A Revisited Model for the Real Time Traffic Management

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Abstract: The real-time traffic management allow to solve unexpected disturbances that occur along a railway line during the normal developement of the traffic. The original timetable is restored through the rescheduling process. Despite the increase of real-time decision support tools for trains dispatchers that enable a better use of rail infrastructure, real-time traffic management received a limited scientific attention. In this paper, we deal with the real time traffic management for regional railway networks, mainly single tracks, in which a centralized traffic control system is installed. The rescheduling problem is presented as a Mixed Integer Linear Programming Model which resolution allows to carry out the rescheduling process in a very short computational time.

1 INTRODUCTION

A railway system is a complex system with many interacting processes that depend on technical devices, human behavior, external environment, and therefore contains many risks of disturbances. The usual method how railways manage their traffic performance is through a carefully designed plan of operations, defining several months in advance routes, orders and timing for all trains. This process, called *off-line timetabling*, is followed by a real-time traffic management which consists in managing disturbances that may occur during the ordinary functioning of the network.

Once a delayed train deviates from its original schedule, it may propagate its delay to other trains due to infrastructure, signaling or timing conflicts. Major disturbances may influence the off-line plan of operations that should be subject to short-term adjustments in order to minimize the negative effects of the disturbances. Possible traffic control actions include changing dwell times at scheduled stops, changing train speeds along lines, or adjusting train orders at junctions, stations and passing points. Other control actions involve major modifications such as changing train routes or even canceling scheduled train journeys. The main goal of the real-time dispatching is to minimize trains delays, while satisfying the traffic regulation constraints, and ensuring compatibility with the current position of each train, see (D'Ariano, 2008).

In this paper we deal with real-time traffic control problem for a regional single-track railway where an operating system called Centralized Traffic Control (CTC) is installed. The CTC provides a centralized control for signals and switches within a limited territory, controlled from a single control console. The command is carried out by the Train Dispatcher (TD).

The train dispatcher observes the status of the territory – i.e. occupation of line sections, location of trains, etc. – in a continuous manner and collects information; meanwhile, he communicates with the upper level decision-makers and the staff in the territory in order to exchange decisions taken. In case of an unplanned event and emergency he takes a decision and makes necessary actions in accordance with the rules and regulations pre-defined by the railway authority, see (İsmail, 1999).

The TD may benefit from appropriate decision support system, such as scheduling algorithms, to perform a real-time simulation and evaluation of traffic under disturbances in order to quickly reschedule train movements and to reduce delays from a global perspective.

It is important to find a good compromise between the solution quality, the time horizon of the traffic prediction, and the computational effort. If a short time horizon is adopted, only few trains, and few conflicts, can be detected and solved with short computation times. On the other hand, a longer time horizon

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leads to a larger number of trains running in the system, in order to eliminate completely the propagation of the disturbance. There is a tradeoff between the size of the time horizon of traffic prediction (bigger time horizon meaning better quality) and the computational time. In fact, in a small time horizon the real-time dispatching does not take into account conflicting trains outside the time horizon. On the other hand, a conflict arising far in the future may not be as relevant as a closer conflict, since other unforeseen events could still affect the further conflict, see (D'Ariano, 2008). In a small time horizon, the computational time is smaller because datas are limited.

Usually the train dispatcher reschedules the involved trains, depending on the known duration of the disturbance. He bases his decisions on his own knowledge, resolving a conflict at a time when it occurs, and then manually rebuilds the timetable, with a considerable waste of time and no certainty that its decisions will lead to an optimal solution.

Building on the formalism given in (Dotoli et al., 2013), we present a model that solves the rescheduling problem for regional passenger transport networks with stations of equal importance, where the CTC system is installed. We formulate the problem as a Mixed Integer Linear Programming Problem (MILP).

