Optimal Control for Forest Management in the Czech Republic

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Keywords: Optimal Control, Forest Structure, Forestation, Biodiversity.

Abstract: This contribution presents initial qualitative results and discussions when addressing the particular dynamic optimization problems in Czech forestry management. First, we analyze the deterministic infinite time horizon optimal control model aimed to determine the optimal paths for plantations of various mixed forests in the Morava region in the Czech Republic. Second, the problem of optimal dynamic path for the subsidy rates is established and its solution via optimal control using the simulated data is suggested. The at foremost aim of the presentation is to present the research topic itself and to discuss the optimization and solution techniques suggested.

1 INTRODUCTION

For 200 years the artificially planted spruce forests have covered the majority of forest land in the Czech Republic. For several last decades the problems with exhausted soil and lack of biodiversity in forests have escalated to intensive need for modifying the established forest structure. The state forestry authorities have introduced a subsidy policy to support the reforestation by original mixed forests and presented the target optimal forest structure in Czech forests. Although the state intentions and the economical support are clear, the particular optimal long run strategies for each forest area are unknown. There are number of possible particular combination of mixed forests to involve, various rotation scenarios to choose from and number of different forestation strategies to adopt.

The decision process appears to consist from two stages: first, the optimal rotation for each particular forest type is identified and then the particular areas of forest land are assigned to each forest type according to selected criterion.

In this contribution we suppose the optimal tree rotation for each forest type is known as well as the production and profit functions for particular forest types and the subject of our research is to address the second stage of the decision process. Concerning the appropriate methods of operations research that would provide the support for this decision making we can distinguish the static and dynamic approach.

The first one is represented by the linear programming (LP) that is a common broadly used technique in the forestry problems (see e.g. review (D'Amours et al., 2008)). Using LP we obtain the optimal forest structure given the particular area and decision criterion. However, there is no information on how to arrive from current situation to the desired (optimal) one. For this reason we consider the dynamic optimization techniques to be more appropriate when identifying the optimal long run strategy for the forest owners in the Czech Republic.

In our research we propose to employ the optimal control (OC) for finding the optimal time path for the state variables - the areas forested by the particular forest types. We expect that for the infinite horizon problem with (current value) profit maximization criterion we obtain a steady state solution representing the optimal target forest structure and the optimal state and control paths determining the appropriate strategies of forestation. Optimal control techniques have been steadily employed in the forestry management research problems during the several last decades for analyzing the potential optimal strategies (for recent contributions mostly related to our problem see (Caparros et al., 2013), (Caparros and Jacquemont, 2003) (Bach, 1999) or (Cerda and Martin Barroso, 2013)). However, due to the complex structure of the dynamic decision problems in forestry some theoretically oriented papers have appeared addressing the particular specifics of forestry decision problems (Sahashi, 2002).

Janová J. and Kadlec J..
Optimal Control for Forest Management in the Czech Republic.
DOI: 10.5220/0004924504660470
In *Proceedings of the 3rd International Conference on Operations Research and Enterprise Systems* (ICORES-2014), pages 466-470
ISBN: 978-989-758-017-8

Apart from the problem discussed above, we intend to solve another dynamic decision problem: to find the optimal subsidy strategy for reaching the desired forest structure in the Czech Republic within the least time. While the first problem concerns the forest owner decision making under given (constant) subsidies, the latter problem searches for the dynamic paths of the values of particular subsidies to optimize the forestation process in the Czech Republic as whole.

After a brief description of the situation and related data in Sec. 2, we introduce the two above mentioned decision problems: the forest owner forestation strategy in Sec. 3 and subsidy strategy in Sec. 4. We present the concept of the optimization models and discuss the possibilities to identify the state equations. We suggest the form of the functions involved and for the theoretical models suggested we provide a qualitative solution and interpretations.

2 SITUATION

The current composition of forests in the Czech Republic together with natural and recommended distribution is summarized in Table1.

