

Multi-objective Optimization of Investment Strategies Based on Evolutionary Computation Techniques, in Volatile Environments

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Keywords: Multi-objective Optimization, Stock Market Forecast, Technical Analysis, Financial Markets, Moving Average, Time Series Prediction.

Abstract: In this document, the use of a multi-objective evolutionary system to optimize an investment strategy based on the use of Moving Averages is proposed to be used on stock markets, able to yield high returns at minimal risk. Fair and established metrics are used to both evaluate the return and the risk of the optimized strategies. The Pareto Fronts obtained with the training data during the experiments conducted outperform both B&H strategy and the classical approaches that consider solely the absolute return. Additionally, the PF obtained show the inherent trade-off between risk and returns. The experimental results are evaluated using data coming from the principal world markets, namely, the main stock indexes of the most developed economies, such as: NASDAQ, S&P500, FTSE100, DAX30 and NIKKEI225. Although, the experimental results suggest that the positive connection between the gains with training and testing data, usually assumed in the single-objective proposals, is not necessarily true for all cases.

1 INTRODUCTION

Besides some unfavourable judgments (Korczak et al., 2002), Technical Indicators (TI) are still widely used as tools to do the technical analysis of financial markets, exploiting the existence of trends to establish potential buy, sell or hold conditions. This study is notoriously tricky for a number of reasons, though (Achelis, 2000) has made a complete reference that fully explains the most important TI's one can identify and use. Anyway, the main difficulty of TI usage is still deciding its suitable parameter values, as number of days of periods, and this, in order to take advantage of the market and improve your likelihood of success.

Thus, evolutionary computation appears as a highly suitable alternative to extend technical analysis of financial markets to tune the parameters of some chosen TI (or set of TI's), so that, the desired goals are achieved, at maximum extent possible. In this environment, what the system should do, can be viewed as some kind of predicting future stock prices. Consequently, in this context, evolutionary computation emerges as a stochastic search technique able to deal with highly complicated and non-linear search spaces.

In the last decade, several financial crises have

occurred with large consequences on the valorisation of financial assets. Therefore emerges the principal motivation for this paper: tune an Investment or Trading Strategy (TS) able to achieve both the highest returns with the minimal risk.

One of the goals of this work is to tune a TS to present the highest returns as existing single objective based approaches, and concurrently reduce the risk. The proposed framework is tested using data from the main stock indexes of the most developed economies, such as NASDAQ, S&P500, FTSE100, DAX30 and NIKKEI225; then the results are presented, and some possible conclusions outlined.

The next section will present the related work using GA and the various TS's currently used in Technical Analyses. In Section 3 the methodology, the roles of the most relevant modules used to build the proposed framework, and the chromosome encoding are outlined. The TS adopted in this study and the metrics used to evaluate the evolved TS are also presented in this section. Section 4 presents the results and the most relevant outcomes are highlighted. Finally, in section five, the conclusions of this study are presented.

2 RELATED WORK

Stock market analysis has been one of the most attractive and active research fields, where many Machine Learning techniques have been used. Generally speaking, one can distinguish two methods for anticipating future stock prices and the time to buy or sell; one is Technical Analysis (Murphy, 1999) and the other is Fundamental Analysis (Graham et al., 2003). Fundamental Analysis look at stock prices using financial statement of each company, economic trend and so on; requires a large set of financial and accounting data, difficult to obtain and both released with some delay and often suffers of low consistency. Technical Analysis numerically analyzes the past movement of stock prices, is based on the use of technical stock market indicators that work on a series of data, usually stock prices or volume, (Achelis, 2000) is accurate, on time, and relativity easy to obtain. Consequently, this work will be focused on the use of Technical Analysis to anticipate future stock price movements.

Many approaches based on evolutionary computation have been proposed and applied to diverse fields of financial to predict worth trends. In an attempt to summarize, in most of the works, the generated returns are exclusively used as the only fitness metric, without accounting for the related risk. Some examples are the use of GAs to optimize TI's parameters (Fernández-Blanco et al., 2008), or to develop TS based on TI's (Bodas-Sagi et al., 2009), (Gorgulho et al., 2011).

