Price-demand Modeling
A Tool to Support Inventory and Production Decisions for Competing Products

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Abstract: Aim of this positioning paper is to explore the existing price-demand models that have been applied in inventory and production management so far and to identify new potential structures that may have been applied in marketing research but have not been touched yet in inventory and production research. Our focus will be on dynamic pricing structures for competing products, since our exploration so far revealed that they have not been studied exhaustively in price-demand inventory literature. Specifically, we propose a price-demand-inventory model framework for optimizing joint pricing, production, inventory and transportation decisions in a supply chain with multiple products, multiple factories, and multiple markets operating in multiple periods. The factories operate in a cooperative environment where these decisions are given centrally so as to maximize the total profit. Competition between the various product types is achieved centrally by setting market specific prices for each product type in each market in each period. To support this price setting we introduce several promising price-demand structures for competing products.

1 INTRODUCTION AND LITERATURE SURVEY

In the realm of dynamic pricing, studies that consider the effect of pricing on the demand of a product have been made for decades, in parallel with the research on inventory and production management. Marketing researchers identify the four determinants of the demand by price, product, place, and promotion, referred to as the classical four Ps of marketing. Among these, price has always been considered as the most significant factor in affecting the demand of a product. Extensive research has been done into formulating the relationship between the demand and the marketing mix variables (Lilien et al., 1992).

In the inventory and production management literature, models have been developed with exogenous demand assumptions where the demands are either planned orders or estimated by forecasting models. Ample attention has been given in literature to stochastic models where the demands are considered to be random variables with known probability distributions. However, most of these stochastic models tend to ignore the underlying determinants of demand.

The past decades, research into models for joint decision making for price and inventory management has received increasing attention. The earliest study that incorporates price as a decision variable into inventory theory is by Whitin (1955) where the total profit is maximized in an EOQ setting. After the 1990’s the joint research in these areas has gained a considerable acceleration due to the increasing pressure to attain flexibility and responsiveness in the supply chains operating under fierce global competition. This motivation grew by the advances obtained in information and communication technology that provided facilitators for the applicability of dynamic pricing strategies, namely: (i) increased availability of demand data that led to better customer segmentation; (ii) introduction of faster and cheaper labelling in retail stores and e-procurement environments; and (iii) development of better decision support tools with higher speed and accuracy. Thus, more and more analytical models were needed for developing optimal management strategies (Elmaghraby and Keskinocak, 2003); (Chen and Simchi-Levi, 2012).

The broad research area of price-demand and inventory management models is comprehensively reviewed with different perspectives in (Chen and

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Simchi-Levi, 2012); (Yano and Gilbert, 2005); (Elmaghraby and Keskinocak, 2003); (Petruzzi and Dada, 1999). In general, the basic choices in joint price-demand and inventory models are identified as the following set of structural attributes. Primarily, the structure of the chosen price-demand model plays a significant role in the setting of the problem and the optimal policy obtained. Other attributes are: (i) incorporating capacity restrictions on the quantity ordered; (ii) using cost structures like fixed ordering costs or price adjustment costs; (iii) incorporating pricing policies like static pricing where the price is fixed for a few periods once it is set; (iv) the structure of the inventory replenishment policy, namely the availability of multiple replenishments in the planning period or a single replenishment like in revenue management; (v) the number of planning periods being single or multiple; (vi) the number of products being single or multiple; and (vii) the structure of the supply chain being competitive, coordinated or cooperative. As for the latter, the perspective of the model is essential: does it support the buyer, the seller, or the system as a whole.

In this study our aim is to explore the existing price-demand models that have been applied in inventory and production management so far and to identify new potential structures that may have been applied in marketing research but have not been touched yet in inventory and production research. In particular, our focus will be on dynamic pricing structures for competing products, since our exploration so far revealed that they have not been studied exhaustively in price-demand inventory literature.

