Integrating Computers, Science, and Mathematics
A Course for Future Mathematics Teachers

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Abstract: A course for prospective secondary mathematics teachers was developed at the University of Delaware, based on professional recommendations to integrate science, technology, engineering, and mathematics in the preparation of teachers of mathematics. Students used GeoGebra, Cabri3D, and Mathematica to model phenomena in the physical, natural and social sciences. They used motion sensors and graphing calculators to study motion. They wrote Python programs to simulate random phenomena. They built a robot and controlled it with a computer program, and made explicit the mathematical and scientific concepts involved in the functioning of the robot. Several forms of formative and summative assessment were conducted during the course. Teachers learned alternative ways of looking at mathematical concepts, and established connections in mathematics and with other areas.

1 INTRODUCTION

Technological environments offer opportunities to students of different skills and levels of understanding to engage with mathematical tasks and activities (Hollebrands, 2007). However, the integration of technology in mathematics teaching has been slower than anticipated due to multiple difficulties teachers face (Hohenwarter, Hohenwarter, & Lavicza, 2008). Having the technology available in schools is not enough to guarantee that teachers will know how to use it to teach mathematics. Future and in-service teachers need to have opportunities to develop the expertise and know-how to be able to incorporate the use of technology in their own teaching (Lawless & Pellegrino, 2007).

The new course *Learning mathematics with technology* offered at the University of Delaware is geared for first-year prospective secondary mathematics teachers. The purpose of the course is to provide future mathematics teachers, early in their teacher preparation program, with the knowledge and experience of technology-based activities that foster mathematical communication, connections, reasoning, and reflection that help students develop better understanding in mathematics. The course integrates modern interactive technologies to emphasize the learning of concepts, problem solving, exploration in mathematics, mathematical modeling, and connections to physics and other sciences. The course is based on current research and theories about how students best learn mathematics (Boaler, 2008), incorporates professional recommendations on the preparation of teachers of mathematics and the type of mathematics they need to experience and learn themselves (Conference Board of the Mathematical Sciences, 2012; National Council of Teachers of Mathematics, 2000; Common Core State Standards, 2010, National Research Council, 2002, 2011) and incorporates best teaching practices. The use of writing to learn mathematics is an important component of the course.

2 CONTENT KNOWLEDGE FOR MATHEMATICS TEACHERS

This course emphasizes pedagogical content knowledge (Shulman, 1986, 1987) for mathematics teachers. The students learn new technology, connections of mathematics with science, and a richer set of connections among high school mathematical concepts. The course implements

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strategies and methodologies to learn and teach mathematics with technology that could be used by future teachers in their own classrooms. Students actively participate in mathematical modeling of physical, biological, and social phenomena (Gordon & Gordon, 2010) using a variety of technology tools, such as TI CBR 2 motion sensors and GeoGebra (International GeoGebra Institute, 2013). They also use hands-on materials and the computer to further develop their own understanding of mathematics concepts, such as conics. Throughout the course, students use Python (Enthought, 2013) and other programming platforms to write their own short computer programs that involve the use of loops (FOR or WHILE) and the use of logical structures such as IF…ELSE.

How to work in cooperative groups was explicitly discussed several times during the semester. One students wrote in his end of class reflection: "At the beginning of class, when we talked about the concept of working together in a group, it helped me better understand how to truly work together and feed off of each other's ideas. Communication as they said, is the most important thing about it and that helped me truly understand the concept."

2.1 Motion Sensors, Velocity, and Acceleration

Students conducted several experiments with moving objects. They captured data for position vs. time with motion detectors and analyzed the corresponding graphs. The very first day they used constant-velocity vehicles (Printz, 2006). They followed with the 7 m jump of a small stuffed toy donkey hanging from a parachute. Students collected distance vs. time data and described how the different phases of the fall were reflected on the graph (before the parachute was completely deployed and after), and how the corresponding parts of the graph represented constant positive acceleration or constant velocity (initial accelerated motion followed by falling at the terminal velocity). One student wrote about the parachute activity on her end of class reflection: "The effect air resistance has on acceleration and velocity of a falling object was a concept today's lesson clarified for me."