In our research, we focus on the Morava region in the east part of the Czech Republic. According to the factors decisive for the forest type choice we will define the homogeneous subregions as highlands and lowlands and assign them the suitable forest types (see Table 2). Subregions were defined in respect to state policy of tree species change from spruce to broadleaves and mixed forest types. Spruce is out of optimal growing conditions in lowlands and highlands where is negatively influenced by fungi, pest and abiotic factors. Suitable forests types were defined with respect to production and tree optimal growing conditions. Mountains subregion is not defined because spruce has optimal conditions there and other tree species are out of production optimum.

3 FORESTATION STRATEGY

The problem is to identify the optimal forestation strategy from the point of view of the forest owners in the Morava region given the current forest structure and the subsidies: 12000 CZK per ha of natural regeneration of desired tree species plus 20000 CZK per ha in the fifth year after successful reforestation. In our model, the criterion of decision making is the economic profit from forests (including the income from selling the timber and obtaining the subsidies for the forest structure changes)[1].

To address the decision problem we introduce infinite horizon OC problem with free terminal points to be solved for each particular subregion.

$$\max \quad V = \int_0^\infty \Pi(t) e^{-\rho t} \tag{1}$$

$$\Pi(t) = \sum_{i=1}^{n} R_i(x_i(t), u_i(t)) - \tag{2}$$

$$-K_i(x_i(t), u_i(t)) \tag{3}$$

$$\text{s.t.} \tag{4}$$

$$u_i(t) = \dot{x}_i(t), \tag{5}$$

$$\sum_{i=1}^{n} x_i(t) = L, \tag{6}$$

$$x_i(t) \geq 0, \tag{7}$$

$$u_l(t) \geq 0,\ 1 \leq l \leq s < n \tag{8}$$

$$x_i(0) \text{ given},\ 1 \leq i \leq n. \tag{9}$$

Here, n denotes the number of forest types appropriate for the given subregion, $x_i(t)$ are the state variables representing the area of land forested by type i in time t. Setting

$$x_n(t) = L - \sum_{i=1}^{n-1} x_i(t), \tag{10}$$

where L denotes the total area of the subregion, we can exclude the constraint (6) from the model. In (8) s is the number of supported forest types and the constraint reflects the fact that once the piece of land is reforested by the new (supported) forest type, it stays in the new status.

Further notations: ρ = discount rate, control variables: $u_i(t)$ = total area reforested at time t of forest type i (in ha per year), s is the number of supported forest types, $\Pi(t)$ = the current profit from the forests in the subregion, $R_i(t)$ = total revenues and $K_i(t)$ = total costs from growing and logging at time t. We assume the revenue function can be split into two terms:

$$R_i(t) = G_i(x_i(t)) + \sigma_i u_i(t),$$

where G_i represents a known function of revenues from logging in the particular forest type while $\sigma_i u_i(t)$ calculates the subsidy from increasing the area forested by type i, where

$$\sigma_i = \begin{cases} \sigma_i > 0 & \text{for supported forest types} \\ 0 & \text{for the others} \end{cases}$$

[1]Note that apart from economic criterion we should mention other benefits from forest planting, that could enter the model. In our research we start with the simple economic (financial) decision criterion, that mostly reflect the objective of running the forest-business in the Czech Republic. Once the basic model is established, it could appear useful to incorporate further criteria and/or constraints.

Table 1: Perceptual composition of tree species in Czech forests.

Composition	Natural	Current	Recommended
Spruce	11.2	51.4	36.5
Fir	19,8	1,0	4,4
Pine	3,4	16,7	16,8
Larch	0,0	3,9	4,5
Other conifers	0,3	0,3	2,2
total conifers	34,7	73,2	64,4
Oak	19,4	7,0	9,0
Beech	40,2	7,7	18,0
Hornbeam	1,6	1,3	0,9
Ash	0,6	1,4	0,7
Maple	0,7	1,3	1,5
Elm	0,3	0,0	0,3
Birch	0,8	2,7	0,8
Linden	0,8	1,1	3,2
Alder	0,6	1,6	0,6
Other broadleaves	0,3	1,6	0,6
Total broadleaves	65,3	25,6	35,6
Unstock area	0,0	1,2	0,0

Table 2: Suitable forests in subregions: S= Spruce, SB= Spruce with beech,SF= Spruce with fir, SDG= Spruce with douglas fir, B= Beech, BL= Beech with larch, BDG= Beech with douglas fir, BO=Beech with oak, OB=Oak with beech, BH=Beech with hornbeam, BF=Beech with fir, Br= Birch, P= Pine.