According to what was stated for the first time in 1952 (Markowitz, 1952), any TS should have the highest possible profit with the feasible minimal risk. Sadly, these two metrics are intrinsically conflicting by virtue of the risk-returns trade-off. Some articles propose the combination of the two conflicting objectives into one single metric, in particular (Bodas-Sagi & al., 2009) use the Chicago Board Options Exchange (CBOE) Volatility Index (VIX) as an estimate of risk. Also, (Schoreels & al., 2006) propose the use of a Capital Asset Pricing Model (CAPM) (William, 1964) system, based on portfolio theory (Markowitz, 1952) to reduce risk through balanced selection of securities. More recently (Pinto et al., 2011) propose and study several alternatives to the classical fitness evaluation functions.

A Multi-Objective system to maximize the total returns and to minimize the risk as the exposure to it is proposed by (Chiam et al., 2009). The framework is tested using data gotten from one stock market,

the Singapore Exchange stock market (Straits Times Index (STI)). Hence, some of the conclusions drawn on this study could be attributed to the market used to test it. Moreover, the metric used to evaluate the return is peculiarly unusual; so, it is difficult, to compare the presented results with the results presented by other alternative applications.

3 METHODOLOGY

The proposed system consists of a Multi Objective Genetic Algorithm coupled with a market return evaluation module that does the fitness evaluation, and this, based on the estimation of the two conflicting objectives, on the chosen market, and on the specified period.

3.1 Strategy and Parameters

The strategy tested on this work was the Moving Average Crossover (MAC), which is based on the use of two Moving Averages (MA), with different periods. One, formed by the MA with the shorter of the two periods is called the "Fast MA", and the other, with the longer period is the "Slow MA". The "Fast MA" reflects changes earlier than does the "Slow MA". A buying (or sell short) signal is generated when the Fast MA crosses over the Slow MA. Conversely, sell (or a buy short) signal is generated when the Fast MA crosses under the Slow MA.

After defining the strategy, it is necessary to define the parameters of the MAC, which in the case are the type of the MA's and the corresponding period. It is important to stress that, for the type of MA to use, the GA has also the freedom to choose between a Simple or an Exponential MA.

Although it is common to tune the parameters of one single TI and then use it to generate buy and sell signals, for both long and short positions, in this article, the option of using a separate set of parameters for each of the possible actions was taken; to specify: "enter long"; "exit long"; "enter short"; and "exit short".

Some pre-processing of the historical data is also done. This applies for instance to the MA periods, which are calculated at program start and are limited to the following set of Simple or Exponential MA's: 1, 4, 8, 12, 14, 16, 20, 24, 28, 32, 36, 40, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200 and 250 days. This set of periods has been chosen because it covers the most widely used, long and short-term MA periods, found

on books and recommended by experts (Achelis, 2000).

3.2 Genetic Encoding

The chromosome must represent the MAC indicator used, this way one MAC chromosome is represented by two genes: one represents the type and the period of the Fast MA and the other does the same for the Slow. These entries are natural numbers in the interval of values between 0 and 65 as it encodes, in one single entry (integer variable) the type of MA and its period. In Table 1 is represented the chromosome structure.

Table 1: Chromosome representation.

Parameters	Enter long position		Exit long position		Exit short position		Enter short position	
	Fast MA	Slow MA	Fast MA	Slow MA	Fast MA	Slow MA	Fast MA	Slow MA
Chromosome	0..65	0..65	0..65	0..65	0..65	0..65	0..65	0..65

3.3 Fitness Evaluation

The fitness evaluation process is concerned with simulating the performance of the each trading agent in the evolving population and calculating the corresponding total returns and the related risk. The resultant fitness values of the trading agent must be evaluated under some established and fair metric, as will be discussed in the next subsections.

3.3.1 Return Metric

The profits generated by a given TS can be measured in different ways, as will be seen next: For instance, the potential profits can be estimated by simply summing the area under the total asset graph during the trading period (Schoreels et al., 2005). Alternatively, another return metric could be the final (total) assets; this means the available capital plus the value of all holdings, at the end of the investment period (Kendall et al., 2003). Sadly, both above metrics have the nuisance that they are always attached to the initial cash invested.

