

A Multi-demand Adaptive Bargaining based on Fuzzy Logic

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Abstract: Nowadays, decisions in estate investment are made by a group of investors with different demands and then how to find an agreement among them become an essential issue. Thus, this paper introduces a fuzzy logic based bargaining model to solve such problems. Moreover, we also do lots of simulation experiments to reveal how bargainers' risk attitude, patience and regret degree influence the outcome of a game, and benchmark our model with the previous one. From these experiments, we can conclude that our model can reflect the human intuitions well, has a higher success rate, and bargains more efficiently than the previous one.

1 INTRODUCTION

Nowadays, many business decisions are not made by just one person but a group of people. They often need to negotiate before making an ultimate decision. For example, in a problem of real estate investment some investors demand to build big houses, while some demand to build economically affordable houses; some insist on using environmental but expensive material, while some support low-cost one; and so on. There are so many inconsistencies among different investors. So, to make a decision accepted by all, they have to bargain with each other.

In such a problem, it is hard to elicit numerical utilities and do quantitative analyses (Zhang, 2010). Thus, some researchers tried to express bargainers' preferences in an ordinal scale (Shubik, 2006; Zhang and Zhang, 2008). However, the information relevant to the bargainers' risk attitudes, a very important factor in bargaining (García-Gallego et al., 2012), is lost (Zhang, 2010).

To deal with this issue, some researchers build new models. For example, Zhang (2010) introduced a new ordinal bargaining model, in which the preference ordering of a bargainer is defined on the player's demands and the risk attitudes of a bargainer can be represented through the ranking of conflicting demands. However, the models of this kind still have some drawbacks. For example, they cannot explicitly represent players' attitudes towards risk; and ignore that bargainers' preferences can be changed because of different risk attitudes.

Thus, further Zhan et al. (2013) introduced another new ordinal bargaining model, in which each bargainer has two preference orderings over his demands: one for reflecting the bargainer's own taste without considering any information about the bargaining, while the other for reflecting not only his own taste but also his thinking about which demand should be insisted on or given up earlier. Thus, his risk attitude can be tasted out by comparing the two preferences. Moreover, in their model, a bargainer's preference could be changed during a bargaining according to his psychological factors about risk, patience and regret. A fuzzy logic system is used to calculate the change of the preference dynamically.

However, Zhan et al. (2013) did not do sufficient empirical analyses upon their model. Moreover, their fuzzy rules are not very intuitive. So, this paper re-analyses the psychological experiments of setting the rules in (Zhan et al., 2013) and simplifies these fuzzy rules. According to these new rules, we do lots of experiments to reveal some insights into the model. In addition, we illustrate our new model by solving a bargaining problem in the real estate investment.

The rest of the paper is organised as follows. Section 2 recaps the bargaining model and its solution concept. Section 3 presents the improved fuzzy reasoning systems. Sections 4 and 5 empirically analyse the influence of input parameters in the fuzzy system and benchmark our solution method with a previous one. Section 6 illustrates the model by solving the problem in the investment in real estate. Section 7 discusses the related work. Finally, Section 8 concludes

the paper with future work.

2 MODEL DEFINITION

This section recaps the bargaining model of Zhan et al. (2013).

Definition 1. A bargaining game is a tuple of $(N, \{X_i, \succ_{i,o}, \succ_{i,d}^{(0)}\}_{i \in N}, \mathcal{A}, FLS)$, where

- N is the set of all the bargainers in this game;
- X_i is the demand set of bargainer i in a propositional language denoted as \mathcal{L} , consisting of a finite set of propositional variables and standard propositional connectives $\{\neg, \vee, \wedge, \rightarrow\}$;
- $\succ_{i,o}$ is bargainer i 's original demand preference ordering, which is a total pre-order on X_i (i.e., satisfying totality, reflexivity and transitivity);
- $\succ_{i,d}^{(0)}$ is bargainer i 's initial dynamic demand preference ordering, i.e., a total pre-order on X_i (i.e., satisfying totality, reflexivity and transitivity);
- \mathcal{A} is bargainers' action function defined as:

$$\mathcal{A}(x^*, \zeta, \lambda) = \begin{cases} \text{move down } x^* \text{ two levels} \\ \text{if } (\zeta \geq 0.7) \text{ and} \\ \quad (\exists x', x'' \notin CDS_i, x^* \succ_{i,d}^{(\lambda)} x', x''), \\ \text{move down } x^* \text{ one level} \\ \text{if } (0.7 > \zeta \geq 0.3 \text{ and} \\ \quad \exists x' \notin CDS_i, x^* \succ_{i,d}^{(\lambda)} x') \text{ or} \\ \quad (\zeta \geq 0.7 \text{ and} \\ \quad \quad \exists x' \notin CDS_i, x^* \succ_{i,d}^{(\lambda)} x'), \\ \text{do nothing} \\ \text{otherwise.} \end{cases} \quad (1)$$

where ζ is the change degree, $x^* \in CDS_i$ (i.e., the conflicting demand set of bargainer i in X_i), and n means the λ -th round of the bargaining game;

- FLS is a fuzzy logic system for calculating the preference change degree.

