represent the number of class-1 and class-2 requests in the server i (including the arriving one).

The performance measures of practical interest of these strategies, such as the loss probability, mean waiting time in the service waiting queue, and mean sojourn time are evaluated via simulation, and compared with the values in case of the following conventional strategies:

(c) IT strategy – Routing to each server in turn according to a fixed order.

(d) RAND strategy – Routing to each server randomly, with the same probability.

Based on the evaluation results, we discerned the most suitable routing strategies for the loss system and waiting system.

Under the PS rule, when a request either arrives at or departs from the system, the remaining sojourn time of other requests will be extended or reduced, respectively. This extension or reduction of the sojourn time is calculated using the number of requests of each class and the priority ratio. Employing a simulation program to execute these events and calculations enables us to analyze the performance of the prioritized limited multi-server PS rule, which is realistic in a time-sharing system (TSS) with a sufficiently small time slot.

In the simulation program, the arrival timer or service timer of each request controls the simulation clock. In each while loop of the simulation program, one of these timers expire, and the abovementioned arrival processing or service end processing is executed. Simultaneously, the time duration until the expiry of the next timer is pushed forward in order to skip the insignificant simulation clock, thereby shortening the total simulation time.

The PS rule, an idealization of quantum-based RR scheduling at the limit where quantum size becomes infinitesimal, has been the subject of many papers (Kleinrock, 1967) (Fayolle and Mitrani, 1980) (Altman, Avrachenkov and Ayesta, 2006) (Haviv and Val, 2008). A limited PS system and a prioritized limited PS system, in which the number of requests receiving service is restricted to a fixed value, have been proposed as well. Further, the performance of these systems has been analyzed (Yamazaki and Sakasegawa, 1987) (Shikata, Katagiri and Takahashi, 2011). Moreover, load-balancing strategies for multi-class multi-server PS systems with a Poisson input stream, and heterogeneous service rates have been investigated (Chen, Marden and Wierman, 2009) (Gupta, Balter, Sigman, and Whitt, 2007) (Altman, Ayesta, and Prabhu 2011). However, routing strategies in a prioritized limited multi-server PS system have not been investigated. Thus, the most suitable routing strategy in the loss system or waiting system remains to be discerned. Moreover, the influence that the service facility capacity may have on the mean sojourn time, the mean waiting time in the service waiting queue, and the loss probability in the prioritized limited multi-server PS system have not been investigated.

2 PRIORITIZED LIMITED MULTI-SERVER PS SYSTEM

In the prioritized limited multi-server PS system, an arriving request enters the dispatcher, which routes this request to each prioritized limited PS server according to a predetermined strategy. Suppose that there are two classes, and an arriving (class-1 or class-2) request, which is routed to server i, encounters $n1_i$ class-1 and $n2_i$ class-2 requests. According to the prioritized limited multi-server PS rule, if ($m * n1_i + n2_i \le C_i$), class-1 requests individually and simultaneously receive $m / (m * n1_i + n2_i)$ of the service facility capacity of server i, whereas class-2 requests receive $1 / (m * n1_i + n2_i)$ of it. When a server meeting this condition ($m * n1_i + n2_i \le C_i$) does not exist, the arriving request will be queued in the corresponding class waiting room prepared in the dispatcher or rejected. Here, m (≥ 1) denotes the priority ratio, and C_i ($\le \infty$), the service facility capacity of server i.

2.1 Routing Strategies

The evaluation model is shown in Figure 1. When a request arrives at the dispatcher, first, it is checked whether there exists a server that satisfies the condition ($m * n1_i + n2_i$) $\le C_i$. Otherwise, the arriving request is rejected or is placed in the service waiting queue for each request prepared in the dispatcher. If there are one or more servers in which the value of ($m * n1_i + n2_i$) is less than the C_i, the arriving request is routed to one of these servers according to the predetermined routing strategy. In the waiting system, when the service for a request ends in one of the servers, another request is taken from the service waiting queue and is routed to this server. The following four routing strategies are considered, and their performances are compared.

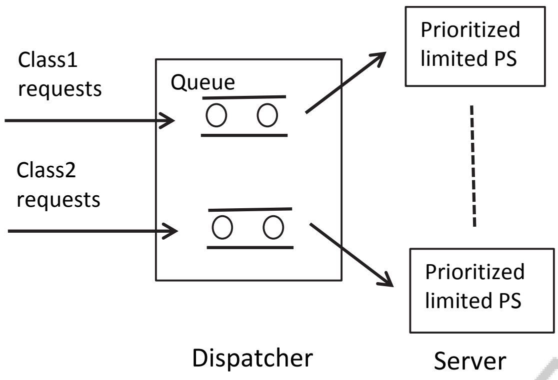

Figure 1: Evaluation Model.