Subregion	Forest type current	Forest type suitable
The higlands	S	SB, SF, SDG, B, BL, BO, Br
The lowlands	S	OB, BH, BL, BDG, BF, P

For the purpose of further estimation of the functions $G_i(x_i), K_i(x_i, u_i)$, we adapt the assumptions made in (Caparros et al., 2013) and set

$$G_i = g_{i0} + g_{i1}x_i + \frac{1}{2}g_{i2}x_i^2, \quad (11)$$

$$K_i = k_i + a_i x_i + b_{i1}u_i + \frac{1}{2}b_{i2}u_i^2. \quad (12)$$

where the total cost function in (12) is composed from the farming costs and reforestation costs. Note that we assume $b_{i1} = b_{i2} = 0$ for the spruce type forest - supposed to be reduced - i.e. decreasing the area of the forest type (after logging the current stand) is costless.

3.1 The Qualitative Solution

We analyze the small scale problem considering just three forest types: $x_1, x_2 =$ areas forested by desired (finantially supported) types and $x_3 =$ the area forested by spruce (supposed to be decreased). According to (10) we apply the substitution $x_3 = L - x_1 - x_2$ and the OC problem is of the form

$$\max \quad V = \int_0^\infty \Pi(x_1, x_2, u_1, u_2)e^{-\rho t} \quad (13)$$

$$\dot{x_1}(t) = u_1(t), \quad (14)$$

$$\dot{x_2}(t) = u_2(t), \quad (15)$$

$$u_1(t) \geq 0, \quad (16)$$

$$u_2(t) \geq 0, \quad (17)$$

$$x_1(0) \geq 0, x_2(0) \geq 0 \text{ given.} \quad (18)$$

The non-negativity constraints (16-17) reflect the fact, that the area forested by desired types may only increase, i.e. once converting after clearcut the forest area to a desired type it remains in the new status. This requirement stems from the aim of the optimization - to permanently change the forest structure and is also supported by the subsidy policy. These constraints together with (18) ensures also the nonnegativity of state variables $x_1(t), x_2(t)$.

Having

$$\Pi = R_1 - K_1 + R_2 - K_2 + R_3 - K_3$$

where

$$
\begin{aligned}
R_1 &= g_{10} + g_{11}x_1 + \frac{1}{2}g_{12}x_1^2 + \sigma_1 u_1, && (19)\\
K_1 &= k_1 + a_1 x_1 + b_{11}u_1 + \frac{1}{2}b_{12}u_1^2, && (20)\\
R_2 &= g_{20} + g_{21}x_2 + \frac{1}{2}g_{22}x_2^2 + \sigma_2 u_2, && (21)\\
K_2 &= k_2 + a_2 x_2 + b_{21}u_2 + \frac{1}{2}b_{22}u_2^2, && (22)\\
R_3 &= g_{30} + g_{31}(L - x_1 - x_2) + && (23)\\
&\quad + \frac{1}{2}g_{32}(L - x_1 - x_2)^2, && (24)\\
K_3 &= k_3 + a_3(L - x_1 - x_2). && (25)
\end{aligned}
$$

Using current value Hamiltonian

$$
\begin{aligned}
H_c &= \Pi + m_1 u_1 + m_2 u_2, && (26)\\
& m_i = \lambda_i e^{\rho t}, i = 1,2 && (27)
\end{aligned}
$$

the maximum principle conditions are:

$$
\begin{aligned}
\frac{\partial H_c}{\partial u_i} &\leq 0 && (28)\\
u_i \frac{\partial H_c}{\partial u_i} &= 0 && (29)\\
\dot{x}_i &= u_i && (30)\\
\dot{m}_i &= \rho m_i - \frac{\partial H_c}{\partial x_i}, i = 1,2. && (31)
\end{aligned}
$$