The bargainers' demands are expressed by logical statements, and every bargainer's original preference ordering and initial dynamic preference ordering are over his demands rather than the agreements of a bargaining game. Because all bargainers' demands may be logically inconsistent in a set, the purpose of a bargaining game is to find an agreement consisting logically consistent statements.

In the bargaining model, the dynamic preference can be changed during a bargaining. Thus, a parameter, called *change degree* (i.e., ζ), is used to capture the degree to which a bargainer wants to change his preference. It is calculated by the fuzzy logic system

FLS , which inputs are bargainers' risk attitude, patience descent degree and regret degree. Accordingly, by action function \mathcal{A}_i , bargainer i will take a proper action to change his preference. That is, after the λ -th round, dynamic demand preference structure $(X_i^{(\lambda)}, \succ_{i,d}^{(\lambda)})$ of bargainer i will be updated to a new one, denoted as $(X_i^{(\lambda+1)}, \succ_{i,d}^{(\lambda+1)})$, by a certain action chosen by action function (1), where its input (i.e., change degree ζ) is determined by the fuzzy logic system.

Let $\{X_i^{(1,\lambda)}, \dots, X_i^{(L_i(\lambda),\lambda)}\}$ be the partition of $X_i^{(\lambda)}$ induced by equivalence relation \sim , which is defined by preference ordering $\succ_{i,d}^{(\lambda)}$, and $L_i(\lambda)$ denotes the height of the hierarchy of bargainer i in the λ -th round (specially, L_i is short for $L_i(0)$). We regard every partition as different levels from high to low, that is, $X_i^{(1,\lambda)}$ is the demands in the highest level in $X_i^{(\lambda)}$ and $X_i^{(L_i(\lambda),\lambda)}$ is the demands in the lowest level in $X_i^{(\lambda)}$. There are two steps in every round: (i) concession, i.e., every bargainer gives up the least preferred demands (i.e., the demands in the lowest level in the current round) if their remaining demands are inconsistent; and (ii) changing the demand preference after concession. So, according to step (i), $X_i^{(\lambda+1)} = X_i^{(\lambda)} \setminus X_i^{(L_i(\lambda),\lambda)}$, and after concession, according to step (ii), $\{X_i^{(1,\lambda)}, \dots, X_i^{(L_i(\lambda),\lambda)}\}$ will be updated to $\{X_i^{(1,\lambda+1)}, \dots, X_i^{(L_i(\lambda+1),\lambda+1)}\}$ through action function \mathcal{A} . Formally, we have:

Definition 2. For bargaining game $G = (N, \{X_i, \succ_{i,o}, \succ_{i,d}^{(0)}\}_{i \in N}, \mathcal{A}, FLS)$, its dynamically simultaneous concession solution (DSCS) is:

$$S(G) = \begin{cases} (X_1^{(\nu)}, \dots, X_n^{(\nu)}) & \text{if } \forall i \in N, X_i^{(\nu)} \neq \emptyset, \\ & \nu < \min\{L_i \mid i \in N\}, \\ (\emptyset, \dots, \emptyset) & \text{otherwise,} \end{cases} \quad (2)$$

where ν is the minimal rounds of concessions of the game, i.e., $\nu = \min\{k \mid \cup_{i=1}^n X_i^{(k)} \text{ is consistent}\}$ ($X_i^{(k)}$ is the set of demands of bargainer i after k rounds of the bargaining). And the agreement of game G is:

$$A(G) = \bigcup_{i \in N} s_i(G), \quad (3)$$

where $s_i(G)$ is the i -th element of $S(G)$.

3 FUZZY LOGIC SYSTEM

This section will present our fuzzy logic system for calculating the preference change degree.

The fuzzy rules we reset are listed in Table 1. Rule 1 means that if a bargainer does not lose too many

Table 1: Fuzzy rules.

| |
|--|
| If regret degree is Low then change degree is Low. |
| If regret degree is Medium then change degree is Medium. |
| If regret degree is High then change degree is High. |
| If patience descent degree is Low then change degree is Low. |
| If patience descent degree is Medium then change degree is Medium. |
| If patience descent degree is High then change degree is High. |
| If initial risk degree is Low then change degree is High. |
| If initial risk degree is Medium then change degree is Medium. |
| If initial risk degree is High then change degree is Low. |

consistent demands, which makes him regret just a little, then his desire to change his preference is low. Other rules can be understood similarly.

In each round of bargaining, when calculating the change degree, the input parameters of the fuzzy rules are:

(i) *Regret degree* (ϑ). Formally, it is calculated by:

$$\vartheta_i^{(\lambda)}(|RC_i^{(\lambda)}|) = \frac{|C_i| - |RC_i^{(\lambda)}|}{|C_i|}, \quad (4)$$

where $|C_i|$ is the number of consistent demands of bargainer i in X_i and $|RC_i^{(\lambda)}|$ is the number of remaining consistent demands of bargainer i after the λ -th round of bargaining.

(ii) *Patience descent degree* (ρ). Formally, it is given by:

$$\rho_i(\lambda) = \frac{\lambda}{L_i}, \quad (5)$$

where λ is the number of completed rounds of bargaining and L_i is the height of the initial dynamic preference hierarchy of bargainer i in the first round.