2.1.1 RST-based Strategy

In this strategy, at the arrival of a request, the sum of the remaining sojourn time of each request in each server is evaluated. An arriving request is routed to the server that has the smallest sum.

2.1.2 NNR-based Strategy

In this strategy, at the arrival of a request, the value of $(m * n1_i + n2_i)$ in each server is evaluated. An arriving request is routed to the server that has the smallest value of $(m * n1_i + n2_i)$.

2.1.3 IT Strategy

In this strategy, the server to which an arriving request is routed is selected in turn according to a fixed order. If $(m * n1_i + n2_i) > C_i$ in the selected server i, the next server is selected according to the same rule.

2.1.4 RAND Strategy

In this strategy, the server to which an arriving request is routed is selected at random with the same probability. If $(m * n1_i + n2_i) > C_i$ in the selected server i, the next server is selected according to the same rule.

2.2 Remaining Sojourn Time

When a request arrives at (or departs from) the system, the remaining sojourn time of each of the requests that are currently receiving service is extended (or shortened). This extension (or reduction) of the remaining sojourn time of each request that is receiving service can be calculated using the number of each class requests and the priority ratio. By tracing these numerical changes in the remaining sojourn time in the simulation program, the loss probability, mean waiting time in the queue, and mean sojourn time of an individual class request in the server are evaluated.

If a class-1 request is routed to a server i, $m / \{m * (n1_i + n2_i)\}$ of the service facility capacity will be allocated to this request thenceforth until the arrival (or departure) of the next request in this server. The sojourn time of an arriving request So is then given by

$$So = Sr * (m * n1_i + n2_i) / m \quad (1)$$

where Sr is the requested service time of an arriving request. Moreover, $m / \{m * (n1_i - 1) + n2_i\}$ (to a class-1 request) or $1 / \{m * (n1_i - 1) + n2_i\}$ (to a class-2 request) of the service facility capacity is given to each request that is receiving service in this server currently. Thenceforth, until the arrival (or departure) of the next request, $m / \{m * (n1_i + n2_i)\}$ (to a class-1 request) or $1 / \{m * (n1_i + n2_i)\}$ (to a class-2 request) of the service facility capacity will be allocated to each request that is being serviced in this server. Therefore, the remaining sojourn time of each request following the arrival of this request, Sa, is then extended as follows:

$$Sa = Sb * \{m * (n1_i + n2_i)\} / \{m * (n1_i - 1) + n2_i) \quad (2)$$

where Sb is the remaining sojourn time of each request immediately before this request arrives. Similarly, at the arrival of a class-2 request, the sojourn time of an arriving request So is given by:

$$So = Sr * (m * n1_i + n2_i) \quad (3)$$

The remaining sojourn time of each request following the arrival of this request, Sa is extended as follows:

$$Sa = Sb * (m * n1_i + n2_i) / (m * n1_i + n2_i - 1) \quad (4)$$

In addition, when the sojourn time of a class-1 request terminates, the remaining sojourn time of each request following the departure of this request from this server Sa reduces to the following:

$$Sa = Sb * \{m * (n1_i - 1) + n2_i\} / (m * n1_i + n2_i) \quad (5)$$

here Sb is the remaining sojourn time of each request immediately before this request departs from this server. At the conclusion of service of a class-2 request, Sa is reduced as follows:

$$Sa = Sb * (m * n1_i + n2_i - 1) / (m * n1_i + n2_i) \quad (6)$$

Tracking these events and calculations enables us to evaluate practical performance measures, such as the loss probability, mean waiting time in the service waiting queue, and mean sojourn time of requests.