As summarized in (Chen and Simchi-Levi, 2012), the most common price-demand structure used in joint price-demand-inventory models is the linear demand $d(p) = b - ap,\ p \in [0, b/a]$, $a > 0$, $b \geq 0$ where demand $d(p)$ is a linearly decreasing function of price p. In spite of its computational simplicity, this structure is questioned in terms of its validity due to the linearity assumption and possibility of obtaining negative demands at high price levels. At this point, the exponential demand structure $d(p) = \exp\{b - ap\}$, $a > 0$, $b > 0$ is introduced as an applicable alternative. The third common structure is the iso-elastic demand $d(p) = ap^{-b}$, $a > 0$, $b > 1$ where the demand elasticity b is the same for all price levels. On the other hand, the logit demand structure $d(p) = N\exp\{-ap\}/(1 + \exp\{-ap\})$ is an alternative to all of the above structures by allowing a fixed potential demand N which is multiplied by the probability of buying a product at a price p. It should be noted that all these models can be extended to reflect the effect of complement or substitute products' prices on the demand of a certain product in a competitive environment.

In a stochastic setting, random demand $d(p, \varepsilon)$ is defined as a function of the price p and a random noise ε. Standard approaches include the additive model $d(p, \varepsilon) = d(p) + \varepsilon$, with $E(\varepsilon) = 0$, and the multiplicative model $d(p, \varepsilon) = d(p)\varepsilon$, with $\varepsilon \geq 0$ and $E(\varepsilon) = 1$. Yet, there are hybrid structures of the additive and multiplicative models (Chen and Simchi-Levi, 2012).

Moreover, we have the price-demand models with intertemporal effect, i.e., the models that incorporate the effect of prices in the previous periods on the current demand of an item. Ahn, et al. (2007) and Gümüş and Kaminsky (2010) provide models with substitute products and multiple periods where the total demand in a period is the sum of the linear demand function of price, the intertemporal effect, and the substitution effect.

In addition to the price-demand models mentioned above, marketing theory includes several other approaches. Among these, market share attraction (MSA) models are used to calculate the demand of a product in competitive environments moderated by $K > 1$ substitute products. Assuming that there is a fixed potential demand D, the demand of substitute product k, $d_k = Dm_k$, for $k = 1, 2, \ldots, K$ is obtained by multiplying D by the market share m_k of product k. Here, the market share m_k is defined as the ratio of the so-called attraction A_k of product k with the total attractions of all products, i.e., $m_k = \frac{A_k}{\sum_{j=1}^{K} A_j}$. Depending on how the attraction caused by the price of an item is formulated, MSA models can have different forms (Lilien et al., 1992).

The above MSA approach is a good example of a technique that has proven its value in marketing theory, but is still of limited importance when it comes to price-demand inventory and production models.

Aim of this positioning paper is to unfold a line of research to change this. Specifically, we propose a price-demand-inventory model framework for optimizing joint pricing, production, inventory and transportation decisions in a supply chain with multiple products, multiple factories, and multiple markets operating in multiple periods. The factories operate in a cooperative environment where these decisions are given centrally so as to maximize the total profit. Competition between the various product types is achieved centrally by setting market specific prices for each product type in each market in each period. To support this price setting we propose several price-demand structures for competing products, many of which are based on MSA.

The remainder of the paper is organized as follows. In Section 2 we recall from literature an illustrative modeling framework for price-demand-inventory models and discuss its merits as well as possible extensions. In Section 3 we propose our price-demand-inventory framework for competing products and present our research agenda. We conclude in Section 4 with a discussion of our approach.

In the next section, let us discuss an interesting modeling framework for price-demand-inventory models.

2 THE GENERALIZED STRUCTURE OF PRICE-DEMAND-INVENTORY MODELS

In their comprehensive review, Chen and Simchi-Levi (2012) provide a general modeling framework for a single product in a periodic review deterministic setting. Specifically, a firm is considered that makes pricing and replenishment decisions of a single product over a finite planning horizon with T periods. Letting the price vector $\boldsymbol{p} = (p_1, p_2, \dots, p_T)$ where p_t denotes the price in period t, $t = 1,2,\dots,T$ and the demand vector $\boldsymbol{d}(\boldsymbol{p}) = (d_1(p_1), d_2(p_2), \dots, d_T(p_T))$ where $d_t(p_t)$ is the demand in period t as a function of the selling price p_t in period t, two mathematical models are introduced that operate recursively.