In the original model the new timetable after the disturbance is obtained by minimizing train delays in all the stations programmed in their path, while considering constraints regarding travel times, stop times at stations, safety standards and network capacity. The model is applied to a limited time horizon that is choosen by the analyst. In order to solve conflicts that may occur in the rescheduled timetable after the time horizon, an iterative heuristic algorithm is applied. The heuristic algorithm solves a conflict at the time when it occurs; priority is given to the train with the highest traveling time, namely the longest presence on the line. The computational time for limited time horizons is of the order of seconds, but the heuristic algorithm requires an elevated computational time that depends on the number of trains and the complexity of the raiway line. The methodology provides a decision support system to the train dispatcher that has to take decisions in order to restore traffic and limit inefficiencies for passengers.

We adapt the previous methodology to regional networks mainly made of single tracks and take into account the constraints imposed by the railway infrastructure and the time constraints imposed by the initial schedule.

The revised model solves all conflicts that arise along the railway line after the occurrence of the disturbance; the heuristic algorithm is therefore no longer applied. The rescheduled timetable is established in a shorter time, then discomfort for passengers is restricted and the quality of the transport service is increased.

To show its effectiveness, we study the problem in a particular section of a railway network located in Southern Italy, see (FSE - Ferrovie del Sud Est, 2013). The FSE network is constituted by single tracks with few double track segments and in some stations only one train can stop or pass through.

The paper is organized as follows. In Section 2 we present the problem formalization. In Section 3 the mathematical model for the resolution of the problem is proposed. In section 4 we present the application of the model to the case study of the FSE railway network. Finally, Section 5 contains some concluding remarks and suggestions for further research.

2 PROBLEM FORMALIZATION

2.1 Initial Scheduling

Definition 1 (Railway Network)**.** A railway network is defined by a set of segments on which trains runs.

Segment (b_i): A segment b is a railway section between two points. We define by $\mathbb{B} = \{b_1, b_2, \ldots, b_{\mathsf{B}}\} = \{b_i\}_{i\in[\![1,\mathsf{B}]\!]}$ the set of segments. B denote the cardinality of the set $\mathbb{B}$. The set of segments is partitioned into the subset $\mathbb{B}^s$ corresponding to segments into a *station*, and $\mathbb{B}^c$ corresponding to the subset of *rail connections* outside stations.

Track (v_j): Let b be a segment $\in \mathbb{B}$. We define by $\mathbb{V}^b = \{v_1^b, v_2^b, \ldots v_{\mathsf{V}^b}^b\} = \{v_j^b\}_{j\in[\![1,\mathsf{V}^b]\!]}$ the set of parallel tracks in b. The set of all tracks in the railway network is denoted by $\mathbb{V}$. V and V^b denote respectively the cardinality of $\mathbb{V}$ and $\mathbb{V}^b$ for a given segment b. Given a track $v \in \mathbb{V}$, we denote by b^v its corresponding segment.

Circulations in a railway network are defined by a set of trains. Train's path is made of an ordered set of movements.

Definition 2 (Trains and Movements)**.** We assume that the train's length is compatible with the length of all tracks that compose the railway line. Trains are thus defined as follows.

Train (t_k): The set of trains using the railway network is denoted as $\mathbb{T} = \{t_1, t_2, \ldots, t_{\mathsf{T}}\} = \{t_k\}_{k\in[\![1,\mathsf{T}]\!]}$. T denotes the cardinality of $\mathbb{T}$.

Train Direction (d^t): Each train is defined by a *direction* parameter expressing the position of its destination station. Let t be a train $\in \mathbb{T}$. We denote by $d^t = 0$ the direction of a train which head goes to the north (a.k.a. "*even trains*") of the railway line, and $d^t = 1$ of a train running to the south (a.k.a. "*odd trains*").

