

# Research Proposal in Probabilistic Planning Search

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**Keywords:** Bayesian Networks, Planning, and Planning Search.

**Abstract:** In planning search, there are different approaches to guide the search, where all of them are focused in have a plan (solution) in less time. Most of the researches are not admissible heuristics, but they have good results in time. For example, using the heuristic-search planning approach plans can be generated in less time than other approaches, but the plans generated by all heuristic planners are sub-optimal, or could have dead ends (states from which the goals get unreachable). We present an approach to guide the search in a probabilistic way in order to do not have the problems of the not admissible approaches. We extended the Bayesian network and Bayesian inferences ideas to our work. Furthermore, we present our way to make Bayesian inferences in order to guide the search in a better way. The results of our experiments of our approach with different well-known benchmarks are presented. The benchmarks used in our experiments are: *Driverlog*, *Zenotravel*, *Satellite*, *Rovers*, and *Freecell*.

## 1 INTRODUCTION

In planning, there are four main approaches to increase the efficiency of planning systems.

First, Blum and Furst developed a novel algorithm called *Graphplan* (Blum and Furst, 1997). This algorithm reduces the branching factor by searching in a special data structure. Furthermore, this algorithm searches for layered plans (parallel plans). This algorithm has three limitations. First, Graphplan applies only to STRIPS language (Fikes and Nilsson, 1994). Second, this planner performs poorly without extra ad hoc reasoning capabilities. Third, the most important limitation of Graphplan is that the quality of the plan is not as good as the speed of the planning of this planner.

There are many planning systems that use the Graphplan algorithm or their own version of this algorithm as SGP (Weld et al., 1998), Blackbox (Kautz and Selman, 1999), IPP (Koehler, 1999), Medic (Ernst et al., 1997), STAN (Fox and Long, 2011), FF (Hoffmann and Nebel, 2011) and others. Furthermore, there is the LPG (Local Search for Planning Graphs) planner (Gerevini et al., 2003), which is the only one that does not use the Graphplan algorithm properly, but it still has a planning-graph approach. Therefore, LPG works with heuristics that exploit the structure of the planning graph.

Second, Kautz and Selman developed a novel method for planning called *planning as satisfiability*

(SAT) (Kautz et al., 1992), which transforms planning problem into a propositional satisfiability problem for which efficient solvers are known. The SAT-PLAN04 (Kautz, 2004) uses STRIPS language as well as PDDL language (Fox and Long, 2003), but SatPlan does not handle any non-STRIPS features other than types, such as derived effects and conditional actions.

Third, Bonet and Geffner developed a new approach based on *heuristic-search planning* (HSP) (Bonet and Geffner, 2001). In this approach, a heuristic is choosing among a set of different heuristics in order to guide the search through the state space. Unfortunately, the heuristics used in this algorithm are not fully admissible. Indeed, these heuristics do not work for all domains.

Hoffman and Nebel, using and improving HSP ideas, developed the FF (Fast-Forward) system (Hoffmann and Nebel, 2011), which is one of the fastest planners in STRIPS language. The FF heuristic implements a relaxed Graphplan algorithm to obtain the minimum distance between the state and the goal state, but this relaxed algorithm is not admissible because the relaxed Graphplan not consider the delete list (which is all the delete effects of all operators). The search algorithm, called Enforced Hill-Climbing (EHC), only does a local search, which can lead to dead ends (states from which the goals get unreachable).

Fourth, Bonet and Geffner used and improved

ideas of *planning with incomplete information* (Gensereth and Nourbakhsh, 1993) to develop another approach based on *planning with incomplete information as heuristic search in belief space* (Geffner, 2011).

*Planning with incomplete information* is distinguished from *classical planning* in the type and amount of information available at planning and execution time. In planning with incomplete information, the initial state is not known, but sensor information may be available at execution time. By contrast, in classical planning the initial state is known and there is no sensor to get knowledge or feedback of the states at execution time.

Planning with incomplete information can be formulated as a problem of search in *belief space*, where belief states can be either set of states or more generally probability distribution over states. Bonet and Geffner made the explicit formulation of planning with incomplete information, in order to extend it to tasks like contingent planning where the standard heuristic search algorithms do not apply. The limitations of this planner are two. The first limitation is that the search algorithm is not optimal like A\*. The second limitation is the complexity of a number of preprocessing and runtime operations in GPT (General Planning Tool) (Bonet and Geffner, 2011) scales with the size of the state space.

The heuristic-search planning approach has good results in execution time, but the plans generated by all heuristic planners are sub-optimal, or could have dead ends (states from which the goals get unreachable). Our approach is different from heuristic-search, we developed a probabilistic method to guide the search in an efficient way, in order to not have dead ends, and also have optimal plans.

The paper is organized as follows. First, we explain classical planning, Bayesian inference and the probabilistic planning search. Then we report results of the experiments and some conclusions are drawn.

## 2 CLASSICAL PLANNING FRAMEWORK

In *Classical Planning* (Russell, 2009), there are only considered environments that are fully observable, deterministic, finite, static (change happens only when the agent acts), and discrete (in time, action, objects, and effects).