(iii) *Initial risk degree*. It is defined as follows:

Definition 3. Let $h_i(x)$ and $H_i(x)$ be the levels of demand x in the original demand preference hierarchy and the initial dynamic demand preference hierarchy, respectively. Specifically, $h_i(x) = 1$ means bargainer i prefers x the most in the original preference and $h_i(x) = L_i$ means bargainer i prefers x the least in the original preference, where $L_i = \max\{h_i(x) \mid x \in X_i\}$. Similar things go for $H_i(x)$. Then the initial risk degree of bargainer i is given by:

$$\gamma_i = \begin{cases} \frac{\sum_{c_i \in CDS_i} (h_i(c_i) - H_i(c_i))}{\sum_{c_i \in CDS_i} h_i(c_i) - \frac{(1+N_i)N_i}{2}} & \text{if } \sum_{c_i \in CDS_i} (h_i(c_i) - H_i(c_i)) > 0, \\ \frac{\sum_{c_i \in CDS_i} (h_i(c_i) - H_i(c_i))}{\sum_{c_i \in CDS_i} h_i(c_i) - N_i L_i + \frac{(N_i-1)N_i}{2}} & \text{if } \sum_{c_i \in CDS_i} (h_i(c_i) - H_i(c_i)) < 0, \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

where CDS_i , N_i and L_i are bargainer i 's conflicting demand set, conflicting demands' number and initial dynamic preference level number, respectively.

The meanings of linguistic terms of the fuzzy variables in Table 1 are as follows. The *low* regret degree means that a bargainer just regrets a little for the demands given up in the previous round. The *medium* regret degree means that a bargainer cares about the demands given up and regrets having insisted on the preference in the previous round. And the *high* regret degree means that a bargainer regrets very much for insisting on the preference in the previous round and more likely changes it because it causes a lot of consistent demands lost. The linguistic terms of other two parameters can be understood similarly.

These linguistic terms of the fuzzy variables are modelled by the following fuzzy member function:

$$\mu(x) = \begin{cases} 0 & \text{if } x \leq a, \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b, \\ 1 & \text{if } b \leq x \leq c, \\ \frac{d-x}{d-c} & \text{if } c \leq x \leq d, \\ 0 & \text{if } x \geq d. \end{cases} \quad (7)$$

For convenience, we represent formula (7) as $\mu(x) = (a, b, c, d)$. Thus, the linguistic terms of regret degrees can be expressed as $\mu_{low}(\vartheta) = (-0.2, 0, 0.2, 0.4)$, $\mu_{medium}(\vartheta) = (0.2, 0.4, 0.6, 0.8)$, and $\mu_{high}(\vartheta) = (0.6, 0.8, 1, 1.2)$. Similarly, we can have $\mu_{low}(\rho) = (-0.2, 0, 0.2, 0.4)$, $\mu_{medium}(\rho) = (0.2, 0.4, 0.6, 0.8)$, and $\mu_{high}(\rho) = (0.6, 0.8, 1, 1.2)$; $\mu_{low}(\gamma) = (-1.4, -1, -0.6, -0.2)$, $\mu_{medium}(\gamma) = (-0.6, -0.2, 0.2, 0.6)$, and $\mu_{high}(\gamma) = (0.2, 0.6, 1, 1.4)$; and $\mu_{low}(\zeta) = (-0.2, 0, 0.2, 0.4)$, $\mu_{medium}(\zeta) = (0.2, 0.4, 0.6, 0.8)$, and $\mu_{high}(\zeta) = (0.6, 0.8, 1, 1.2)$.

We use the standard Mamdani method (Mamdani and Assilian, 1975) to do fuzzy reasoning as follows:

Definition 4. Let A_i be a Boolean combination of fuzzy sets $A_{i,1}, \dots, A_{i,m}$, where $A_{i,j}$ is a fuzzy set defined on $U_{i,j}$ ($i = 1, \dots, n; j = 1, \dots, m$), and B_i be fuzzy set on U' ($i = 1, \dots, n$). Then when the inputs are $\mu_{A_{i,1}}(u_{i,1}), \dots, \mu_{A_{i,m}}(u_{i,m})$, the output of such fuzzy rule $A_i \rightarrow B_i$ is fuzzy set B'_i defined by:

$$\mu_i(u') = \min\{f(\mu_{A_{i,1}}(u_{i,1}), \dots, \mu_{A_{i,m}}(u_{i,m})), \mu_{B_i}(u')\}, \quad (8)$$

where f is obtained through replacing $A_{i,j}$ in A_i by $\mu_{i,j}(u_{i,j})$ and replacing "and", "or", "not" in A_i by "min", "max", " $1 - \mu$ ", respectively. And the output of all rules $A_1 \rightarrow B_1, \dots, A_n \rightarrow B_n$, is fuzzy set M , which is defined by:

$$\mu_M(u') = \max\{\mu_1(u'), \dots, \mu_n(u')\}. \quad (9)$$

The result what we get is still a fuzzy set. To defuzzify the fuzzy set, we need the following centroid method (Mamdani and Assilian, 1975):

Definition 5. The centroid point u_{cen} of fuzzy set M given by formula (9) is:

$$u_{cen} = \frac{\int_{U'} u' \mu_M(u') du'}{\int_{U'} \mu_M(u') du'} \quad (10)$$

Actually, u_{cen} in above is the centroid of the area that is covered by the curve of membership function μ_M and the horizontal ordinate.