Therefore, an alternative metrics exists that considers its relative value and is known as Return on Investment (ROI). This metric is a ratio and represents the money gained or lost on an investment relative to the amount of money invested. ROI is usually expressed as a percentage, and for one period, by definition, is calculated according with equation 1. "Profit" is the amount of money gained or lost and "Initial_Investment" is the money invested.

$$\begin{aligned}
 ROI &= \frac{Profit}{Initial_Investment} \\
 &= \frac{Final_Assets - Initial_Investment}{Initial_Investment} \quad (1) \\
 &= \frac{Final_Assets}{Initial_Investment} - 1
 \end{aligned}$$

ROI still has the trouble that, for multi-period investment, it is difficult to compare it with the results one would get in one single period. Therefore, a metric that could be compared with similar alternative investments should be used instead. This way, in this article, the Annualized ROI, will be used. The Annualized ROI is nothing more than the "Geometric Average of the Ratio of the Returns" also known as the "True Time-Weighted Rate of Returns". Mathematically, for an investment lasting for N periods, with full reinvestment, is computed as exposed in equation 2; in this equation, N is the number of periods, more exactly, the number of years, the investment lasts.

$$Annualized(ROI) = \sqrt[N]{(ROI + 1)} - 1 \quad (2)$$

3.3.2 Risk Metrics

Risk is usually seen as the volatility or the uncertainty of the expected returns over the investment period. Therefore, the linked risk of any investment technique can be estimated in several ways, as will be examined subsequently.

The most traditional risk metric is inherited from statistics and from Markowitz Mean-Variance Model (Markowitz, 1952), and consists in the use of the variance of the results as a gauge for the risk. This variance can be calculated using the standard deviation or the variance between the returns, this statistical measure of the dispersion of the results is usually named, in finance, as volatility.

Instead, risk can be computed as the exposure to it (Weissman, 2005). Specifically, it can be measured by the proportion of trading days when a position is maintained open on the market, and is, mathematically, the ratio between the time the agent is on the market and the total trading time available. Essentially, staying longer in the market corresponds to a higher exposure to risk, like market crashes and other disastrous events, while shorter periods on the market correspond to a lower risk exposure and greater liquidity (as the capital is engaged for a smaller time).

Alternative metrics for the risk can be found on the literature, as, for instance, the use of some risk-adjusted return metric, as the Sharpe ratio, Sortino ratio, Sterling ratio (SR), Calmar ratio (CR) or also

VIX which compute the net profitability after discounting the associated risk (Korczak et al., 2004). In short, the preceding risk metrics are in reality alternative methods to combine into one single objective (or metric) the two conflicting objectives faced on this kind of problems (risk and return).

Therefore, in the remaining of this paper, the risk exposure will be used as the risk metric.

3.4 Optimisation Kernel

This study is concerned with the Evolutionary Optimization of a TS treated as a multi-objective problem, so the Optimisation Kernel is based on a version of a state of the art multi-objective evolutionary algorithm: Non Dominated Sorting Genetic Algorithm 2 (NSGAI) (Deb et al, 2002). NSGAI parameters are as follows: population size 500, the crossover probability fixed to 0.8 and parents selected by tournament selection. Each run on training data continued for 300 generations and the probability of real mutation set to 0.1.

3.5 The Investment Simulator

The Investment Simulator or Market Return Evaluation Module simulates an investment in the user specified index including long and short positions. Stock market index, which it could buy (“go long”), sell it and stay out of the market (“Out”) or even sell if it didn’t own any (“go short”) hoping to profit from a decline in the price of the assets between the sale and the repurchase.

Since daily data was available, the training consisted in formulating an TS, give to the agent some initial cash to spend, and every day simulate the performance of the agent; having it to buy or sell (“long” or “short”) the total cash available, if the conditions defined on its encoded strategy are met.

Transaction costs were not included in the simulation, as dividends not too. Environment is also assumed discrete and deterministic in a liquid market.

4 RESULTS

A multi-objective evolutionary optimization of a TS is studied in this essay what involves the maximization of a Return Metric and the minimization of the related Risk Metric. In this kind of problems the optimal solutions exist in the form of a set of tradeoffs known as the Pareto-optimal set

(PF); and any objective belonging to a solution in the optimal set cannot be improved without degrading at least one of the others objectives.

An example of a possible PF is illustrated in Figure 1, and this represents clearly the risk-return trade-off or Efficient Frontier always faced in this kind of problems.