The condition (29) is satisfied either with $u_i = 0$ or $\frac{\partial H_c}{\partial u_i} = 0$. The first one represents the situation

$$\frac{\partial H_c}{\partial u_i} = \sigma_i - b_{i1} + m_i \leq 0$$

which means that the profit from increasing the area of type i is smaller than the profit from preserving the additional area unit and using it otherwise. In this situation the area forested by type i remains constant, i.e. $\dot{x}_i = u_i = 0$.

The latter solution branch

$$\frac{\partial H_c}{\partial u_i} = \sigma_i - b_{i1} - b_{i2}u_i + m_i = 0 \qquad (32)$$

is the case of increasing the area forested by type i by u_i ha per year.

To obtain the control and state paths the particular forms of the functions involved are needed. We intend to obtain these by regression analysis using the real world data.

4 SUBSIDY STRATEGY

In the previous section the subsidy rates were given constants. Now we assume these rates can be changed during time and we state the question how to control the subsidies rates to obtain the desired forests structure in the least time possible.

Using optimal control formulation we obtain:

$$
\begin{aligned}
\min \quad & W = \int_0^\infty 1 dt && (33)\\
\dot{x}_i(t) &= f_i(u_1(t), \ldots, u_n(t), x_1(t), \ldots, x_n(t)) && (34)\\
u_i(t) &\geq 0, && (35)\\
x_i(0) &\geq 0, \quad \text{given} && (36)
\end{aligned}
$$

where the control variable $u_i(t)$ is the subsidy rate for the forest type i, while the state variable $x_i(t)$ is as in the previous problem the forest area of type i.

The state equations (34) reflect the influence of the subsidy rates and present composition of the forest onto the area change for particular forest type. Reflecting the particular decision making of the forest owner, these equations are uneasy to be obtained. Since there is a lack of real data on how the decision maker react to change of the subsidies, we can not simply use regression analysis. Here, we suggest that interesting theoretical information could be obtained using the data from simulation of the decision maker's behavior. These simulation are supposed to be based on the linear programming approach that appeared to be a valid tool to support the profit criterion optimization problems in forestry (e.g. (D'Amours et al., 2008)). Particularly, we formulate a linear programming problem of profit maximization given the present values of future streams of subsidies s_i, revenues r_i and costs c_i per 1 ha of forest type i:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{n} x_i \cdot (r_i + s_i - c_i) && (37)\\
\text{s.t.} \quad & && (38)\\
& \sum_{i=1}^{n} x_i = L, && (39)\\
& \sum_{i=1}^{k} a_{ij} x_i \leq b_j && (40)\\
& \sum_{i=1}^{k} d_{ij} x_i \geq e_j, 1 \leq k \leq n && (41)\\
& x_i(t) \geq 0, 1 \leq i \leq n. && (42)
\end{aligned}
$$

The decision variables are the areas forested by particular types and constraints (40-41) reflect existing area limits for the particular forest types and their combinations.

Solving the linear programming model (37-42) repeatedly for different subsidies rates s_i, we generate the simulated data set which can be used to estimate the relation among rate of subsidies and the change of the forest structure, and in this way, to help establish the state equation (34).

5 CONCLUSIONS

Although the operations research techniques offer promising decisions support help within the forestry industry, its real applications by forest managers and authorities is infrequent in the Czech Republic. Partially this is due to the natural unwillingness of the decision makers to deal with mathematical models or even their results which is supported by often failures of theoretical models' when addressing the real world problems.

In our research we focused on identification of particular decision problems in forestry that are of importance for Czech forestry industry and that are complex enough that the decision support is desirable. We suppose that our results will not provide the ready-made solution, but may be of help when making the final decision. In this contribution we focused on presenting the selected problems and the first approaches to deal with them.

ACKNOWLEDGEMENTS

The research is supported by the Czech Science Foundation grant nr. 13-25897S.

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