Decision variables
x_t: Order quantity in period t
p_t: Price in period t
Auxiliary variables
$d_t(p_t)$: Demand as a function of the selling price p_t in period t
I_t: Inventory level in period t
z_t: Binary variable showing whether an order is placed in period t
Parameters
k_t: Fixed ordering cost in period t
c_t: Variable ordering cost in period t
h_t: Unit holding cost in period t
q_t: Upper bound on the order quantity in period t
$\underline{p_t}$, $\overline{p_t}$: Lower and upper bounds on the price in period t

For given optimal total costs for ordering, production, and inventory holding $C(\boldsymbol{d}(\boldsymbol{p}))$, with $\boldsymbol{d}(\boldsymbol{p}) = (d_1(p_1), d_2(p_2), \dots, d_T(p_T))$, Model 1 is used to optimize the prices in each period as shown below:

$$\text{Maximize Profit} = \sum_{t=1}^{T} p_t d_t(p_t) - C(\boldsymbol{d}(\boldsymbol{p}))$$

such that

$$p_t \in \left[\underline{p_t}, \overline{p_t}\right],\ t = 1,2,\dots,T.$$

Model 2 is used to find the optimal total costs for ordering, production, and inventory holding, $C(\boldsymbol{d}(\boldsymbol{p}))$ as shown below:

$$C(\boldsymbol{d}(\boldsymbol{p}))\text{=Minimize } \sum_{t=1}^{T} k_t z_t + c_t x_t + h_t I_t$$

such that

$$I_t = I_{t-1} + x_t - d_t, \quad t = 1,2,\dots,T \tag{1}$$

$$x_t \le q_t z_t, \quad t = 1,2,\dots,T \tag{2}$$

$$I_0 = 0,\ \ z_t \in \{0,1\},\ \ I_t, x_t \ge 0 \tag{3}$$

Although the above framework is inspiring, it lacks a number of features that are frequently encountered in practice:

- Capacity considerations to fulfill the demand and/or options to take care of backlog are missing. Instead, demand is always satisfied completely per each period and may not be met partially or postponed
- The model is single product so that competition between various product types is not accounted for
- The model is single market
- The model does not distinguish between producers or production sites
- Transportation is not included

Since −according to our findings− the above framework is representative for the state of the art in deterministic price-demand-inventory modelling we judge it worthwhile to endeavor a research journey in order to discover how the above shortcomings can be mended.

3 A PRICE – DEMAND – INVENTORY FRAMEWORK FOR COMPETING PRODUCTS

In this section we introduce a price-demand-inventory model framework for optimizing joint pricing, production, inventory and transportation decisions in a supply chain with multiple products, multiple factories, and multiple markets operating in multiple periods. The factories operate in a cooperative environment where these decisions are given centrally so as to maximize the total profit. Competition is between the various product types that in every period aim for generating demand in each of the multiple markets. Competition is achieved centrally by setting market specific prices for each product type in each market in each period. The formulation of the model is as follows:

Indices

i: index of factory (*i=1,2,…,I*)
j: index of market (*j=1,2,…,J*)
k: index of product type (*k=1,2,…,K*)
t: index of time period (*t=1,2,…,T*)

Parameters

u_{ikt}: Production costs per unit for product type k produced at factory i in period t

h_{ikt}: Inventory holding costs per unit for product type k stored at factory i in period t

c_{ijkt}: Transportation costs per unit for product type k sent from factory i to market j in period t

s_{jkt}: Stockout costs per unit for product type k to be sold in market j in period t

f_{it}: Production capacity of factory i in period t

Variables

X_{ijkt}: Amount of product type k sent from factory i to market j in period t

Y_{ikt}: Amount of product type k produced at factory i in period t

I_{ikt}: Inventory of product type k stored at factory i in period t

P_{jkt}: Price per unit of product type k in market j in period t

Auxiliary function

$D_{jkt}(P_{j1t}, P_{j2t}, \dots, P_{jKt})$: Quantity of product type k demanded by market j in period t given all prices $P_{j1t}, P_{j2t}, \dots, P_{jKt}$