*Classical Planning* (Nilsson, 2010) is defined as a tuple  $\Sigma = \langle S, S_0, S_G, A \rangle$ , where  $S$  is a finite state space,  $S_0$  is an initial situation given by a single state  $S_0 \in S$ ,  $S_G$  is a goal situation given by a non-empty

set of states  $S_G \subseteq S$ , and  $A$  is a set of finite actions. These actions  $A(s) \subseteq A$  are applicable in each state  $s \in S$ , and every action  $a \in A(s)$  maps each state  $s \in S$  into another state  $s_a = f(a, s)$  in a deterministic way. Indeed, there are positive action costs  $c(a, s)$  for doing an action  $a \in A(s)$  in a state  $s \in S$ .

A plan (*solution*) for a deterministic problem is a sequence of actions  $\{a_0, a_1, \dots, a_k\}$  that generates a state trajectory  $\{s_0, s_1 = f(s_0, a_0), \dots, s_{k+1} = f(s_k, a_k)\}$  such that each action  $a_i \in A(s_i)$  is applicable in state  $s_i \in S$  and  $s_{k+1} \in S_G$  is a goal state. A plan  $\pi$  is optimal when the total cost  $\sum_{i=0}^k c(s_i, a_i) \geq 0$  of doing an action  $a_i \in A(s)$  in a state  $s_i \in S$  is minimal.

In Classical Planning is used a transition function  $f(s_i, s_{i+1}, a_i)$  to guide the search (from a state  $s_i$  to another state  $s_{i+1}$  using an action  $a_i$ ) to reach a *solution*. The most successful planners use heuristics to have a *solution* in less time as do FF, HSP, and others. But, these planners do not have the same results in *length* (of the *solution*) as in *time*. In order to deliver better results in *length*, we propose a novel approach to guide the search in Planning using *Bayesian inference*.

## 3 BAYESIAN NETWORK AND INFERENCE

*Bayesian networks* (Russell, 2009) are directed acyclic graphs whose events (nodes) are represented by random variables which can have several states with a probability of happening.

Figure 1 depicts a Bayesian network (Russell, 2009) where an event has an effect in several events, and one or more events can have an effect on one event.

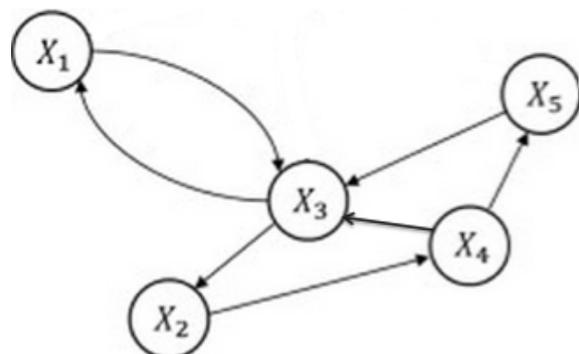


Figure 1: Bayesian network.

*Bayesian Inference* (O’Hagan et al., 2004) is a method of inference that update the probability of a hypothesis (belief) using Bayes’ rule as additional evidence is acquired.

The process of inference has two steps:

1. Update of the belief given an evidence
2. Propagation of the evidence through the network

Morawski defined a novel model of Bayesian network, which consists of a tree with one *father* with several *children* in a hierarchical way (Morawski, 1989). Indeed, the tree has a father-children relationship, where the father sends information to his son ( $\pi$ ) and a son to his father ( $\lambda$ ). Figure 2 depicts the Morawski model of a Bayesian network.

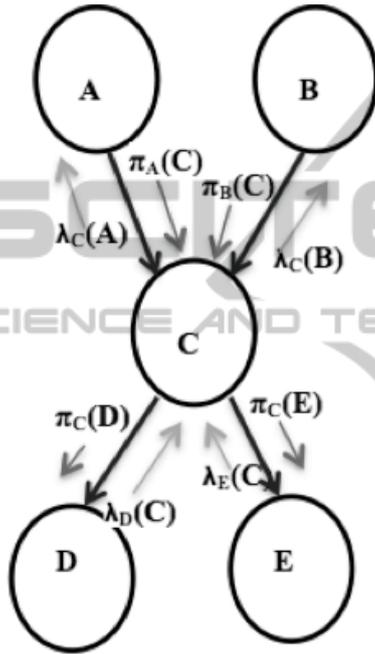


Figure 2: Evidence propagation in a Bayesian network.

In this cause-effect relationship between father and children we have both evidence: diagnostic and causal evidence, where these evidences use the *chain rule* given conditional independence to be estimated. Furthermore, we have a node *belief* which uses the causal and diagnostic evidence.

