Weighted Sum-rate Maximization for Multi-user Mimo-OFDM Downlink with ZF-DPC Methods

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Abstract: Multi -user MIMO techniques were born due to the urge of high data rates and spectral efficiency in 4G systems. For scenarios with a large number of users to be served in one cell, high capacity gains can be achieved by transmitting independent data streams to different users sharing the same time -frequency resources through the use of MIMO precoding. Linear precoding is employed in MU-MIMO communication system to improve the system capacity and to minimize the receiver complexity. The previous works on optimization algorithm to design a linear precoder to maximize the system capacity is assumed to have perfect channel state information (CSI) at the base station (BS).However the CSI available at the BS is imperfect due to channel estimation errors. With enough channel state information (CSI) at the transmitter, MIMO precoding allows to increase multi-user diversity gain. However, without a correct precoding vector selection, the interference between users can seriously degrade the overall network data rate. In a close-loop configuration, the base station (BS) receives from each user the preferred precoding vector and modulation and coding scheme (MCS).To achieve the highest multi -user diversity gains and avoid users interference, the BS needs to recalculate the precoding vector and MCS for each user. Weighted sum rate maximization is also considered, and qualification of throughput difference between two strategies is performed. In this process, it is shown that allocating the user powers in direct proportional to user weights asymptotically maximizes weight sum rate. The goal of this paper is to investigate the performance and complexity of state -of-the - art methods for calculation of precoding vectors such as zero –forcing (ZF) or mean square error (MMSE) and Dirty paper Coding(DPC).

1 INTRODUCTION

Multiple input -multiple output (MIMO) techniques are essential features in 3GPP LTE and LTE -A systems in order to achieve high data rates and high system capacity. When a large number of users need to be served in one cell, high capacity gains can be achieved by transmitting independent data streams to different users sharing the same time - frequency resources. This is called multi - user MIMO (MU - MIMO) and it can be realized through the use of MIMO precoding. Several precoding techniques applicable to the LTE standard have been introduced and discussed in the past few years (Zhou et al., 2009); (Cho et al., 2010); (Schwarz et al., 2010); (PHILIPS, 2007); (Schwarz et al., 2010); (Ribeiro et al., 2008); (Liu et al., 2012).

In a closed-loop configuration, the receiving user obtains downlink channel state information (CSI) by calculating three values that are feed backed to the base station (BS): channel quality indicator (CQI), rank indicator (RI), and precoding matrix indicator (PMI). With this information, the BS becomes aware of the channel quality of the users and can therefore choose the proper transmit modulation and coding schemes (MCS) for each of them. On the other hand, the PMI shows the precoding vector preferred by the user according to a certain criterion, for example, mutual information. However, these CSI feedback values reported by each user do not consider the interference created to the rest of the users on the same time-frequency resources. The base station should recalculate the PMI and MCS in order to avoid user interference. If we directly apply the CSI values feed backed by the users in a MU-MIMO transmission, the system performance can be degraded significantly due to interferences between users.

Dirty paper coding (DPC) was proved to achieve the capacity region of the multiple antenna broadcast channel (BC). However, implementation of DPC requires significant complexity at both transmitter and receiver, and the problem of finding practical dirty paper codes close to the capacity limit is still open. On the other hand, linear precoding is a low complexity but suboptimal transmission technique (with complexity roughly equivalent to point-to-point multiple- input–multiple-output (MIMO)) that is able to transmit the same number of data streams as a DPC-based system. Linear precoding therefore achieves the same multiplexing gain (which characterizes the slope of the capacity versus SNR curve) as DPC, but incurs an absolute rate/power offset relative to DPC.

2 SYSTEM MODEL

We consider an MU-MIMO system with one BS and *K* mobile station *(*MS), where the BS is equipped with *M* antennas and each MS with *N* antennas. The point-to multipoint MU-MIMO system is employed in downlink transmission. We consider the channel as the flat fading MIMO channel with Rayleigh distribution. Assuming that the transmitted signal is linearly precoded at the base station, the vector of the received signals at the K receivers is given by:

$$y_k = H_k W x + n_k \quad (1)$$

Where, H is the KXM channel matrix, x is the vector consisting K independent streams of data With zero mean and normalized variance and n is and additive white Gaussian noise vector. Due to errors introduced by channel estimation, reciprocity mismatch, quantization or delay we assume that the base station has only an estimate of the true channel response H that we denoted by $\widehat{\mathrm{H}}$.In the channel estimation model that we consider here