Problem formulation:

$$\text{Maximize} \sum_{j,k,t} P_{jkt} \sum_i X_{ijkt}$$

$$-\left(\sum_{i,k,t} u_{ikt}\ Y_{ikt} + \sum_{i,k,t} h_{ikt}\ I_{ikt} + \sum_{i,j,k,t} c_{ijkt}\ X_{ijkt}\right)$$

$$-\sum_{j,k,t} s_{jkt}\left(D_{jkt}(P_{j1t}, P_{j2t}, \dots, P_{jKt}) - \sum_i X_{ijkt}\right)$$

such that

$$\sum_k Y_{ikt} \le f_{it}, \quad \forall\, i, t \tag{4}$$

$$\sum_i X_{ijkt} \le D_{jkt}(P_{j1t}, P_{j2t}, \dots, P_{jKt}), \forall\, j, k, t \tag{5}$$

$$I_{ik(t-1)} + Y_{ikt} - \sum_j X_{ijkt} = I_{ikt}, \forall i, k, t \tag{6}$$

$$X_{ijkt}, Y_{ikt}, I_{ikt}, P_{jkt} \ge 0, \ \forall\, i, j, k, t \tag{7}$$

Here, the objective maximizes profit of all factories together. Restriction (4) gives capacity constraints per factory per period. Restriction (5) specifies the demand per market per product type per period given all product type prices in the market and period under consideration. Restriction (6) gives the inventory balance constraints on product type level for all factories.

Crucial in the above model framework are the price-demand functions $D_{jkt}(P_{j1t}, P_{j2t}, \dots, P_{jKt})$ specifying the amount of product type k demanded by market j in period t given all product type prices in that same market and period.

Choices for D_{jkt} form the cornerstone for the research announced in this positioning paper. So, let us give them due attention. For brevity of writing we suppress the indices j and t. Thus, we write D_k instead of D_{jkt} and P_k instead of P_{jkt}. Contrary to the price-demand functions discussed in Section 1, we are interested in those functions that model *competition* between products.

A computationally simple choice that we will examine is:

(1) the linear competitive price-demand model :

$$D_k = a_k - b_{kk}\ P_k + \sum_{\substack{m \\ m<>k}} b_{km}\ P_m \tag{8}$$

where a_k and b_{km} are nonnegative constants. This choice results in a non-linear programming problem with linear constraints and a quadratic (first term of the) objective. This demand model may fit for a price sensitive market in which total demand of all product types together is fluctuating. However, in some markets in practice, total product demand is fixed. For instance in a health insurance market, where all citizens have to be insured, the total demand is fixed. Hence the demand D_k depends solely on the market share. For those markets we will examine the following choice:

(2) the market share attraction (MSA) competitive price-demand model with fixed total demand:
$D_k = Dm_k$ with D denoting total market demand and with the market share of product type k being given by $m_k = \frac{A_k}{\sum_{j=1}^{K} A_j}$ where the attraction is a function of all product type prices: $A_k = A_k(P_1, P_2, \dots, P_K)$. Of special interest is linear attraction given by:

$$A_k = 1 - c_{kk} P_k + \sum_{\substack{m \\ m<>k}} c_{km} P_m \tag{9}$$

where the c_{km} are nonnegative constants. This choice results in a non-linear programming problem which is computationally harder than the previous choice. Clearly, MSA is also an interesting approach when total market demand is not fixed. Therefore, we will also examine a third choice for D_k:

(3) the market share attraction (MSA) competitive price-demand model with fluctuating total demand:
$D_k = D_{AV} m_k$ with m_k as in *(2)* and with the total market demand D_{AV} depending on the average price:

$$D_{AV} = D(1 - \frac{\varepsilon}{K} \sum_{m} P_m) \tag{10}$$

where ε is a (small) positive constant and D denotes total priceless market demand. The idea behind this choice is that total market demand D_{AV} is affected unfavorably by the average price level.

4 OUR RESEARCH APPROACH

For each of the above competitive price-demand models we start by examining the computational tractability of the corresponding non-linear programming problem. We experiment with heuristic search methods such as multi-start local search. Gauging our approach on small problems we scale up to larger ones. Next, we interpret the results in order to discover simple approximate heuristic rules. These results may indicate in what market situations our models are valuable tools. We conclude our research by facing the challenge of parameter calibration.

Clearly, the above competitive price-demand models are also interesting in other settings. For instance, they can serve as a decision support tool in electronic reverse auctions (ERAs), where competing product supplier agents may place price bids which are evaluated by the various markets. In their bid evaluations, the market players may base their product demands on one of the above price-demand models. These bid evaluations may be fortified by using recently developed machine learning techniques (Den Boer, 2013). It is our intention to include settings like these in our research.

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