4 EXPERIMENTAL ANALYSIS

This section will analyse how bargainer's risk degree, patience descent degree and regret degree in the fuzzy logic based model affects the outcome of a bargaining game. We will use the measure of the average level number of remaining demands in bargainers' outcome in initial dynamic preference. A smaller average level number means a higher average level (i.e., a bargainer gains more what he prefers) and a large average level number means a lower average level (i.e., a bargainer gains less what he really wants). In all experiments, we run 1000 times bargaining under the setting that every bargainer's action function is formula (1) and the fuzzy rules are those in Table 1.

Now we do two experiments to investigate the effect of attitude towards risk in two dimensions: (i) the effect upon the average rounds to achieve agreements and (ii) the average preference levels of remaining demands in certain bargainer's outcome. We randomly generate 10 demands in different preference levels for two bargainers and arbitrarily label N (changing from 0 to 10) of them as their conflicting ones.

In the first experiment, the bargaining is carried out in the fuzzy logic based model with both bargainers' risk degrees are fixed in the three cases of $(\gamma_1, \gamma_2) = (1, 1)$, $(\gamma_1, \gamma_2) = (1, -1)$, and $(\gamma_1, \gamma_2) = (-1, -1)$ to model: (i) one risk seeker encounters another risk seeker, (ii) one risk seeker encounters one risk averser, and (iii) one risk averser encounters another risk averser, respectively.

From Figure 1, we can see that the average rounds to reach agreements is the lowest when one risk averser encounters another risk averser in a bargaining game; and the one is the highest when one risk seeker encounters another risk seeker. Moreover, comparing the “- × -” type of line with the “- * -” type of one and the “- * -” type of line with the “- · -” type of one, we can see that if a bargainer chooses to be a risk seeker, no matter his opponent is a risk seeker or a risk averser, the bargaining will cost more time and the bargainer will get fewer demands than when he chooses to be risk averse.

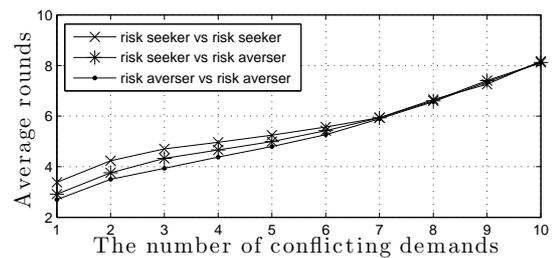


Figure 1: Average rounds of reaching agreements with the number of conflicting demands about effect of risk degree.

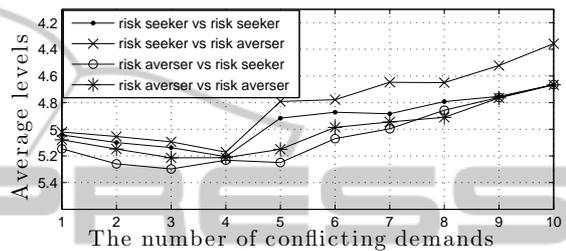


Figure 2: The average preference levels of remaining demands in the first bargainer's outcome with the number of conflicting demands for different risk attitudes.

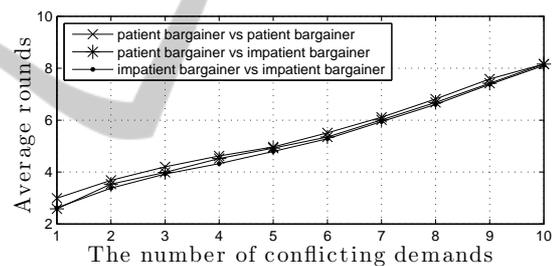


Figure 3: Average rounds of reaching agreements with the number of conflicting demands about effect of patience descent degree.

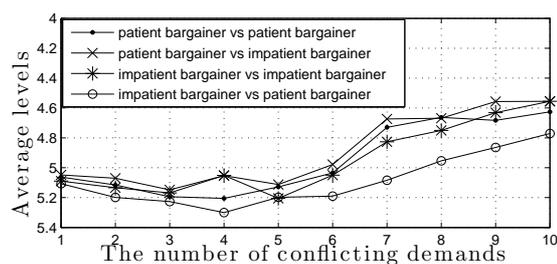


Figure 4: Average preference levels of remaining demands in the first bargainer's outcome with the number of conflicting demands about effect of patience descent degree.

In the second experiment, we also model the cases similar to the first experiment, but the average preference levels of remaining demands in each bargainer's outcome are different. So, we carry out four cases as showed in Figure 2, and just draw the first bargainer's situation. From Figure 2, comparing the “- · -” type

of line with the “-o-” type of one and the “-x-” type of line with the “-*” type of one, we can see that if a bargainer is risk seeking, no matter his opponent is risk seeking or averse, his average preference levels of remaining demands is higher than that when choosing to be risk averse. That is, a risk seeker can gain more demands that he prefers.