*Diagnostic evidence* is estimated, in equation 1, using  $\lambda$  information of an event. In this expression,  $Children(f_i)$  stands for the children of the event (father node)  $f_i$  and  $\lambda_k(f_i)$  stands for the probability of an event  $k$  happening given the event (father node)  $f_i$ , which is sent from the event  $k$  to the event  $f_i$ .

$$\lambda(f_i) = \prod_{k \in Children(f_i)} \lambda_k(f_i) \quad (1)$$

*Causal evidence*, which is defined in equation 2, uses  $\pi$  information of an event. Furthermore, this evidence is estimated using the *conditional independence* and *rule chain*. In this expression,  $s_i$  stands for

a state of the event  $S$ ,  $States(f)$  stands for the states of the event  $f$ ,  $P(s_i|f_k)$  stands for the probability of happening state  $s_i$  given his father(s)  $f_k$ ,  $\pi_S(f_k)$  stands for the probability of happening the event  $f_k$  sent from the event  $f_k$  to the event  $S$ , and  $father(S)$  stands for the fathers of the event  $S$ .

$$\pi(s_i) = \sum_{k \in States(f)} P(s_i|f_k) \prod_{f \in father(S)} \pi_S(f_k) \quad (2)$$

The *belief* of an event  $e_i$  is estimated, in equation 3, using the  $\lambda$  and  $\pi$  information (of his son and father), where  $\alpha$  is a normalization constant to maintain in balance the joint probabilities, i.e. to maintain the sum of all beliefs (in a Bayesian network) equal to 1.

$$\beta(e_i) = \alpha \pi(e_i) \lambda(e_i) \quad (3)$$

Bayesian inference is used to do inference between conditional events, which could be helpful in a planning problem search to reach to an optimal solution in a probabilistic way.

## 4 PROPOSAL PROBABILISTIC PLANNING

We propose a novel Probabilistic Planning Search, which introduces the use of probability to guide the search space in a non-relaxed way. Furthermore, we propose a model for planning with probabilistic search where a goal is achieved taking account the probability to reach a *goal* state.

The model for the probabilistic planning is characterized by a quintuple  $\Sigma = \langle S, S_0, S_G, A, P \rangle$  where  $S$  is a finite state space,  $S_0$  is a non-empty set of possible initial states  $S_0 \in S$ ,  $S_G$  is a non-empty set of goal states,  $A$  is a set of actions with  $A(s)$  denoting the actions applicable in state  $s$   $A(s) \in A$ , and  $P$  is a *probability transition function* such that  $P(s_k|a_i, s_i)$  denotes the non-empty set of probabilities of passing from the an initial state  $s_i$  using an action  $a_i$  to the state  $s_k$ .

In our approach we try to establish a basic form to search in the Bayesian network using Morawski model. We defined the causal and diagnostic evidence based on the amount of children states and on the distance from this state to the goal, respectively. Intuitively, we reduce the branch factor with the causal evidence because it takes account the less amount of children states; and reduce the plan length because we take account the minimum cost to reach the goal. This probability transition function is used to guide the search, where the maximum probability is chosen in each state.

## 4.1 Bayesian Inference

The state space, where nodes are states, can be modeled as a *Bayesian network* (with a father-children structure) with events replaced by states  $s \in S$  and arrows (which establish cause-effect relationship between events) are replaced by actions  $a \in A(s)$ .

Morawski used the belief equation of an event  $e_i$  to estimate the probability of this event  $e_i$  given other events that occur in the past. In the same way, we use this belief equation to estimate the probability to find the goal state  $g \in S$  given an action  $a \in A(s)$  chosen a shorter way. In this probabilistic model, we estimate the *probability transition function* as equation 4.

$$P(s_k | a_i, s_i) = \alpha \lambda_{\text{children}(s_i, a_i)}(s_k) \pi_{s_i}(s_k) \quad (4)$$

In these expressions,  $\lambda_{\text{children}(s_i, a_i)}(s_k)$  stands for the probability of happening of the *son* state  $\text{children}(s_i, a_i)$  given the state  $s_k$ ,  $\pi_{s_i}(s_k)$  stands for the probability of happening  $s_k$ , and  $\alpha$  is a normalization constant to maintain the sum of the *total probability* equal to 1.

The *probability transition function* estimates the shorter branching factor to reach a *goal* state. This equation uses causal and diagnostic evidence. These evidences can be estimated using cost equations of reaching the goal state and branching.

$$c(g; s) := \begin{cases} 0 & g \in s \\ \min_{a \in A(s)} [1 + c(s_{\text{prec}}; s)] & \text{otherwise} \end{cases} \quad (5)$$

The main costs used to update the *probability transition function* are the maximum and total cost. For the estimation of these costs, we used the Bellman-Ford algorithm (Vector, 2003), equation 5, to estimate the shorter length cost to reach a *goal* state  $g \in S$  from state  $s \in S$ . In this expression,  $s_{\text{prec}}$  stands for the *precondition* states (or *prior* states).