3 SIMULATION FLOW

Simulation flow of the prioritized limited multi-server PS system is shown in Figure 2. The simulation program is developed in C programming language. In this program, the simulation clock is controlled by the arrival timer or service timer of each request that is receiving service. At the arrival of each class request, the time duration until the next arrival of the request is set into the arrival timer according to the predetermined arrival time distribution. Further, the service time (e.g., remaining sojourn time) of each arriving request calculated using the equation (1) or (3) is set into the service timer. Moreover, the arrival time of each request is memorized in the corresponding variable. The sojourn time of each request in the server is evaluated using this data and service end time. In addition, the waiting time in the service-waiting queue is evaluated using this data and service start time. In the while loop of this simulation program, the extension or shortening of the remaining sojourn time of each request mentioned in the Section 2.2 is executed on the expiry of one of the arrival timers or service timers. Moreover, the service timer or arrival timer with the next smallest value is detected, and the time duration of this timer is subtracted from all the remaining timers.

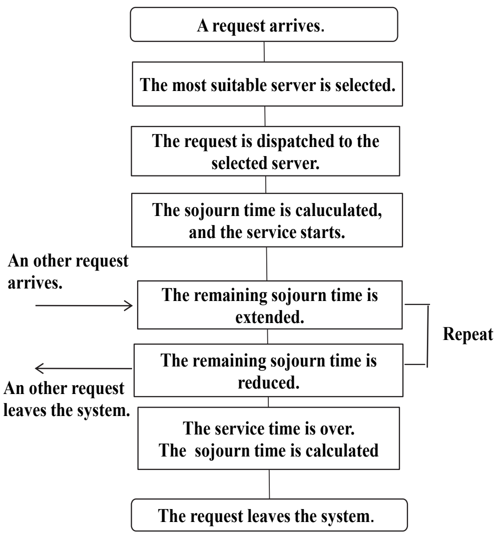

Figure 2: Simulation flow.

Therefore, in the next while loop this timer expires. Simultaneously, the simulation clock is pushed forward by this time duration in order to skip the insignificant simulation clock. The while loop is repeated until the number of arriving requests attains a predetermined value.

4 EVALUATION RESULTS

In the evaluation, the priority ratio m is assumed to be two, and the two-stage Erlang inter-arrival distribution and the two-stage hyper-exponential service time distribution are considered. Two servers are prepared, and these servers have the same performance and service facility capacity. Evaluation results are obtained as the average of ten simulation results. About 80,000 requests were produced for each class in each run.

4.1 Limited Multi-Server PS System

4.1.1 Loss System

In this system, only the class-2 requests are served. Figure 3 compares the loss probabilities in the loss system in case of the RST-based, NNR-based, IT, and RAND strategies. Here, S or A represents the mean requested service time or arrival rate respectively. The range of markers includes 95% of the reliability intervals obtained from the ten simulation runs. The logarithm of the loss probability increases linearly as the service facility capacity decreases. The loss probability value in case of the RST-based strategy is slightly lesser than its value in case of the NNR-based strategy, and is lesser than its value in case of the IT or RAND strategy.

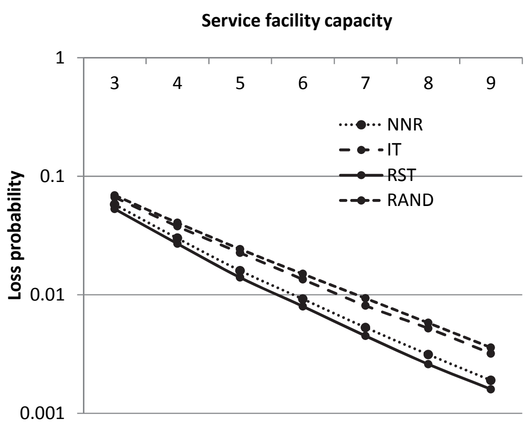

Figure 3: Loss probability versus service facility capacity /A=0.8, S=2).

The loss probability in case of the RAND and IT strategies shows almost the same value.

Figure 4 compares the mean sojourn time of the RST-based, NNR-based, IT, and RAND strategies. The mean sojourn time increases as the service facility capacity increases. The mean sojourn times of the RST-based and NNR-based strategies are almost the same, and are significantly lesser than their corresponding values obtained by the IT or RAND strategies. The mean sojourn time in case of the RAND strategy approaches the value obtained by the IT strategy as the service facility capacity decreases. Based on the above-mentioned results, it may be inferred that the RST–based strategy, which realizes small loss probability and mean sojourn time, is the most suitable routing strategy for the loss system of the limited multi-server PS system.

4.1.2 Waiting System

Figure 5 and 6 compares the mean waiting time in the service waiting queue, and mean sojourn time (not includes the waiting time in the service waiting queue) in the waiting system, respectively, for the RST-based, NNR-based, IT, and RAND strategies. The mean sojourn time in case of the IT strategy is lesser than its corresponding value in the RAND strategy, and approaches the value obtained by the RAND strategy as the service facility capacity decreases. The mean waiting time increases as the service facility capacity decreases.