Accordingly, we can conclude a risk seeking bargainer can gain fewer but more favorite demands than a risk averse one in the fuzzy logic based model. This often happens in real life. For example, in stock markets, a high income often comes with a high risk.

Now we turn to analyse the influence of patience descent degree and regret degree by doing other two groups of experiments. Each contains two experiments similar to those ones in the previous subsection.

Figures 3 and 4 show the influence of the patience descent degree, while Figures 5 and 6 show the effect of regret degree. Similarly to the analyses in the last subsection, from the four figures, we can conclude that a patient bargainer can gain more favourite demands than an impatient one, and a difficult-regretting one gains more favourite demands than an easy-regretting one. However, as showed in Figures 3 and 5, both parameters cannot alone influence the average rounds of reaching agreements obviously, but both are positive correlated with the change degree. So, we do another group of experiments to see how the two influence the bargaining together. And the data is shown in Figures 7 and 8. Comparing Figures 3, 5 and 7, we can see that the two can together influence the outcome of bargaining more obviously than single one does.

5 BENCHMARK WITH SCS

This section empirically analyses how well the fuzzy logic based model and its solution concept (i.e., DSCS) works against the one of Zhang (2010) (i.e., SCS). We will also carry out two groups of experiments to analyse how the outcome qualities change with the numbers of conflicting demands and bargainers, respectively. In addition to success rate, average rounds, and average level in outcome, we will introduce four more indexes to evaluate an outcome of a bargaining game: the number of demands in agreement, the number of consistent demands in agreement, and the highest and the lowest levels of demands in agreement. In both experiments, we run 1000 times bargaining under the setting that every bargainer’s action function is formula (1) and the fuzzy rules are those in Table 1.

In the first experiment, 10 demands are randomly

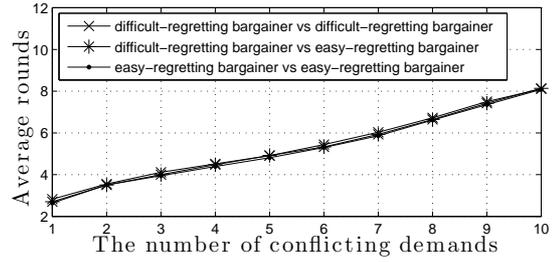


Figure 5: Average rounds of reaching agreements with the number of conflicting demands about effect of regret degree.

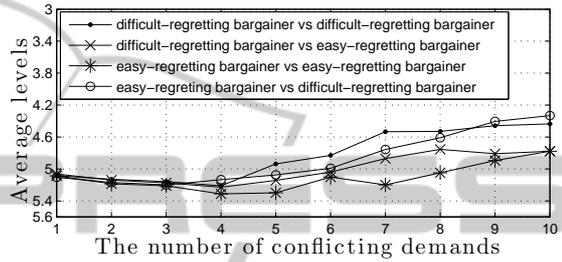


Figure 6: Average preference levels of the demands in bargainer 1's outcome with the number of conflicting demands about effect of regret degree.

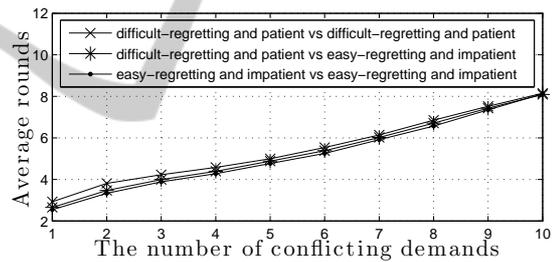


Figure 7: Average rounds of reaching agreements with the number of conflicting demands.

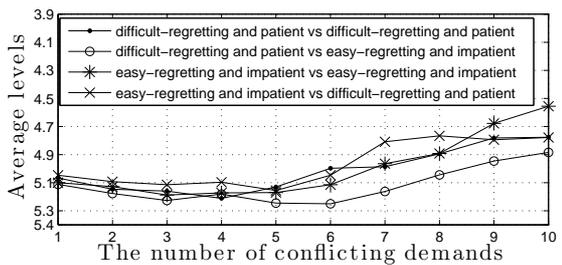


Figure 8: Average preference levels of the demands in bargainer 1's outcome with the number of conflicting demands.

put in different preference levels for two bargainers and arbitrarily label $N \in [0, 10]$ of them as their conflicting demands. Figure 9 shows that the success rate of DSCS is higher than that of SCS, especially when the conflicting demands are increasing, such as when the number of conflicting demands is 8, the success rate of our model is about 10% higher. Figure 10

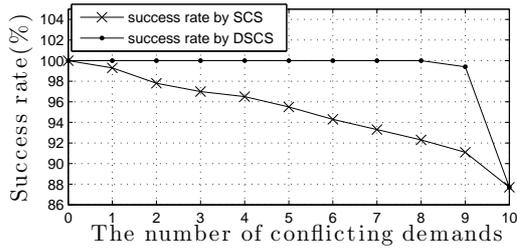


Figure 9: Success rate with the number of conflicting demands.

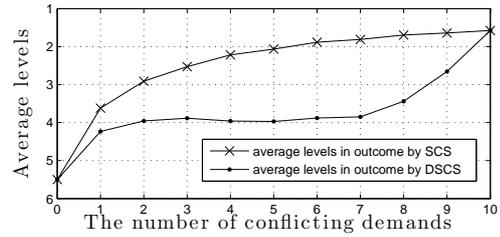


Figure 13: Average preference levels of the demands in the first bargainer's outcome with the number of conflicting demands.