For the estimation of the *total cost* of the branching factor, in equation 6, it is defined as the sum of the costs  $c(g_i, s)$  of each *goal* state  $g_i \in G$ , where  $G$  stands for the set of *goal* states  $G \in S$ .

$$c_{\text{total}}(G; s) := \sum_{g_i \in G} c(g_i; s) \quad (6)$$

The estimation of the *maximum cost* of branching factor, in equation 7, is defined as the maximum cost  $c(g_i, s)$  of each *goal* state  $g_i \in G$ .

$$c_{\text{max}}(G; s) := \max_{g_i \in G} [c(g_i; s)] \quad (7)$$

The estimation of  $\lambda_{s_i}(\neg s_k)$ , which is the probability of *not* happening the state  $s_i$  (in a shorter way) given the goal state  $s_k$ , is defined as equation 8. We divided the maximum cost of branching factor by the total cost of branching factor, because it represents the largest-search-cost possibility divided by all possibilities.

$$\lambda_{s_i}(\neg s_k) = \frac{c_{\text{max}}(s_k; s_i)}{c_{\text{total}}(s_k; s_i)} \quad (8)$$

The complement of  $\lambda_{s_i}(\neg s_k)$  is  $\lambda_{s_i}(s_k)$ , which is estimated as equation 9.

$$\lambda_{s_i}(s_k) = 1 - \lambda(\neg s_k) \quad (9)$$

The estimation of  $\pi_{s_i}(\neg s_k)$ , which is the probability of *not* happening state  $s_k$  in a shorter way, is in equation 10. In this expression,  $\text{Children}(s_i)$  stands for the son states of the state  $s_i$ .

$$\pi_{s_i}(\neg s_k) = \frac{1}{\text{Children}(s_i)} \quad (10)$$

The complement of  $\pi_{s_i}(\neg s_k)$  is  $\pi_{s_i}(s_k)$ , which is estimated in equation 11.

$$\pi_{s_i}(s_k) = 1 - \frac{1}{\text{Children}(s_i)} \quad (11)$$

Both equations (9) and (11) are used to estimate the probability of the shorter branching factor of a state space (modeled as Bayesian network). Therefore, using our approach, we would expect that shorter plans can be found compared with a heuristic approach. We discuss below the results of our approach in different domains compared with other planner approaches.

## 5 EXPERIMENTS & RESULTS

We compared the performance of our approach with other planner approaches for some well-known benchmark problems (STRIPS like) of the AIPS-2002 competition set. We considered Blackbox, which is the combination of both approaches Graphplan and SAT; HSP, which uses the heuristic approach; Metricff, which uses and improved the same ideas from HSP approach; and LPG, which is a planning-graph approach (which is different to Graphplan algorithm, but it used in this planner the same graph-search idea). Furthermore, all the experiments of our probabilistic approach where using a backward approach and the Breadth-first search algorithm.

For each benchmark, it was estimated for each approach the mean percentage error (MPE) between

the plan-length results of such approach and the plan-length minimum of all approaches in each instance.

## 5.1 Driverlog Benchmark

The *Driverlog* domain is a variation of *Logistics* where the trucks need drivers, furthermore some paths can only be traversed by truck, and others only on foot.

Figure 3 depicts the plan-length curves (in logarithmic scale with base 10) on *Driverlog* instances, which their size scales from left to right.

Our approach, in these *Driverlog* instances, solved most of the problems with less steps in the plan (*solution*). Our approach dominates both approaches LPG and HSP. Only in few instances both Blackbox and Metric-ff (in speed version) obtained better plan-length results.

In terms of mean percentage error (MPE), our approach has less error than the other approaches. In comparison with the best results for each approach,

our approach has 0.050 of MPE, Blackbox has 0.060 of MPE, Metric-ff has 0.163, HSP has 0.297, and LPG has 0.436 of MPE.

## 5.2 Zenotravel Benchmark

The *Zenotravel* domain is a transportation problem, where objects must be transported by airplanes, and each airplane has an associated fuel level.

Figure 4 depicts the plan-length curves (in logarithmic scale with base 10) on *Zenotravel* instances, which their size scales from left to right.

Metric-ff approach and our approach solved the *Zenotravel* instances in less steps than the others, but Metric-ff cannot perform one instance in this domain. Furthermore, the Blackbox approach can give a solution to any instance in this domain. Only in three instances our approach has equal or better results in comparison with Metric-ff approach.

In terms of mean percentage error (MPE), Metric-ff and our approach have less error than the other ap-