On the other hand, the mean sojourn time decreases as the service facility capacity decreases. Both the mean waiting time and mean sojourn time of the RST-based and NNR-based strategies is lesser than the corresponding values in the IT or RAND strategy.

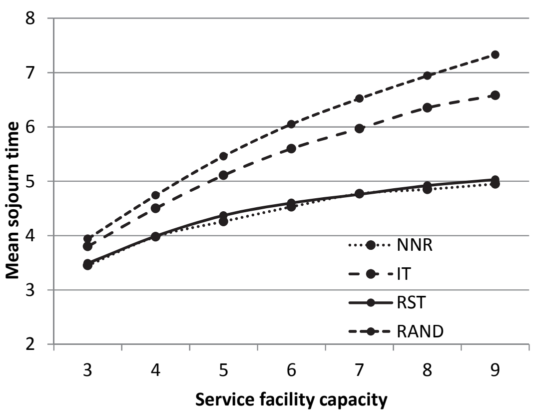

Figure 4: Mean sojourn time versus service facility capacity (A=0.8, S=2).

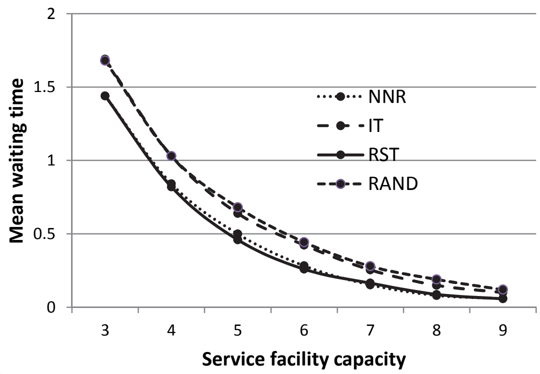

Figure 5: Mean waiting time versus service facility capacity (A=0.8, S=2).

There are few differences in mean waiting time and mean sojourn time between the RST-based and NNR-based strategies. It is evident from the details mentioned above that the NNR-based or RST-based strategies demonstrate the best performances in the waiting system. On the other hand, the calculation algorithm of the normalized number of requests in the NNR-based strategy is easier than that of the sum of the remaining sojourn time in the RST-based strategy. Therefore, it may be inferred that the NNR-based strategy is the most suitable routing strategy for the waiting system of the limited multi-server PS system.

4.2 Prioritized Limited Multi-Server PS System

In the evaluation of the performance of the prioritized limited multi-server PS system, the arrival rate and mean requested service time of each class request are assumed to have the same value.

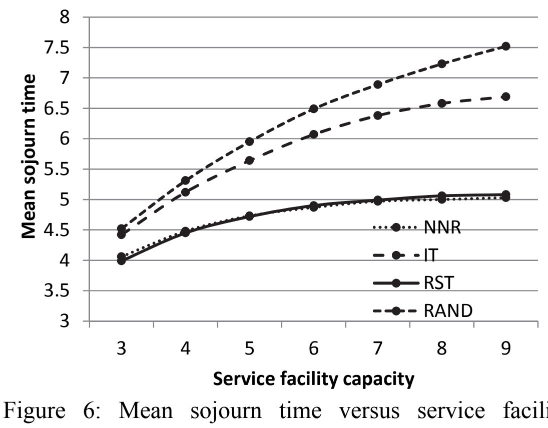

Figure 6: Mean sojourn time versus service facility capacity (A=0.8, S=2).

4.2.1 Loss System

Figure 7 compares the loss probability of the class-1 request (shown as round markers) and class-2 request (shown as cross markers) in case of the RST-based, NNR-based, IT, and RAND strategies. The logarithm of the loss probability increases linearly as the service facility capacity decreases. The loss probability of the class-1 request of the RST-based and NNR-based strategies is almost the same, and is smaller than the value in case of the IT or RAND strategy. The loss probability of the class-2 request of the NNR-based strategy is smaller than its value in other strategies.

Figure 8 compares the mean sojourn time of the class-1 request (shown as round markers) and class-2 request (shown as cross markers) in case of the RST-based, NNR-based, IT, and RAND strategies. The mean sojourn time increases as the service facility capacity increases. The mean sojourn times of the class-1 request of the RST-based and NNR-based strategies are also the same, and are smaller the corresponding values obtained by the IT or RAND strategy.