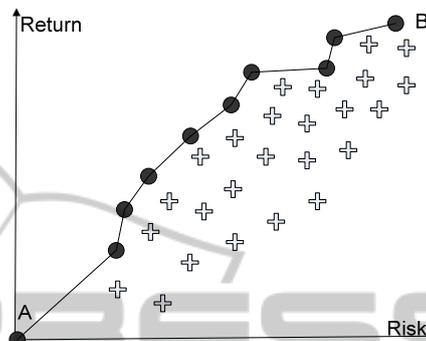


Figure 1: Risk Return Trade-off.

On this illustration, each point denotes a Strategy evolved by the GA. The black circles and the white crosses represent non-dominated and dominated solutions respectively. The set formed by the former solutions is the Pareto optimal solution set because their returns cannot be improved further without compromising risk. In the context of single objective optimization where return is the only goal, the evolutionary process will ultimately drive the solutions towards the extreme point B. This is not applicable to conservative investors, who may prefer a lower risk at a cost of lower returns. Point A represents the extreme case of a conservative investor with zero returns due to his total risk adversity.

4.1 Training and Testing Data Sets

The system was tested using historical daily prices from the stock indexes: S&P 500, FTSE 100, DAX 30, NIKKEI 225 and NASDAQ.

The period of time chosen for training was from 3 Jan. 2000 to 31 Dec. 2007. This time period was assumed sufficient to evolve a competitive population as it exhibited significant movement, including several boom and crash periods. For out of sample and testing period, two years of data was used, and it was from 2 Jan. 2008 to 31 Dec. 2009.

4.2 Analysis of the Training Performance

Figure 2 present the PF's evolved for the 5 indexes tested in this study, in one of the experimental runs performed. Though the various solutions sets vary in terms of Pareto dominance and optimality, all clearly illustrate the inherent trade-off between return and risk. Furthermore, the trading agents evolved are able to generate high returns in open positions less than 100% of the trading period, for instance, the observable annualized ROI near or above 10% with risk exposure around 0.6.

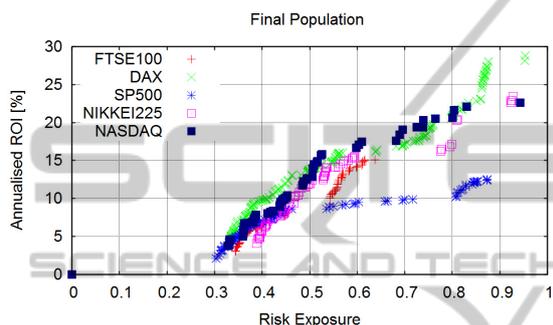


Figure 2: Evolved Pareto Fronts for the five Indexes Tested.

In Financial Computing when analyzing the performance of a given TS, it is common to compare it against the “Buy & Hold” (B&H) and “Sell & Hold” (S&H) strategies. When the ROI performance of the evolved TS (see figure 2) is compared against both B&H and S&H approaches (see B&H and S&H annualised ROI calculation on Table 2), during the training period, it is easy to conclude that, in this context, both B&H and S&H strategies are undoubtedly suboptimal. It is also important to remind that both, B&H and S&H, strategies correspond to a risk exposure of 1 (one); since the capital is all time engaged.

In Figure 3 is presented an example of the eight-year financial data used to optimize the strategy, in the current case is the FTSE100 index. The line labelled “Buy & Hold” characterizes the performance of the B&H strategy; this same line is coincident with the current index evaluation at close price. On this same illustration, the performance of the S&H strategy is exposed by the curve tagged “Sell & Hold”. An example of the trading performance of one of the optimized strategies is also shown on this figure, by the line labelled “Trained Chromosome”. On the same illustration the X axis is the time, and on the Y axis is the assets evaluation.

Table 2: Annualized ROI for B&H and S&H strategies in the training period.