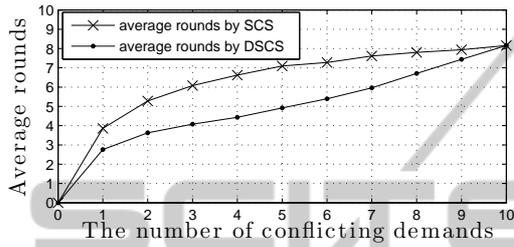


Figure 10: Average rounds of reaching agreements with the number of conflicting demands.

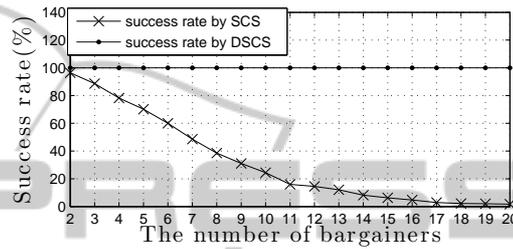


Figure 14: Success rate with the number of bargainers.

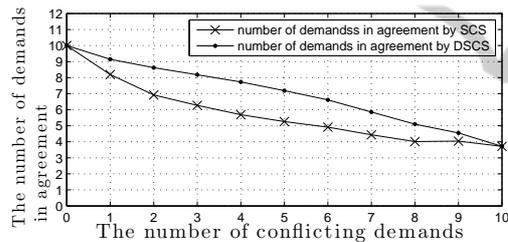


Figure 11: The number of demands in agreement with the number of conflicting demands.

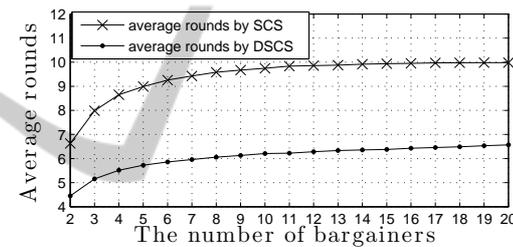


Figure 15: Average rounds of reaching agreements with the number of bargainers.

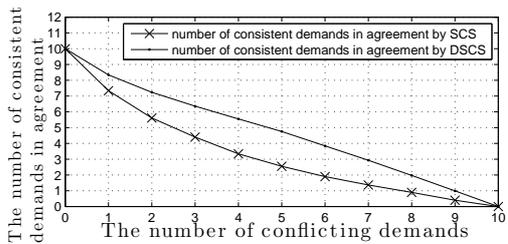


Figure 12: The number of consistent demands in agreement with the number of conflicting demands.

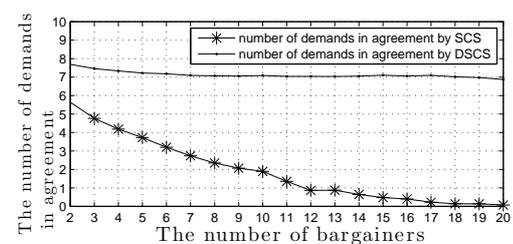


Figure 16: The number of demands in agreement with the number of bargainers.

shows that in DSCS the average rounds of reaching agreements are about two rounds less than than of SCS. Figures 11 and 12 show that in DSCS both the number of demands in agreement and the number of consistent demands in agreement are larger. Figures 13, 19 and 20 show that when the number of conflicting demands increase, the average/the highest/the lowest preference level in a bargainer's outcome in DSCS will be lower than that of SCS.

In the second experiment, we randomly generate

10 demands in different preference levels for M bargainers (inbetween 2 and 20) and arbitrarily select 4 of them as the conflicting demands of all the bargainers. The bargaining will proceed in both models. Figure 14 shows that DSCS can keep a high success rate of bargaining even when the number of bargainers increases, while the success rate will decrease obviously with SCS. Figure 15 shows that DSCS can also keep lower rounds of reaching agreements than SCS. Moreover, Figures 16 and 17 show that more con-

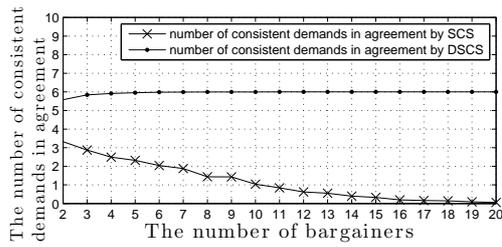


Figure 17: The number of consistent demands in agreement with the number of bargainers.

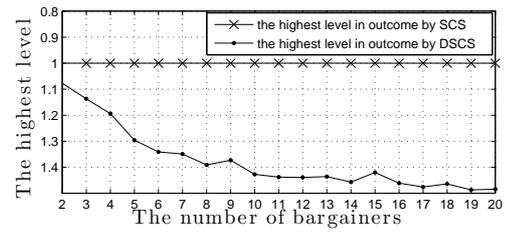


Figure 21: The highest level of the demands in the first bargainer's outcome with the number of bargainers.