On the other hand, the mean sojourn time of the class-2 request in case of the RST-based strategy is slightly lesser than its value in case of the NNR-based strategy, and is lesser than its value in case of the IT or RAND strategy. Based on the above-mentioned results, it may be inferred that the NNR-based is the most suitable routing strategy in the loss system of the prioritized limited multi-server PS system in cases where the loss probability is a higher priority than the sojourn time. On the other hand, in cases where the sojourn time is a higher priority than the loss probability, the RST-based strategy is the most suitable routing strategy.

4.2.2 Waiting System

Figure 9 shows the relationship between the mean sojourn time and service facility capacity of the class-1 requests and class-2 requests in case of the RST-based, NNR-based, IT, and RAND strategies.

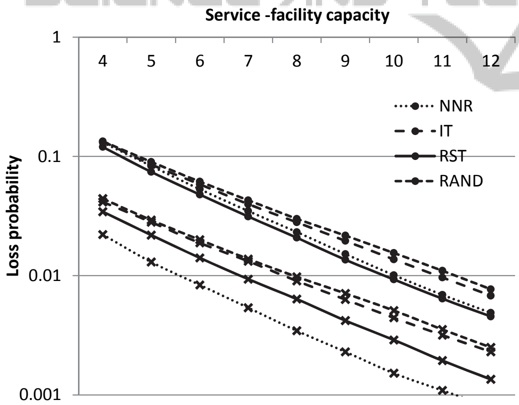

Figure 7: Loss probability versus service facility capacity (A=0.8, S=2).

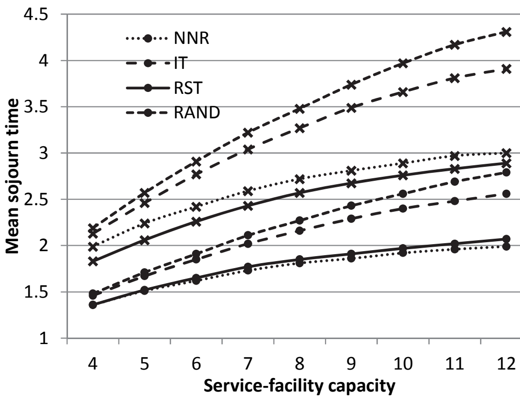

Figure 8: Mean sojourn time versus service facility capacity (A=0.8, S=1).

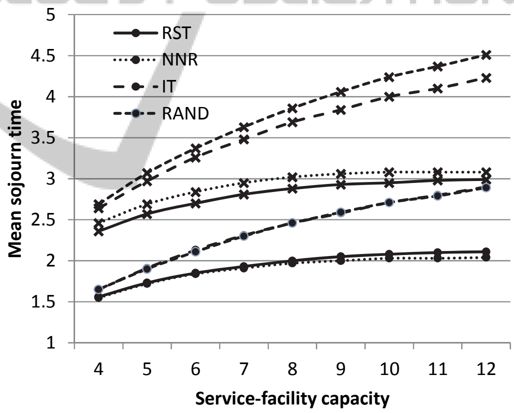

Figure 9: Mean sojourn time versus service-facility capacity (A=0.8, S=1).

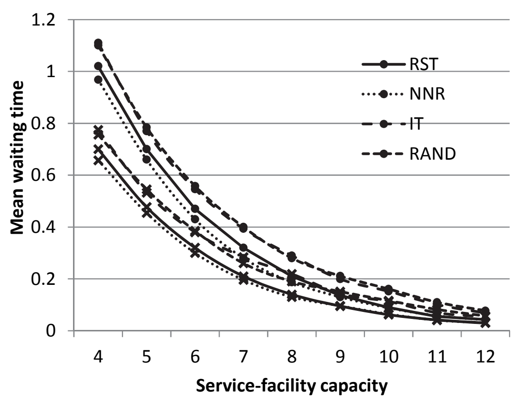

Figure 10: Mean waiting time versus service-facility capacity (A=0.8, S=1).