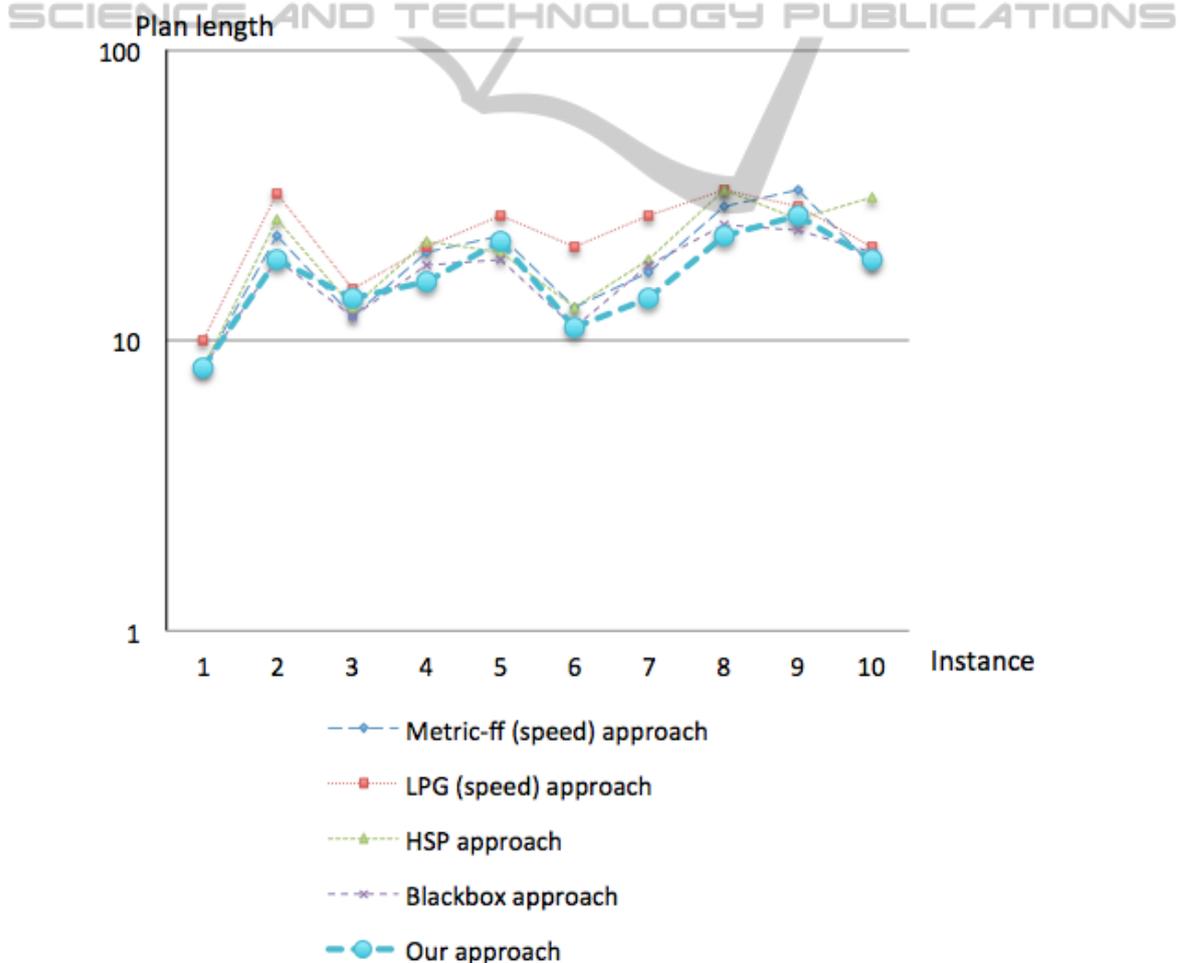


Figure 3: Plan-length curves on Driverlog instances.

proaches. In comparison with the best results for each approach, Metric-ff has 0.118 of MPE, our approach has 0.224 of MPE, HSP has 0.443, LPG has 0.679 of MPE, and Blackbox cannot perform any instance in this domain.

### 5.3 Satellite Benchmark

In *Satellite* domain, a number of satellites must take some observations using some instruments. The actions that a satellite can do are turning in the right direction, switching the instruments on or off, calibrating the instruments, and taking images.

Figure 5 depicts the plan-length curves (in logarithmic scale with base 10) on *Satellite* instances, which their size scales from left to right.

In terms of mean percentage error (MPE), Blackbox Metric-ff and our approach have less error than the other approaches. In comparison with the best results for each approach, Blackbox has 0.057 of MPE, Metric-ff has 0.092 of MPE, our approach has 0.269 of MPE, LPG has 0.289 of MPE, and HSP has 0.593.

### 5.4 Rovers Benchmark

In *Rovers* domain, a number of planetary rovers must analyze some samples, and take images. The actions of a rover include navigating, taking samples, calibrating the camera and taking images, and communicating the data.

Figure 6 depicts the plan-length curves (in logarithmic scale with base 10) on *Rovers* instances, which their size scales from left to right.

In this domain, Metric-ff, HSP and our approach have a better performance than the other approaches. Metric-ff and HSP weakly dominates our approach having equal or better results.

In terms of mean percentage error (MPE), Metric-ff, HSP and our approach have less error than the other approaches. In comparison with the best results for each approach, Metric-ff has 0.016 of MPE, HSP has 0.067, our approach has 0.270 of MPE, Blackbox has 0.347, and LPG has 0.357 of MPE.