Movement (μ_p): A movement indicates the request for a track by a train. Let t be a train $\in \mathbb{T}$. We define by $\mathbb{M}^t = \{\mu_1^t, \mu_2^t, \ldots, \mu_{\mathrm{M}^t}^t\} = \{\mu_p^t\}_{p \in [\![1,\mathrm{M}^t]\!]}$ the ordered set of movements of train t. M^t denotes the cardinality of $\mathbb{M}^t$. $\mu_{first}^t = \mu_1^t$ and $\mu_{last}^t = \mu_{\mathrm{M}^t}^t$ denote respectively the first and the last element in $\mathbb{M}^t$. We define by $\mathbb{M}$ the set of all movements on the railway line and by $\mathbb{M}^{last}$ the set of the last movements of each train.

Train movements are defined by several parameters.

Movement Direction: All movements μ of a train t share the same direction as their train, denoted as d^μ:

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t, d^\mu = d^t \tag{1}$$

Track and Segment of a movement (b_μ, v_μ): Each movement μ of a train is scheduled in an unique segment $b_\mu \in \mathbb{B}$. We denote by $\mathbb{M}^b$ the set of movements scheduled in the same segment $b \in \mathbb{B}$. Each movement $\mu \in \mathbb{M}$ must be scheduled in a track of the segment b_μ, denoted as v_μ, according to the following constraint:

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t, v_\mu \in \mathbb{V}^{b_\mu} \tag{2}$$

Reference Schedule Times ($\alpha_\mu^{\mathrm{ref}}, \delta_\mu^{\mathrm{ref}}, \gamma_\mu^{\mathrm{ref}}$): Each train movement μ is associated to reference times corresponding to its initial schedule. Let $t \in \mathbb{T}$ be a train and $\mu \in \mathbb{M}^t$ one of its movements. We define three reference times $\alpha_\mu^{\mathrm{ref}}, \delta_\mu^{\mathrm{ref}}, \gamma_\mu^{\mathrm{ref}} \in \mathbb{N}$, where:

- $\alpha_\mu^{\mathrm{ref}}$ is the starting time of μ as established in the initial schedule, expressed in minutes taking as reference a time $T_0 \in \mathbb{N}$.
- $\delta_\mu^{\mathrm{ref}}$ is the duration of μ (expressed in minutes) if it occurs in a rail connection, i.e. the minimum running time defined in the initial schedule. This quantity is equal to 0 if the movement occurs in a station. Formally:

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t, b_\mu \in \mathbb{B}^s \Rightarrow \delta_\mu^{\mathrm{ref}} = 0 \tag{3}$$

- $\gamma_\mu^{\mathrm{ref}}$ is the duration of a movement μ (in minutes) if it occurs in a station, i.e. the minimum stopping time defined in the initial schedule. This quantity is equal to 0 is the movement occurs in a rail connection. Formally:

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t, b_\mu \in \mathbb{B}^c \Rightarrow \gamma_\mu^{\mathrm{ref}} = 0 \tag{4}$$

Using such notations, the time interval during which a movement μ reserves its track can be expressed as $[\![\alpha_\mu^{\mathrm{ref}}, \alpha_\mu^{\mathrm{ref}} + \delta_\mu^{\mathrm{ref}} + \gamma_\mu^{\mathrm{ref}} = \beta_\mu^{\mathrm{ref}}]\!]$. Two consecutive movements must be scheduled according to these intervals, thus we have:

$$\begin{gathered}\forall t \in \mathbb{T}, \forall p \in [\![1, \mathrm{M}^t[\![,\\ \alpha_{\mu_{p+1}}^{\mathrm{ref}} = \alpha_{\mu_p}^{\mathrm{ref}} + \delta_{\mu_p}^{\mathrm{ref}} + \gamma_{\mu_p}^{\mathrm{ref}}\end{gathered} \tag{5}$$

In order to reduce the size of the initial problem we could use only one variable $\zeta = \gamma + \delta$ to represent movements duration. However, even if such formulation would reduce the number of initial variables, the size of the problem after the presolve phase would remain unchanged, since modern solvers are able to detect such redundant variables. For clarity, we decided thus to keep using two different variables γ and δ to represent movement duration respectively in a rail connection and in a station.