	NIKKEI 225	FTSE 100	S&P500	DAX30	NASDAQ
B&H Absolute Return	-3695.08	- 206.00	13.14	1316.56	-1478.87
B&H ROI [%]	- 19.44%	- 3.09%	0.90%	19.50%	- 35.80%
B&H Annualized ROI [%]	-2.67%	-0.39%	0.11%	2.25%	-5.39%
S&H Absolute Return	3695.08	206.00	-13.14	-1316.56	1478.87
S&H ROI [%]	19.44%	3.09%	- 0.90%	- 19.50%	35.80%
S&H Annualized ROI [%]	2.25%	0.38%	-0.11%	-2.67%	3.90%

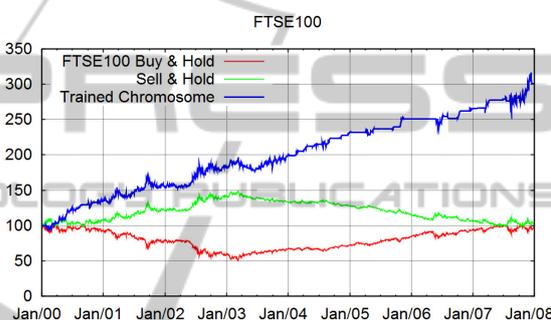


Figure 3: Example of daily closing prices and the performance of one trained agent, for FTSE100 index, in the training period.

In order to have a better insight about the data and results, 30 (thirty) experimental runs were performed, the results collected, and then, discrete intervals of 0.1 of risk exposure considered. With this data, plots like the one shown in Figure 4 were gotten.

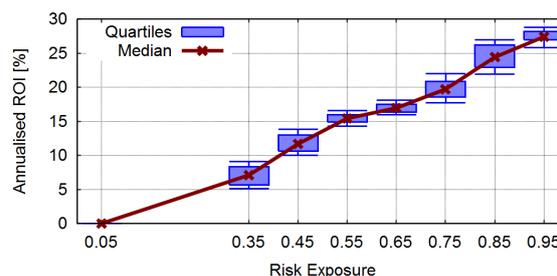


Figure 4: Annualised ROI in discrete intervals of 0.1 Risk Exposure, observed with DAX index.

Figure 4 plots an example of the observed distribution of the Annualized ROI in function of the risk exposure. This illustration shows the First Quartile of data (Q1), the Third Quartile of data (Q3), as well the Median, with the whiskers located

respectively at 10% and 90% of data, and this for the results observed, with training data. Again, the risk-returns trade-off is evident, since the average of the Annualized ROI increases for higher levels of risk exposure. The lack of solutions in the risk exposure range of 0.1 to 0.2 can be due to the difficulty in optimizing the chosen TI to exploit the price movements in order to create strategies in this region. Similar plots, identical the one shown, were also observed for the further indexes also tested in this study.

4.3 Correlation Analysis of Training and Testing Performance

The results presented in the previous subsection showed that it is possible to tune a TS to attain attractive returns at various levels of risk exposure. Despite this, the great effectiveness of any approach will depend on being able to extend these interesting returns to unseen data, which is usually recognized as its generalization performance.

In order to evaluate the engine generalization performance, the available trading data is portioned into two independent sets of data, this means: training and testing data sets, as explained in subsection 4.1. In the training phase of the evolutionary process, the TS will be trained, tuned and evaluated using only training data. After being trained, the developed strategies obtained in the final generation will be then applied to the testing data set and its generalization performance is evaluated. This is an indicator of the framework real effectiveness in getting good results using unseen data.

The plot of the risk-returns PF's for the training data gotten in one of the experimental run is presented in Figure 5. The marks labelled "Pop_Train" represent the final population evolved after 400 generations, while the points tagged "Pop_Tst" represent the results of this same population when applied to the testing data set.

The example shown on Figure 5 is for the NIKKEI index, but similar plots were observed with the further indexes also tested.

Again, in this plot, the risk-returns trade-off is evident with the training data. However, such correlation disappears when the same strategy is applied to the testing data. For instance, annualized ROI of 15% are realizable at a risk level of 0.6 with the training data, while big losses are gotten at the same level of risk with the testing data. This low relation between training and testing results was also observed in previous studies (Korczak et al., 2004), (Chiam et al, 2009).