Students also captured distance vs. time data for a bouncing basketball and analyzed the corresponding graph (Cory, 2010), to determine when the velocity was positive or negative, when was the velocity increasing or decreasing, when was it zero, etc. Especially challenging for them was to interpret what was happening to the velocity when the ball hit the ground.

2.2 Vanishing Points in a Rowing Competition

Students watched a video of a rowing competition (Allain, 2013) and then used GeoGebra and their knowledge of perspective and vanishing points (Figure 1 and Figure 2) to devise a way to determine who was winning the competition. Then they wrote a letter to a fictional television producer explaining the advantages of their method to make it easier for television viewers to see wheter a scull was moving faster than another.

Figure 1: Vanishing point at $t = 34$.

Figure 2: Vanishing point at $t = 38$.

2.3 Fitting Curves to Sets of Points

Students learned ways in which GeoGebra provides an alternative point of entry to the topic of fitting linear and quadratic curves to two or three points. First they played a game in pairs. One player would position two or three points on the screen, and the other player would use sliders corresponding to the coefficients of linear or quadratic functions and adjust the value of the slider to hit as many points as possible. Students expressed that using sliders allowed them to understand better the role of each of the coefficients, and the advantages and disadvantages of using different representations for quadratic functions, such as using the coordinates (h, k) of the vertex in the equation $y = a(x - h)^2 + k$ or using the general equation $y = ax^2 + bx + c$. In another activity students used sliders to experimentally fit a parabola to a set of data representing annual growth of redwood trees vs. rainfall.

Later, students used sliders also to fit other types of curves such as exponential functions. Sliders were used in addition to analytical ways to determine the parameters of the exponential function. One activity was to fit an exponential model to the growth of the population in the United States during its first century.

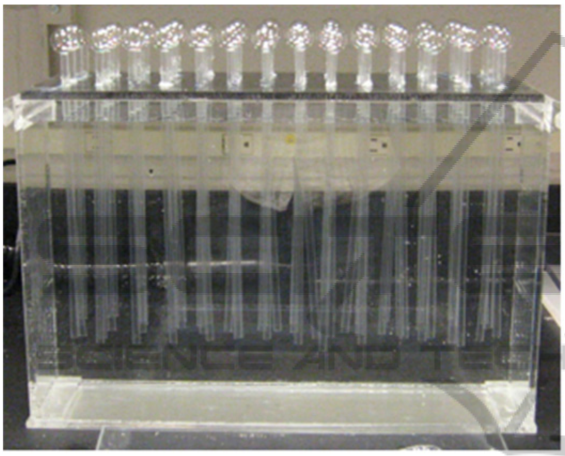

Figure 3: The Bubble Board.

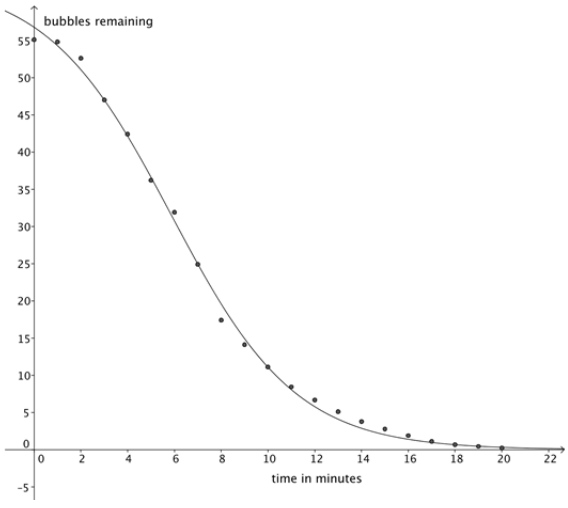

Figure 4: Fitting a logistic curve to the bubble data.

In another activity students generated 56 soap bubbles simultaneously with a bubble board (see Figure 3) (Hammons, Flores, Pelesko, Biehl, 2012) and recorded how many bubbles remained after intervals of one minute. Then they were given the data of the averages of nine such experiments and were asked to use GeoGebra to fit different mathematical models to the data (linear, quadratic, polynomial, exponential, and logistic) and discuss the advantages and disadvantages of each of the models. Students found that a logistic model

$$y = \frac{63.1}{1 + 0.11e^{0.37x}} \tag{1}$$

fit the data pretty well in the case of the bubbles (Figure 4).