Definition 3 (Security Constraints)**.** Since several trains run at the same time on a railway network, several constraints must be verified to ensure the security of circulations.

Track Occupation Constraints: A track cannot be occupied by two trains at the same time. Such restriction can be expressed formally by constraining any pair of movements using the same track to be scheduled on disjoint timing intervals:

$$\begin{gathered}\forall t_1, t_2 \in \mathbb{T}, \forall \mu_i \in \mathbb{M}^{t_1}, \forall \mu_j \in \mathbb{M}^{t_2},\\ v_{\mu_i} = v_{\mu_j} \Rightarrow [\![\alpha_{\mu_i}^{\mathrm{ref}}, \beta_{\mu_i}^{\mathrm{ref}}]\!] \cap [\![\alpha_{\mu_j}^{\mathrm{ref}}, \beta_{\mu_j}^{\mathrm{ref}}]\!] = \varnothing\end{gathered} \tag{6}$$

Safety Times (Δ_m, Δ_f): The safety time is the separation time that has to elapse between two movements μ_i, μ_j on the same *track* (i.e. between a train leaving one track and another one entering the same track). We denote by $\Delta_m \in \mathbb{N}$ the safety time required if trains *meet* and $\Delta_f \in \mathbb{N}$ if one train is *following* the other one. Δ_m and Δ_f are expressed in the same time units as γ^{ref} and δ^{ref}. These time delays must occur between the end of the first movement (denoted as $\beta_{\mu_i}^{\mathrm{ref}}$) and the start of the other one (denoted as $\alpha_{\mu_j}^{\mathrm{ref}}$). Formally, safety time constraints can be expressed as follows:

$$\forall b \in \mathbb{B}, \forall \mu_i, \mu_j \in \mathbb{M}^b$$

$$\begin{array}{rcl} v_{\mu_i} & = & v_{\mu_j} \\ \wedge\ d^{\mu_i} & = & d^{\mu_j} \end{array} \Rightarrow \left\{ \begin{array}{rcl} \alpha_{\mu_j}^{\mathrm{ref}} & \geq & \beta_{\mu_i}^{\mathrm{ref}} + \Delta_f \\ \vee\ \alpha_{\mu_i}^{\mathrm{ref}} & \geq & \beta_{\mu_j}^{\mathrm{ref}} + \Delta_f \end{array} \right. \tag{7}$$

$$\begin{array}{rcl} v_{\mu_i} & = & v_{\mu_j} \\ \wedge\ d^{\mu_i} & \neq & d^{\mu_j} \end{array} \Rightarrow \left\{ \begin{array}{rcl} \alpha_{\mu_j}^{\mathrm{ref}} & \geq & \beta_{\mu_i}^{\mathrm{ref}} + \Delta_m \\ \vee\ \alpha_{\mu_i}^{\mathrm{ref}} & \geq & \beta_{\mu_j}^{\mathrm{ref}} + \Delta_m \end{array} \right. \tag{8}$$

Note such constraints renforce the constraint (6) since they induce a separation delay between the time intervals of two movements on the same track.

A rail transport service on a railway line is graphically represented by a cartesian graph, representing the safety constraints.