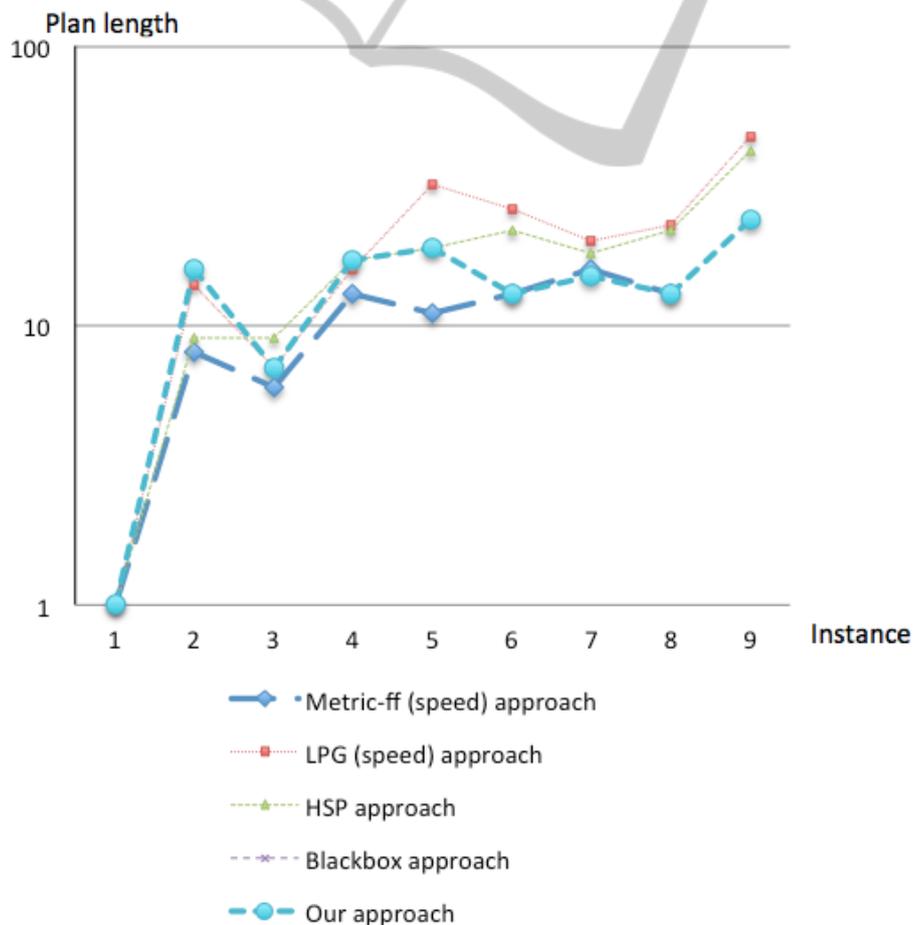


Figure 4: Plan-length curves on Zenotravel instances.

## 5.5 Freecell Benchmark

The *Freecell* domain is a well-known domain of discrete puzzles, where all cards are randomly divided into a number of piles. The objective of the game is to move all cards onto four different piles for each suit, where the cards are arranged upwards from the ace to the king.

Figure 7 depicts the plan-length curves (in logarithmic scale with base 10) on *Freecell* instances, which their size scales from left to right.

In this domain, both LPG and HSP approaches do not have a solution for all tested instances, and it is not clear which approach is best.

In terms of mean percentage error (MPE), Metric-ff and our approach have less error than the other approaches. In comparison with the best results for each approach, Metric-ff has 0.083 of MPE, our approach has 0.166 of MPE, HSP has 0.38 of MPE, LPG has 0.517 of MPE, and Blackbox has 0.526.

## 5.6 Overall Results

Metric-ff and our approach have the best performance in all the domains tested. Therefore, they have less mean percentage error in plan-length results in general.

Our probabilistic approach has better results than Blackbox (which is the combination of Graphplan and Sat approach), LPG (planning-graph approach), HSP (heuristic-search approach), but our approach cannot have overall better results in comparison with Metric-ff (a heuristic approach that uses a relaxed Graphplan algorithm as a heuristic and a novel Enforced Hill Climbing search algorithm).

Our approach, as the other approaches, has difficulties to work with complex problems (i.e. domains with many actions, predicates, objects and goals). In our experiments, the complexity level of each well-known benchmark is increased, where we can see that the level of complexity depends on the benchmark