2.4 Conics

In the United States often students' first encounter parabolas and other conics through equations such as $y = x^2$. Although textbooks mention that such curves can be obtained by slicing a cone, students usually do not have the opportunity to connect the different characterizations of conics. For example, for the ellipse, the curve can be described by the equation; using the locus definition (the sum of the distances to two fixed points is constant); by slicing a cone with a plane such that the angle of cut is bigger than the angle at the vertex of the cone; or in terms of the ratio of the distance from a point to a line and to a fixed point (focus-directrix). In this course students had the opportunity to establish connections among the different ways of characterizing conics using hands-on models, GeoGebra, and Cabri3D. One student expressed in her exit ticket: "Cabri3D helped me understand conic sections better. For the parabola, it helped me see that the distance to the directrix and focus is equal."

2.5 Python Programs to Simulate Random Phenomena

The instructor conducted several short sessions where a situation was presented, and then the class as a whole, using plain English, described the steps that needed to be included in a computer program to simulate the situation. For example, one die is tossed 1000 times. The program should keep track of how many Heads and Tails have occurred. After each toss the program determines who is in the lead and keeps track how many times each player has been in the lead. At the end the total number of outcome for heads and tails are displayed, and also how many times each player was in the lead, and how many times they were tied. The instructor then shared a program that would simulate the tossing of the die and keeping track of the outcomes in the desired way.

Students also did some thought experiments about situations involving random events and then compared their results with those generated by a computer. For instance, students were asked to conduct a thought experiment of tossing a fair coin 100 times and fill a 10 by 10 table with the outcomes. Students thought simulations resulted in very fragmented tables, where tails and heads alternate frequently. Students were then given tables generated with *Mathematica* (Figure 5) to compare the distributions of the number of squares in the largest "island of randomness" (Flores, 2006) in their thought experiments and those generated by the computer.

H	H	T	H	T	T	T	H	H	H
T	H	T	T	T	T	H	H	H	H
H	H	T	T	T	T	H	T	T	T
T	T	T	T	T	H	H	H	T	H
H	H	T	H	T	T	T	T	T	T
H	H	T	T	T	T	T	T	T	T
H	T	H	H	H	T	T	T	H	H
T	H	T	H	H	H	T	H	T	T
H	T	H	H	H	T	H	T	T	T
T	T	T	H	T	H	T	H	T	T

Figure 5: Islands of randomness.

As one of their projects, students were asked to work individually or in pairs to write their own program to simulate a situation involving randomness. Two students wrote a program to simulate random sampling with replacement of marbles of three different colors (Figure 6).

```
import random

def MARBLE():
    T = 0
    L1 = []
    while T < 10:
        T = T + 1
        M = random.randrange(1,13)
        if M > 0 and M < 5:
            print "WHITE"
        else:
            if M > 4 and M < 9:
                print "RED"
            else:
                if M > 8 and M < 13:
                    print "BLUE"
        L1.append(M)
    print sorted(L1)
```

Figure 6: Random sampling with replacement.

2.6 A Robot and a Feedback Loop

The final project for the course is building a robot controlled by a program that includes a feedback loop. The instructor demonstrated in class a bumper car. The bumper car moves forward until it bumps into a wall. It moves back for one second, and turns for 1.5 seconds, and then repeats the cycle. Thus, inside the loop we want the bumper to do either of two things. Change direction and rotate if it bumps against an object, else keep moving forward if not.

Programs in Lego Mindstorms are constructed by dragging icons, rather than typing code. For example, the switch icon will open the IF… THEN… ELSE part of the program. If the touch sensor is pressed, the motor will execute the two command on the upper part (move in reverse, and turn). Else, the motor will continue moving forward (Figure 7).

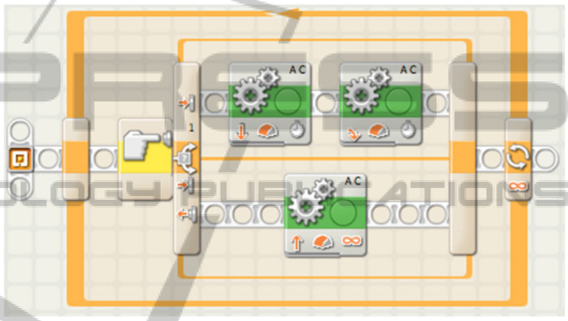

Figure 7: A program with a feedback loop.