Graphic Timetable: The graphic timetable is a cartesian graph that represents the situation of a railway schedule for a day, see (Vicuna, 1989). The diagram shows all movements, scheduled and safety times.
The time line is plotted on the x axis. The railway line (space) is plotted on the y axis. Inside stations, tracks are represented by dashed lines parallel to the x axis. Trains are represented by an oblique broken line which orientation indicates train direction. For instance, train lines are oriented from bottom to up for *even trains* that travel from south to north (i.e. to the station on the upper end of the y axis).
Figure 1 represents the graphic timetable of three trains t_1, t_2 and t_4. Train t_1 is directed to the South, while t_2 and t_4 travel in the opposite direction. The railway line is made of 5 segments: single-tracked stations b_1 and b_3, double-tracked station b_5 and single-tracked rail connections b_2 and b_4. Track sets for the five segments are defined by: $V^{b_1} = \{v_1\}$, $V^{b_2} = \{v_1\}$, $V^{b_3} = \{v_1, v_2\}$, $V^{b_4} = \{v_1\}$, $V^{b_5} = \{v_1\}$. Train's paths are defined by sets: $M^{t_1} = (\mu_1^{t_1}, \mu_2^{t_1}, \mu_3^{t_1})$, $M^{t_2} = (\mu_1^{t_2}, \mu_2^{t_2}, \mu_3^{t_2})$, $M^{t_4} = (\mu_1^{t_4}, \mu_2^{t_4}, \mu_3^{t_4})$. All trains cross station b_3 that is single-tracked. As shown, safety times are applied when two trains occupy the same *track* of a segment. Δ_f is the safety time that has to elapse between the end of the movement $\mu_2^{t_2}$ and the beginning of $\mu_2^{t_4}$, where t_2 and t_4 travel in the same direction. Δ_m is the safety time that has to elapse between the end of $\mu_2^{t_4}$ and the beginning of $\mu_2^{t_1}$, where t_1 and t_4 travel in opposite direction.

2.2 Disturbances Issues

When a disturbance occurs along the railway network, that compromises the normal traffic operation, a rescheduling process must be accomplished taking into account time constraints imposed by the initial schedule.

Definition 4 (Disturbance)**.** A disturbance denotes the deviation of a train $t_d \in \mathbb{T}$ from its original schedule due to an unforeseen situation, concerning one of its movements $\mu_d \in \mathbb{M}^{t_d}$.

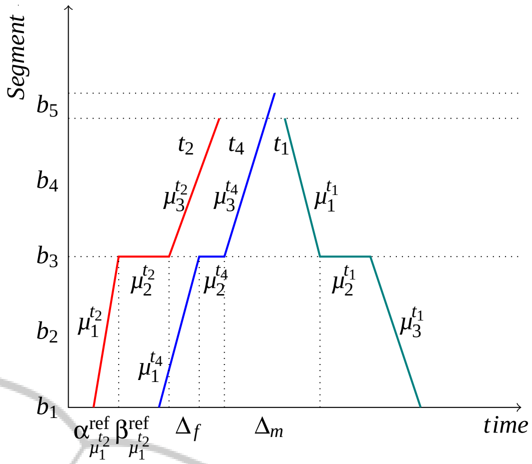

Figure 1: Graphic Timetable.

Disturbance Duration and Reference Time: When a disturbance occurs, a *disturbance reference time* T_d is defined as the first time on which the disturbance has an impact on the schedule of the others trains, i.e. the reference ending time of the disturbed movement. If only one disturbance affects one movement $\mu_d^{t_d} \in \mathbb{M}^{t_d}$ of a train t_d along the railway line, T_d is defined as:

$$T_d = \beta^{\text{ref}}_{\mu_d}$$

We denote by Δ_d the disturbance duration expressed in minutes. The impact of a disturbance on the movement is expressed by an increase of the value of parameter δ or γ depending on the nature of the segment on which the disturbance occurs. For instance, if $b^{\mu_d} \in \mathbb{B}^c$, $\delta'_{\mu_d} = \delta^{\text{ref}}_{\mu_d} + \Delta_d$. Conversely, if $b^{\mu_d} \in \mathbb{B}^s$, $\gamma'_{\mu_d} = \gamma^{\text{ref}}_{\mu_d} + \Delta_d$.
If two independent disturbances affect two different trains along the line, T_d is defined as the minimum final time of movements affected by the perturbation. Let t_{d_1} and t_{d_2} be two trains affected by the disturbance. Let $\mu_{d_1} \in \mathbb{M}^{t_{d_1}}$ and $\mu_{d_2} \in \mathbb{M}^{t_{d_2}}$ be the perturbed movements. T_d is defined as:

$$T_d = min\left\{\beta^{\text{ref}}_{\mu_{d_1}}, \beta^{\text{ref}}_{\mu_{d_2}}\right\}$$