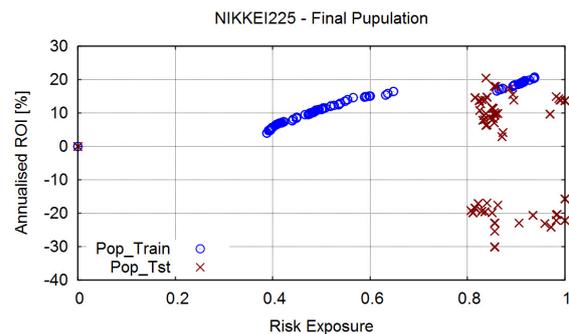


Figure 5: Pareto Fronts observed with training and testing data.

The most evident conclusion from this figure is that positive returns with the training data do not necessarily match positive returns with the testing data.

Hence, it urges the need to better understand how the training and testing data correlate together, in order to examine the generalization performance of the evolved TS's. This suggests that a correlation analysis between the four variables involved should be conducted; to name: training ROI, training risk, testing ROI and testing risk.

To better clarify the results, 30 independent experimental runs were performed and with the results observed in these experimental runs, the graphs shown in Figure 6 where build. On this graphs the variables are plotted and its potential correlations can be visually inspected. Once more, the plot of training ROI and training risk accurately shows the risk-returns trade-off. While an almost random plot is obtained when the testing returns against the testing risk are plotted, therefore this suggests the existence of low correlation between training ROI and testing ROI.

Contrasting to the traditional theory in single-objective approaches where higher training returns are coupled with higher testing returns, this relationship is missing from these plots. Instead, higher training returns correspond to increased volatility in the observed testing returns; this is clearly observable in the graph of Figure 7. This figure plots the quartiles of data (Q1-Q3), the median, as well the whiskers located respectively at 10 and 90% of the observed results, when the training returns are divided in discrete intervals of 5%. On this figure the median of the testing returns does not boost when the values of training returns increase. In its place, there is a visible increase in the variance of the results, denoted by largest vertical lines (both whiskers and boxes).

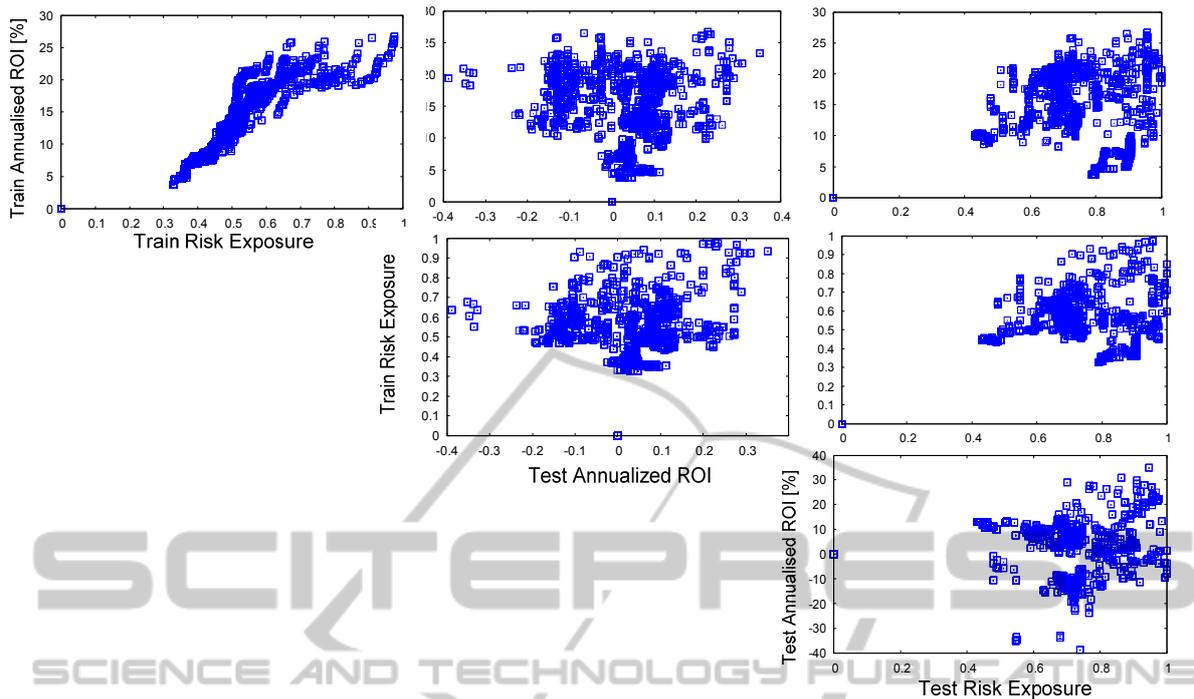


Figure 6: Plots showing the correlation between training returns, training risk, testing returns and testing risk.