The mean sojourn time of the class-2 requests of the RST-based strategy is slightly lesser than the corresponding values obtained by the NNR-based strategy, and significantly lesser than that value obtained by the IT and RAND strategies. The mean sojourn time of the class-1 requests in case of the NNR-based strategy is the same as that value in case of the RST-based strategy, and is significantly lesser than that obtained by the IT and RAND strategies. Figure 10 shows the relationship between the mean waiting time in the service waiting queue and service facility capacity of the class-1 requests and class-2 requests in case of the RST-based, NNR-based, IT, and RAND strategies. The mean waiting time of the class-1 requests and class-2 requests in case of the NNR-based strategy is almost the same as that obtained by the RST-based strategy, and is significantly lesser than that value of the IT and RAND strategies.

Based on the above-mentioned results, it may be inferred that the RST-based and NNR-based strategies are the suitable routing strategies in the waiting system of the prioritized limited multi-server PS system.

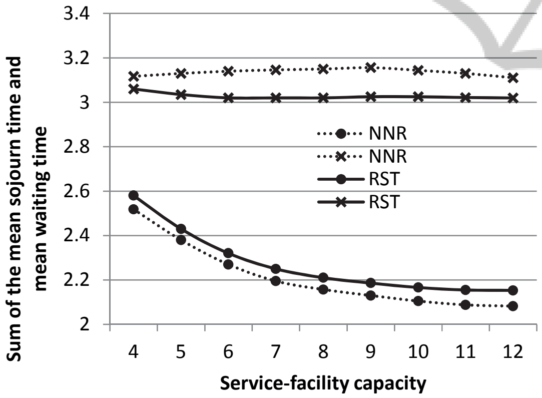

Figure 11: The sum of the mean sojourn and mean waiting time versus service-facility capacity (A=0.8, S=1).

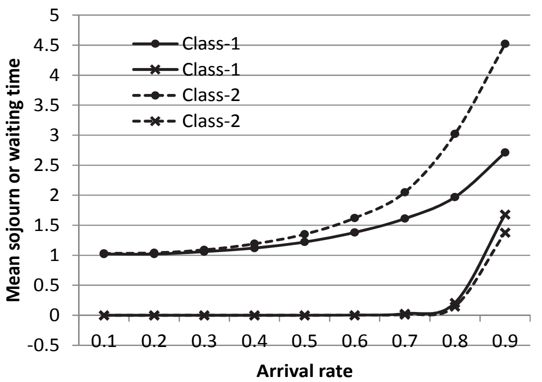

Figure 12: Mean sojourn or waiting time versus arrival rate (S=1, SFC=8).

Figure 11 compares the sum of the mean sojourn time or waiting time in the service waiting queue of the class-1 request (shown as the round marker) and class-2 request (shown as the cross marker) in case of the two (NNR-based and RST-based) strategies which realize the higher performance than the other two (IT and RAND) strategies. The sum of the mean sojourn time or waiting time of the class-1 request in the NNR-based strategy is smaller than that value in the RST-based strategy. On the other hand, the sum of the mean sojourn time and waiting time of the class-2 request in the NNR-based strategy is larger than that value in the RST-based strategy.

Figure 12 shows the relationship between the mean sojourn time (shown as round markers) or mean waiting time in the service waiting queue (shown as cross markers), and the arrival rate in case of the NNR-based strategy. Here, SFC means the service facility capacity. As the arrival rate increases, the differences between the mean sojourn time and mean waiting time in case of the class-1 request and class-2 request increases.

5 CONCLUSION

Routing strategies of an arriving request in the prioritized limited multi-server PS system are proposed, and the practical performance measures, such as the loss probability, mean sojourn time, and mean waiting time in the service-waiting queue are evaluated via simulation for each strategy. In the simulation program, by tracking the extension (or shortening) of the remaining sojourn time of each request that is receiving service at the arrival (or departure) of a request, the performance measures, e.g., the loss probability, mean waiting time in the service waiting queue, and mean sojourn time in these strategies, can be evaluated.

Based on the evaluation results, in the loss system of the limited multi-server PS system, it may be inferred that the RST-based strategy realizes the best performance. On the other hand, in the waiting system, the NNR-based strategy realizes the best performance along with the simple routing control mechanism. In the prioritized limited multi-server PS system, NNR-based or RST-based strategy may be said to be the most suitable routing strategy in the case of both loss and waiting systems. It also may be inferred that the RST–based strategy is best for reducing the sojourn time, and NNR-based strategy is best to reduce the loss probability.

In the future, we intend to study the influence of the service time distribution on the routing strategy of a prioritized limited multi-server PS System, and routing strategies in a prioritized limited multi-server RR system.

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