One team of students chose to build a Robogator (Figure 8). Students described the structure of the program to control the Robogator in plain English:

IF an object is detected within 60 cm of the ultrasonic sensor

THEN the legs will move the robot forward infinitely

ELSE the robot does not move

IF the object is within 30 cm

THEN the jaws will open and close infinitely

Figure 8: Robogator.

Students also made explicit the scientific and mathematical concepts involved with the robot functioning: Velocity, speed, normal force, momentum, friction, centripetal acceleration, torque, feedback loops in nature depending on sensors. They unpacked each one of these concepts further, for example,

"Torque: The Robogator is applying torque in order to move because its legs apply a force when rotating around an axis, which would be the leg joint to the motor. All of these legs are using the same amount of torque to move at a constant speed."

3 COURSE ASSESSMENT

Several forms of assessment were used throughout the course, both formative and summative, ranging from short answers in "exit tickets", informal verbal interviews, to a pre- and post-survey about conics, and a survey of the class as a whole. Students' learning was assessed through a midterm exam, written assignments, and their reports and in-class presentations of the final projects on Python programs and Lego robots.

For the most part, students found the use of technology for modeling exciting and illuminating. After the first session a student wrote on her reflection:

"It was surprising to begin experiments on the first day of class; it set a captivating tone for the rest of the semester. I was intrigued by the different approaches each pair of students took. I look forward to this class in order to improve my analysis skills. Hands-on activities are an extremely helpful tool in learning, and I am ready to have this class for my future teaching career."

At the end of the course, students were given a card where they wrote on one side what to keep in the course, and on the other what needed to be changed or deleted. Seven out the ten participants explicitly recommended to keep the use of GeoGebra.

Students participated actively in cooperative groups as they tackled complex tasks. Many had not worked in groups before in mathematics, and found the experience of learning from each other valuable. One student stated that the group work and collaboration with others "was good for learning how others think and helped me think about math."

Students found the course directly relevant for their future as secondary mathematics teachers. In addition, three peer tutors, more advanced mathematics education students, who participated in the course, also found this experience much more relevant than the required computer science course they took. Two of the peer tutors conducted an independent study related to the course. H. Kretz gave a survey about conics at the beginning of the course and then again after the topic had been covered. She found that in the beginning students described conics using only formulas and equations, with no mention of actually cutting a cone, or other geometric properties. For example, one of the students said "Each conic section is dependent on the equation." When asked how they would teach the topic of conics, most students described also approaches based on symbols, and only one student mentioned the use of computers as a tool for teaching conics. One student wrote "I would draw a conic on the white board and I would show a parabola on a calculator with $y = x^2$ and the textbook to try looking at different equations for conics". At the end of the course, their descriptions of the conics were more elaborated and they incorporated additional geometric elements and more connections. Likewise, at the end, the vast majority of the students mentioned the use of computers for teaching conics. A. Restrepo focused on students' attitudes and dispositions towards the group work, use of computers, and hands-on activities. After the midterm she found that 90% of the students thought that the hands-on activities helped them understand and recall better.

4 CONCLUSIONS

Our course was quite successful in introducing prospective teachers to alternative ways of looking at mathematical concepts, emphasize connections within mathematics, and with other areas. Van Voorst (1999) found in a course for in-service teachers that the use of interactive technology helped mathematics teachers see mathematics "less passively, as a set of procedures, and more actively as reasoning, exploring, solving problems, generating new information, and asking new questions" (p. 2). We found similar changes with the future teachers. Furthermore, our course illustrates a point emphasized in professional recommendations on the preparation of prospective teachers, that teachers should have the opportunity to experience content courses taught in the same way they are expected to teach in High School, courses that provide multiple points of entry to mathematical concepts through the use of computers, hands-on materials, and other tools. Students had also the opportunity to experience the advantages of working in cooperative groups and start developing their skills for making the group work more productive. The course will be taught again in the next academic cycle.

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