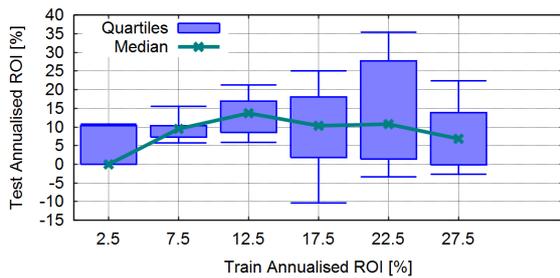


Figure 7: Statistical distribution of testing returns at discrete intervals of the training returns for DAX index.

In conclusion, the positive correlation typically implicit in conventional single-objective approaches, to do the optimization of TS's, between training and testing returns, is not necessarily true for all cases.

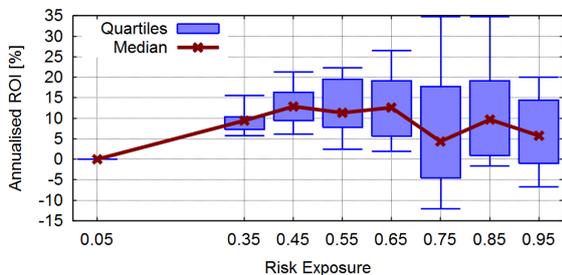


Figure 8: Distribution of testing returns at discrete intervals of training risk for DAX index.

Similar conclusions can be extracted from the plot shown in Figure 8 where the testing returns observed in the 30 independent runs are resumed at discrete intervals of 0.1 training risk. Again, the median of the testing returns does not increase when the training risk increases.

Although, a steady increase is clearly observable in the variance of the test returns is clear from the plots (Figures 8 and 9), what confirms the claim that higher training returns correspond to increased volatility in the test returns results.

Figure 9 shows the number of solutions gotten in each interval of test risk exposure (scale at left) together with the Std. Dev. gotten with both the training data and testing data (scale at right). The apparent drop in the test results volatility for risk

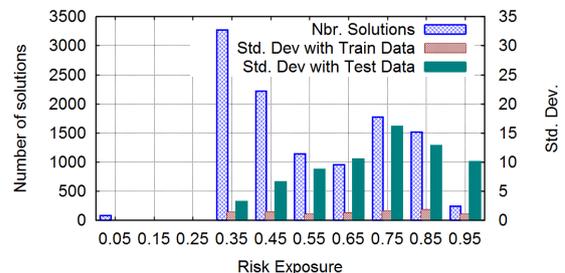


Figure 9: Number of Solutions and Standard deviation of the testing returns at discrete intervals of 0.1 risk exposure for DAX Index.

level above 0.8 is statistically irrelevant as there are few solutions in this region. The plots presented were built with the DAX results, but similar plots were also observed for the remaining four indexes also tested in this study.

4 CONCLUSIONS

This document presented and investigated a multi-objective evolutionary approach to do the optimization of a set of TS's. In this work, fair and established metrics were used to both evaluate the return and the related risk. Both metrics were simultaneously optimized and a popular TI frequently used by real-world professionals was used as the building block of the core strategy. Furthermore, the TS's were trained, and afterwards tested, using data coming from five main stock indexes, representative of the world most developed economies. The PF's obtained by the algorithm using testing data correctly depict the intrinsic trade-off between risk and return.

The low correlation between training returns and testing returns conducted to deceptive results when the testing results are analyzed, what suggests a low potential in the framework generalization capability. Consequently, the experimental results suggest that the positive connection between training and testing returns usually assumed in conventional single-objective approaches may not necessarily hold true for all cases.

Anyway, some interesting conclusions can be extracted, namely the conclusion that higher training returns correspond to increased volatility in the testing results. The MAs have the disadvantage of being a trend follower indicator, so the signals one can get from such indicator are always with some delay. Further tests should be accomplished, using other TI and the achieved results should be seen as a benchmark to further improvements with the use of other TI, or even the use of multi TI strategies.

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