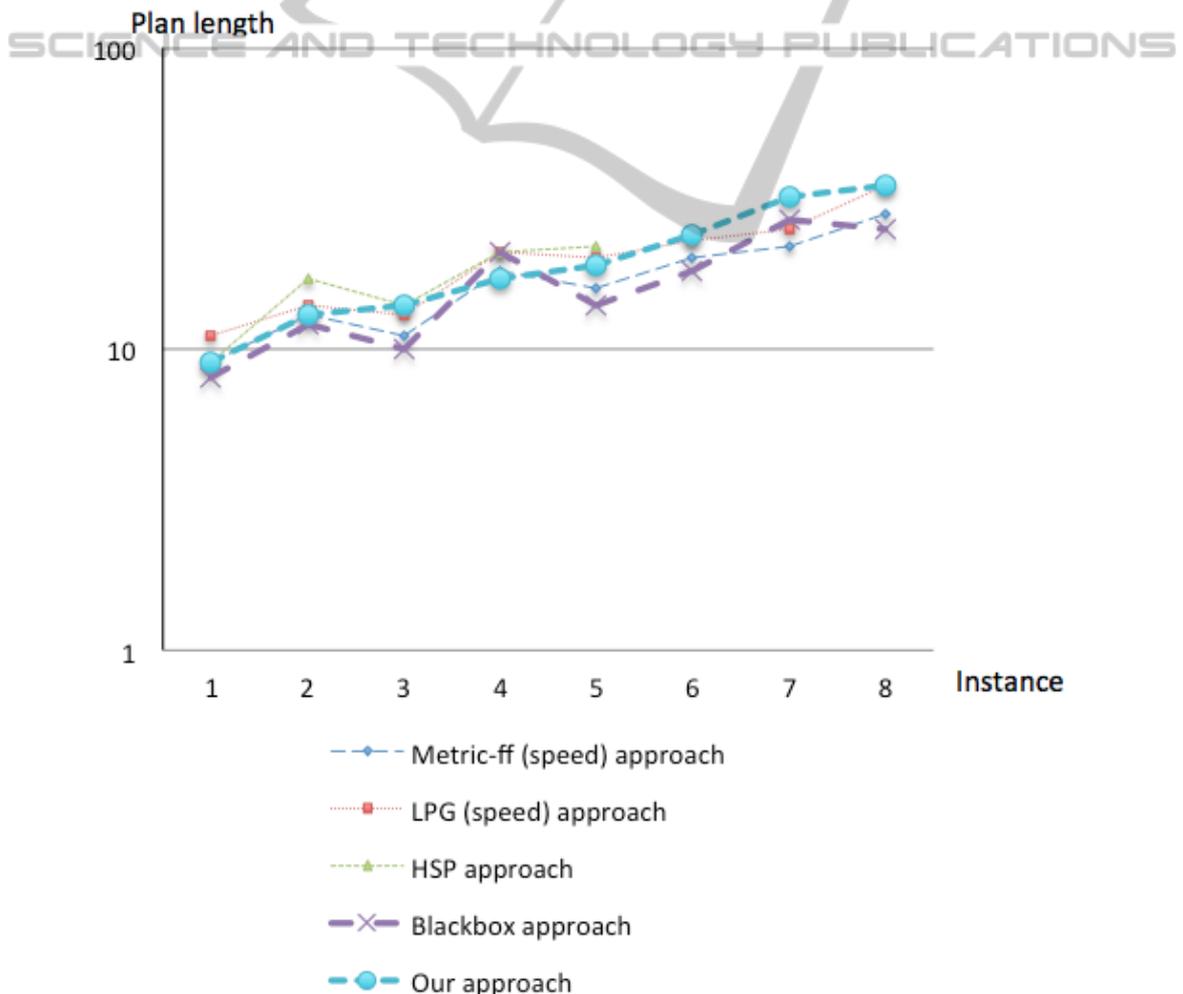


Figure 5: Plan-length curves on Satellite instances.

used.

## 6 CONCLUSIONS

We have presented and modeled a novel probabilistic planning search approach. This probabilistic approach has good results in comparison with the other approaches. The most important contribution is the “*probabilistic search approach*” idea.

We extended the *Bayesian network* idea to *state space*. The state space was modeled as a father-children network structure. Using this network we could make inferences using our Bayesian inference equations.

Furthermore, we defined explicitly probabilistic functions in order to define the Bayesian inference equations, which are based on well-known rules as both *Bayes’ rule* and *conditional independence*.

In our experiments with different benchmarks we found that our approach has good results in terms of

*plan length*.

Our future work focuses on the *search algorithm* that works better with our *Bayesian inferences* in order to get *optimal* results, which means to have plans with less length in less time.

The benchmarks have different variables (states), which have an influence on data distribution. Indeed, this affects our Bayesian inferences, in order to have better results in our future work we will study the data distribution of different well-known benchmarks and we will determine the Bayesian inference equations that work best with.

## ACKNOWLEDGEMENTS

This research was supported by ITESM under the Research Chair CAT-100.