$$\mathrm{H} = \widehat{\mathrm{H}} + \widetilde{\mathrm{H}} \quad (2)$$

Where given the estimated channel matrix$\widehat{H}$, we assume that the estimation error matrix $\widetilde{\mathrm{H}}$ has K*M independent elements with zero mean and estimation error variance denoted byσ_e^2. Also we assume that $\widetilde{\mathrm{H}}$ is independent of the data vector x and the Gaussian noise vector n**.**

$$W = [T_1 T_2 \ \dots\dots T_k] \quad (3)$$

$$x = [\sqrt{P_1} x_1 \sqrt{P_2} x_2 \ \dots\dots \sqrt{P_k} x_k]^T \quad (4)$$

Where **x** is the transmitted symbol vector with *K* data streams, W is the precoding matrix with *K* Precoding vectors, and $[\ \cdot]^{\mathrm{T}}$ denotes the matrix transposition.

$$\widetilde{H}_k = H_k W \quad (5)$$

The channel matrix $\widetilde{H}_k$can be assumed as the virtual channel matrix of user *k* after precoding. At the receiver, a linear receiver$\widetilde{G}_k$ is exploited to detect the transmit signal for the user *k*. The detected signal of the k^{th} user is

$$\hat{x}_k = \widetilde{G}_k y_k \quad (6)$$

The linear receiver $\widetilde{G}_k$ can be designed by ZF or MMSE criteria, and linear MMSE will obtain better performance. In order to simplify the analysis, the power allocation is assumed as $\rho = p/k/N0$ and linear MMSE MIMO detection is used in this paper as

$$\widetilde{G}_k = \left[(H_k W)^H + K.\sigma_e^2.I + \frac{1}{\rho}.I\right]^{-1} \quad (7)$$

Where I is $N \times N$ identity matrix.

$$h_k = H_k.T_k \quad (8)$$

$$\mathrm{SINR_k} = \frac{\left|(\widehat{\mathrm{H}}_\mathrm{k} + \widetilde{\mathrm{H}}_\mathrm{k})\mathrm{T_k}\right|^2}{\sum_{\mathrm{i=1,i\neq k}}^{\mathrm{k}}\left|(\widehat{\mathrm{H}}_\mathrm{k} + \widetilde{\mathrm{H}}_\mathrm{k})\mathrm{T_i}\right|^2 + \frac{1}{\rho}} \quad (9)$$

3 HIGHEST SNR SUM-RATE CALCULATIONS

In this section, we compute the affine approximations to the dirty paper coding sum rate and the linear precoding sum rate at high SNR. In the following section, these expressions are used to quantify the sum rate degradation incurred by linear precoding relative to DPC.

3.1 Dirty Paper Coding

The sum rate by DPC, which achieves the sum capacity, can be written by the duality of the MIMO broadcast channel (BC) and the MIMO multiple-access channel (MAC)

$$C_{DPC}\,(H,P) = \sum_k max_{u(Q_k)\le P} log_2 \left| I + \sum_{k=1}^{K} H_k^H \ Q_k H_k \right| \quad (10)$$

Where Q_k represents the $N \times N$ transmit covariance matrices in the dual MAC. No closed –form solutions to (10) is know to exist, but it has shown that $C_{DPC}\,(H,P)$ convergence to the capacity of the point-point MIMO channel with transfer matrix H whenever $M \geq KN$:

Theorem1: When $M \geq KN$ and H has full row rank

$$lim_{P\to\infty}\left[C_{DPC}(H,P) - log_2.I + \frac{P}{KN}\,H^H.H^I\right] = 0 \quad (11)$$

Using this result we can make a few important observations regarding the optimal covariance matrices at high SNR. Since

$$log_2 I + \frac{P}{KN}\sum_{k=1}^{K} H_k^H.H_k = \; log_2\, I + \frac{P}{KN}H_k^H.H \qquad (12)$$

Choosing each of the dual MAC covariance matrices as $Q_k = \frac{P}{KN}I$ in (10) achieve the sum capacity at asymptotically high SNR.Thus uniform power allocation across the KN antennas in the dual MAC is asymptotically optimal an approximation for the sum rate can be defined as:

$$C_{DPC}(H,P) \cong KN\,log_2 P - KN\,log_2 KN + \; log_2|\,HH^H \quad (13)$$

Where $\cong$ refers to equivalence in the limit (i.e., the difference between both sides converges to zero as $P \to \infty$).Since the MIMO BC and the $M \times KN$ point-to-point MIMO channel are equivalent at high SNR (Theorem 1), the high SNR results developed in (PHILIPS, 2007) directly apply to the sum capacity of the MIMO BC channel.