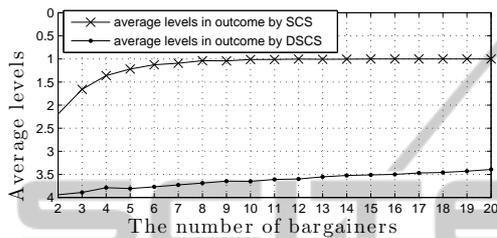


Figure 18: Average preference levels of the demands in the first bargainer's outcome with the number of bargainers.

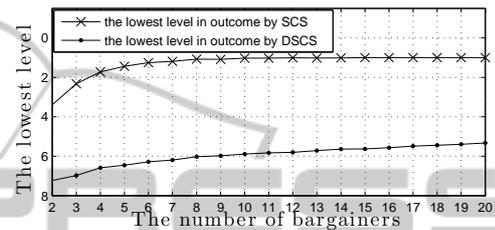


Figure 22: The lowest level of the demands in the first bargainer's outcome with the number of bargainers.

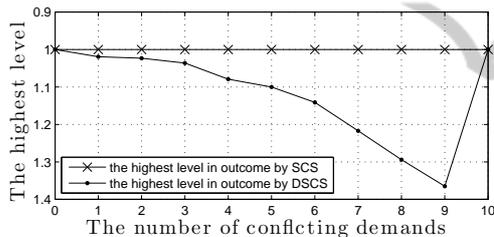


Figure 19: The highest level of the demands in the first bargainer's outcome with the number of conflicting demands.

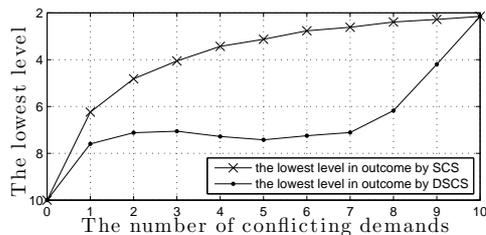


Figure 20: The lowest level of remaining demands in the first bargainer's outcome with the number of conflicting demands.

sistent demands can be saved in the final agreement even when the bargainers increase in DSCS. Figures 18, 21 and 22 show that when the number of bargainers increase, the average/the highest/the lowest preference level in a bargainer's outcome in DSCS will be lower than that of SCS.

Although the average levels of demands are a little lower than SCS, even when the number of conflicting demands or bargainers increases, DSCS can still re-

flect bargainers' cognitive factors of risk, regret, patience, keep a high success rate and a high efficiency, and get more consistent demands in an agreement.

6 AN INVESTMENT PROBLEM

This section illustrates our model by solving the bargaining problem of the real estate investment between two investors. Investor 1 wants building large-scale apartments (LA), using environmental but expensive material (EEM), expanding the green area (GA), building artificial lake (AL), fitment outsourcing (FO), building a big club house (CH), opening communal facilities to the public (OP), property management outsourcing (PMO). Investor 2 wants EEM, GA, FO and OP; but opposes LA, AL, CH and PMO. Thus, their demand sets are:

$$X_1 = \{EEM, GA, LA, FO, AL, CH, PMO, OP\},$$

$$X_2 = \{\neg PMO, \neg LA, EEM, \neg CH, GA, \neg AL, FO, OP\}.$$

Table 2 shows two investors's original preferences over their own demands, which just reflect their own favorites rather than the other side's situation. However, when going to the bargaining, they will worry about their conflicting demands and thus adjust the preferences to form initial dynamic ones, hoping to reach an agreement more easily meanwhile keep their demands as many as possible. In this example, Investor 1 demands LA but Investor 2 demands $\neg LA$, which is a contradiction. Similarly, we can get their

Table 2: Original and dynamic preferences

| Rank | Investor 1 | | Investor 2 | |
|------|------------|---------|------------|---------|
| | original | dynamic | original | dynamic |
| 1 | EEM | EEM | ¬PMO | ¬PMO |
| 2 | GA | LA | ¬LA | EEM |
| 3 | LA | AL | EEM | ¬LA |
| 4 | FO | GA | ¬GH | GA |
| 5 | AL | CH | GA | ¬CH |
| 6 | CH | FO | ¬AL | FO |
| 7 | PMO | PMO | FO | OP |
| 8 | OP | OP | OP | ¬AL |

Table 3: Dynamic bargaining proceeding.

| Round | Rank | Investor 1 | | Investor 2 | |
|---------|------|------------|---------|------------|---------|
| | | original | dynamic | original | dynamic |
| Round 1 | 1 | EEM | ¬PMO | EEM | EEM |
| | 2 | LA | EEM | GA | ¬PMO |
| | 3 | AL | ¬LA | LA | GA |
| | 4 | GA | GA | AL | ¬LA |
| | 5 | CH | ¬CH | FO | FO |
| | 6 | FO | FO | CH | ¬CH |
| | 7 | PMO | OP | PMO | OP |
| Round 2 | 1 | EEM | EEM | EEM | EEM |
| | 2 | GA | ¬PMO | GA | GA |
| | 3 | LA | GA | FO | ¬PMO |
| | 4 | AL | ¬LA | LA | FO |
| | 5 | FO | FO | AL | ¬LA |
| | 6 | CH | ¬CH | CH | ¬CH |
| Round 3 | 1 | EEM | EEM | EEM | EEM |
| | 2 | GA | GA | GA | GA |
| | 3 | FO | ¬PMO | FO | FO |
| | 4 | LA | FO | LA | ¬PMO |
| | 5 | AL | ¬LA | AL | ¬LA |
| Round 4 | 1 | EEM | EEM | EEM | EEM |
| | 2 | GA | GA | GA | GA |
| | 3 | FO | FO | FO | FO |
| | 4 | LA | ¬PMO | LA | ¬PMO |

conflicting demand sets: $CDS_1 = \{LA, AL, CH, PMO\}$ and $CDS_2 = \{¬LA, ¬AL, ¬CH, ¬PMO\}$.