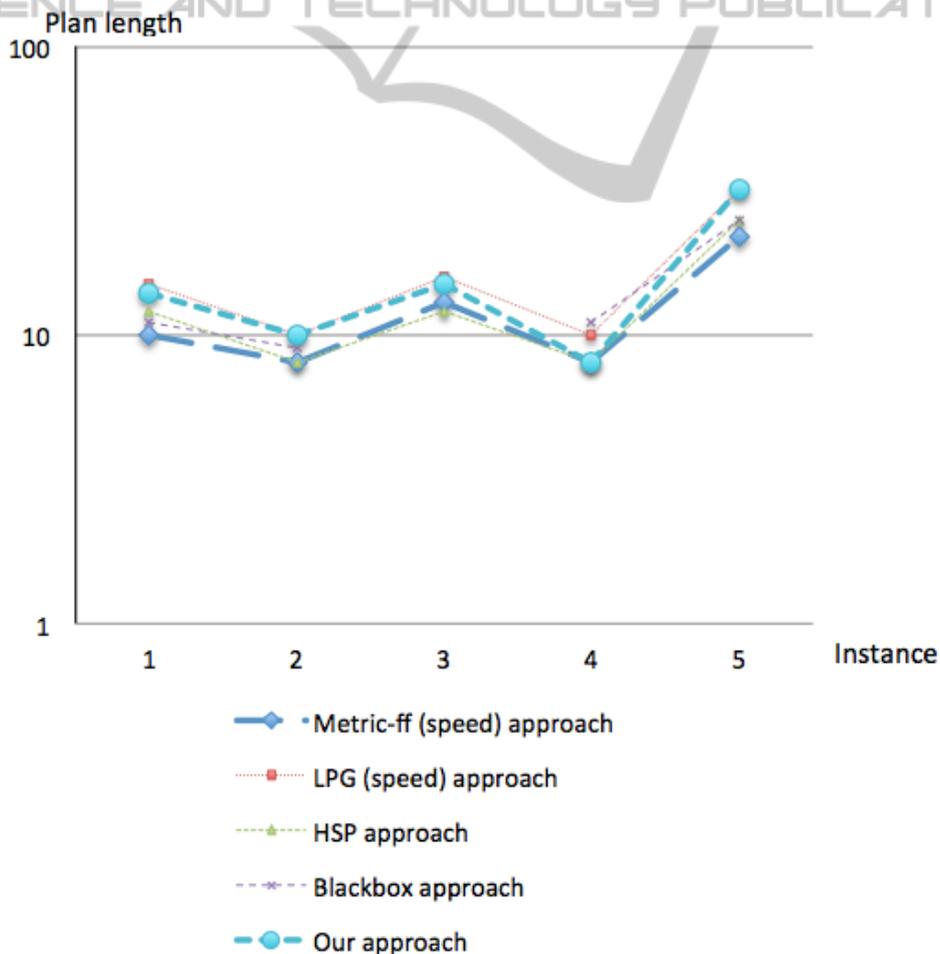


Figure 6: Plan-length curves on Rovers instances.

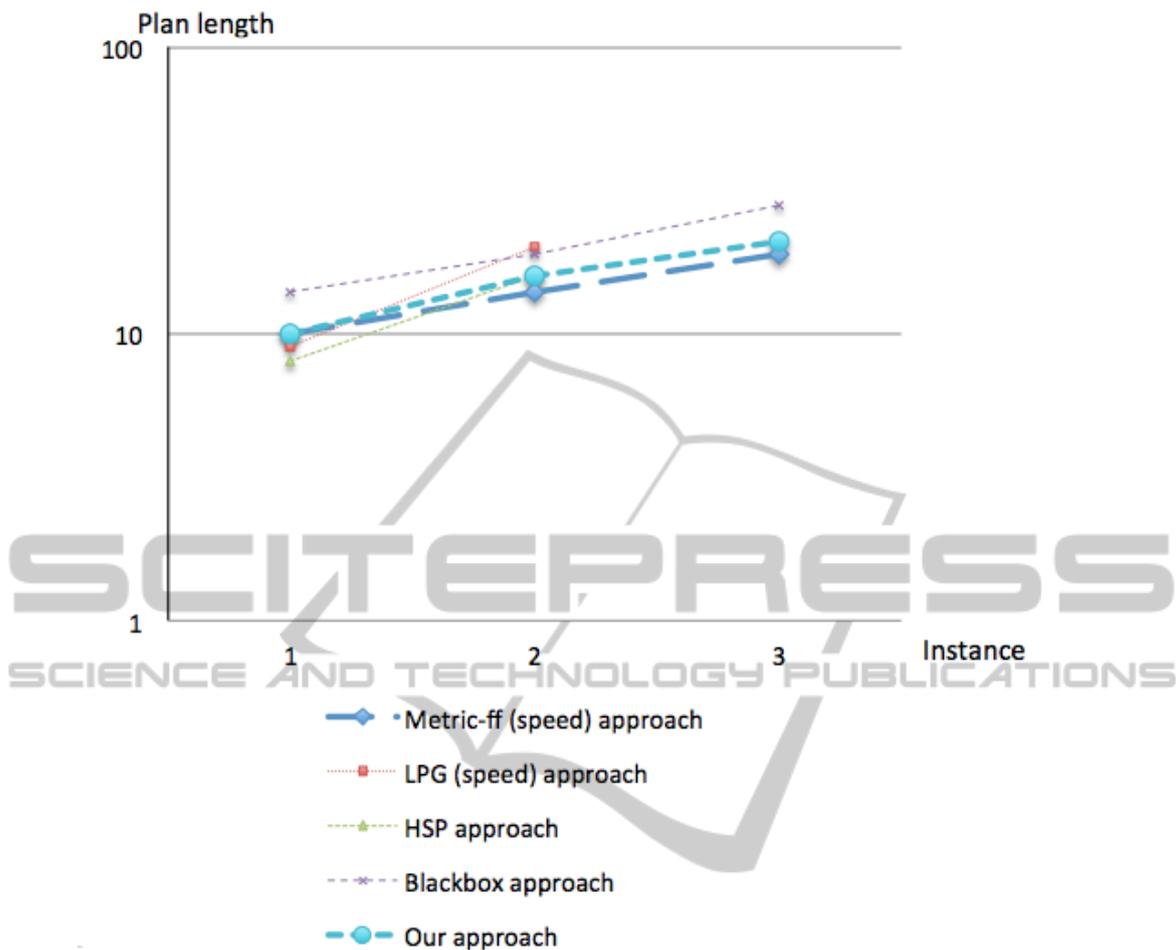


Figure 7: Plan-length curves on Freecell instances.

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