4 RESULT ANAYSIS

Figure 1 plots the ZF and DPC throughputs for two five receiver systems. In a five-transmit-antenna/five receiver system (M = K = 5,N = 1), The figure shows that it gives accurate results throughout the entire SNR range. Throughput curves for a (M = 10,K = 5,N = 1) system are also shown.

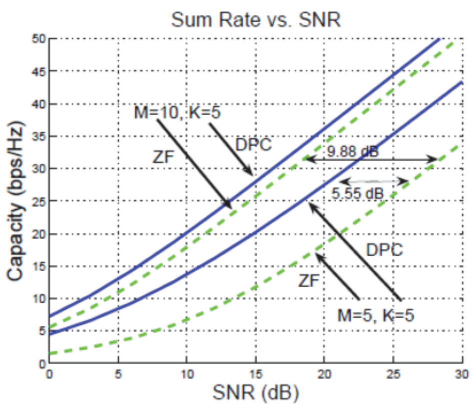

Figure 1: DPC vs ZF at High SNR.

The ZF power penalty is only 1.26 dB, which is reasonably close to the asymptotic penalty of 1.67 dB, increasing the number of transmit antennas from 5 to 10 shifts the sum capacity curve by 5.59 dB, but improves the performance of ZF by 9.88 dB. This is because ZF gains the increase in the sum capacity, along with the significantly decreased ZF penalty due to the increased number of transmit antennas (5.55 dB to 1.26 dB). Thus adding transmit antennas has the dual benefit of increasing the performance of DPC as well as reducing the penalty of using low-complexity ZF.

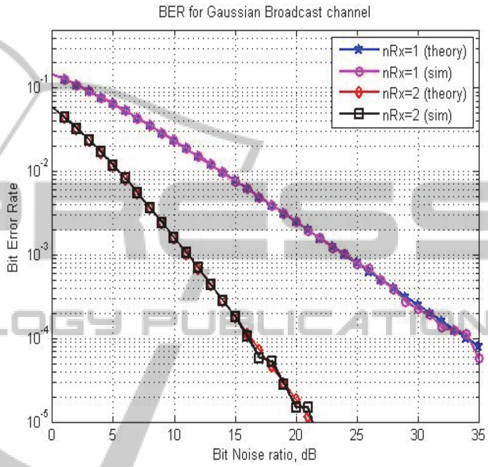

Figure 2: BER for Gaussian error BC channel.

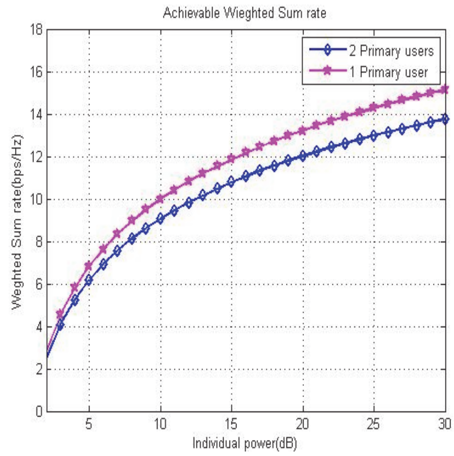

Figure 3: Achievable weighted sum-rate.

5 CONCLUSIONS

We have investigated the difference between the throughputs achieved by DPC relative to those achieved with linear precoding strategies. When the aggregate number of receive antennas is equal or slightly less than the number of transmit antennas, linear precoding incurs a rather significant penalty relative to DPC, but this penalty is much smaller when the number of transmit antennas is large relative to the number of receive antennas. Additionally, one interesting finding is that allocating power directly proportional to user weights is asymptotically optimal for DPC at high SNR. This simple yet asymptotically optimal power policy may prove to be useful in other setting such as opportunistic scheduling.

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