From Table 2, by formula (6), we can obtain two investors' risk degrees $\gamma_1 = 0.364$ and $\gamma_2 = -0.267$. Investor 1 is risk-seeking because he moves up his conflicting demands *LA*, *AL*, *CH* and *PMO* from the original preference to the initial dynamic one. Rather, Investor 2 is risk-averse because he downgrades the conflicting demand *¬PMO*, *¬LA*, *¬CH* and *¬LA*.

Now we show how our model solves it. During the bargaining, the changes of preference and parameters are shown in Tables 3 and 4, respectively. There are two steps in the first round of bargaining. Firstly, as shown in Table 2, there are some contradictions in two investors' demands, so both give up the demands in the lowest level in their dynamic preferences, that is *OP* of investor 1 and *¬AL* of investor 2. Then, the model will be updated into a new one shown in the left table in the first row (denoted as Round 1). Secondly, by the parameters' calculation functions (4),

Table 4: Parameters.

| Parameters | Round 1 | Round 2 | Round 3 | Round 4 |
|------------------------------|----------------|----------------|----------------|----------------|
| $(\vartheta_1, \vartheta_2)$ | (0.25,0) | (0.25,0.25) | (0.25,0.25) | (0.25,0.25) |
| (ρ_1, ρ_2) | (0.125,0.125) | (0.25,0.25) | (0.375,0.375) | (0.5,0.5) |
| (γ_1, γ_2) | (0.364,-0.267) | (0.364,-0.267) | (0.364,-0.267) | (0.364,-0.267) |
| (ζ_1, ζ_2) | (0.31,0.46) | (0.34,0.46) | (0.37,0.47) | (0.38,0.47) |

(5) and (6), we can obtain $\vartheta_1 = 0.25$, $\rho_1 = 0.125$, $\gamma_1 = 0.364$, $\vartheta_2 = 0$, $\rho_2 = 0.125$, and $\gamma_2 = -0.267$, respectively. Thus, by fuzzy rules in Table 1, based on Mamdani method (see Definition 4), we can obtain $\zeta_1 = 0.322$ and $\zeta_2 = 0.376$ in this round. Then, by their action function (formula (1)), their initial dynamic preferences are updated into new ones as shown in the right table in the first row (denoted as Round 1*). According to the second choice of action function (formula (1)), *LA*, *AL*, *CH*, *PMO* of investor 1 and *¬LA*, *¬AL*, *¬CH* of investor 2 are declined. Similarly, we can understand the rest of rounds similarly. The game ends after the 4th round because two investors have nothing in contradictory.

From Table 3, we can see that by the dynamically simultaneous concession method (see Definition 2), the outcome of the game is: $S_1(G) = \{EEM, GA, FO, LA\}$ and $S_2(G) = \{EEM, GA, FO, ¬PMO\}$. So, their agreement is: $S_1(G) \cup S_2(G) = \{EEM, GA, FO, LA, ¬PMO\}$.

7 RELATED WORK

Like Zhang (2010), Bao and Li (2012) also build an axiomatic bargaining model, in which the preference over outcomes is ordinal. However, unlike the model of Zhan et al. (2013), their model does not reflect the bargainers' risk attitudes and patience, which are very important factors for bargaining in real life. Moreover, they did not conduct any simulation experiment to analyse their model, but we do in this paper.

In (Kolomvatsos et al., 2012), a fuzzy logic based model is also introduced for a buyer to decide to accept or reject a seller's offer according to the proposed price, the belief about the seller's deadline, the demand relevancies, and so on. They also do a lot of simulation experiments to show their model's capability, but did not show how the risk attitudes change the bargainers' preferences like what we did.

In the bilateral negotiation model of Zuo and Sun (2009), fuzzy logic is used for offering evaluation. Moreover, they distinguish three attitudes of bargainers in concession: greedy, anxious and calm. They also test how different concession strategies influence agreements. However, they did not compare their solution with the others like what we do in this paper.

8 CONCLUSIONS

This paper improves the fuzzy logic based bargaining model of Zhan et al. (2013). Moreover, through empirical analysis we figure out how human psychological characteristics about risk, patience and regret influence the outcome of a bargaining; and show how the fuzzy logic based model outperforms the model of Zhang (2010) in terms of success rate and agreement reaching efficiency. In addition, we use our model to solve a bargaining problem of estate investment problem. Many could be done in the future. For example, it is interesting to integrate more human psychological characteristics into our model to solve certain problems, and carry out more theoretic and empirical analyses on the extended model.

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