Figure 2 represents a railway line made of three station (b_1, b_3 and b_5) and two single-tracked rail connections (b_2 and b_4). A disturbance occurs in b_2 and affects only the movement $\mu_1^{t_2} \in \mathbb{M}^{t_2}$. Reference time T_d coincides with $\beta^{ref}_{\mu_1^{t_2}}$. The dashed line shows the movement $\mu_1^{t_2}$ after the end of the rescheduling process. The recheduled crossing time $\delta^{eff}_{\mu_1^{t_2}}$ is equal to the reference time $\delta^{ref}_{\mu_1^{t_2}}$ increased by the disturbance duration Δ_d. The effective path of movement $\mu_1^{t_2}$ interferes with the

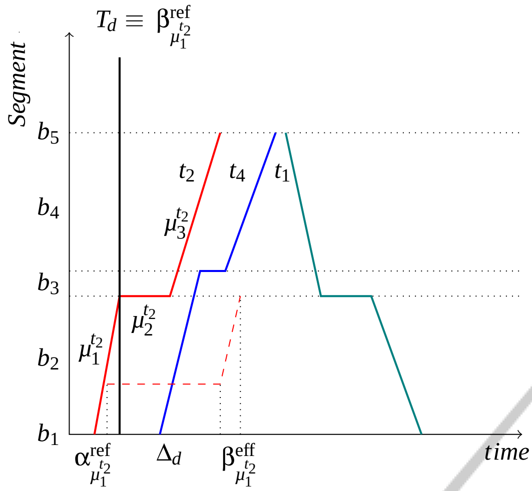

Figure 2: Disturbance of t_2.

path of another movement in the segment b_2 that has to be rescheduled.

Effective Schedule Times: In order to take into account the effect of the disturbance on the subsequent movements on the railway network, we introduce for any movement its *effective scheduled times* denoted by $\alpha^{\text{eff}}, \delta^{\text{eff}}$ and $\gamma^{\text{eff}} \in \mathbb{N}$. Obviously, movements which final date β is scheduled *before* T_d are not altered. Formally:

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t s.t. \beta^{\text{ref}}_\mu \le T_d, \begin{cases} \alpha^{\text{eff}}_\mu = \alpha^{\text{ref}}_\mu \\ \delta^{\text{eff}}_\mu = \delta^{\text{ref}}_\mu \\ \gamma^{\text{eff}}_\mu = \gamma^{\text{ref}}_\mu \end{cases} \quad (9)$$

Effective Time Constraints: These new scheduled times must allow to *absorb* the perturbation by delaying the initial reference times, according to their respective segment types, following constraints (3) and (4). Formally, these previous equations become:

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t s.t. \beta^{\text{ref}}_\mu > T_d,$$

$$b_\mu \in \mathbb{B}^s \Rightarrow \begin{cases} \alpha^{\text{eff}}_\mu & \ge & \alpha^{\text{ref}}_\mu \\ \delta^{\text{eff}}_\mu & = & \delta^{\text{ref}}_\mu = 0 \\ \gamma^{\text{eff}}_\mu & \ge & \gamma^{\text{ref}}_\mu \end{cases} \quad (10)$$

$$b_\mu \in \mathbb{B}^c \Rightarrow \begin{cases} \alpha^{\text{eff}}_\mu & \ge & \alpha^{\text{ref}}_\mu \\ \delta^{\text{eff}}_\mu & \ge & \delta^{\text{ref}}_\mu \\ \gamma^{\text{eff}}_\mu & = & \gamma^{\text{ref}}_\mu = 0 \end{cases} \quad (11)$$

Effective schedule time must also obviously follow the sequencing constraint (5) and safety constraints (6), (7) and (8).

Time Horizon (H): The time horizon H is the term planning in which the rescheduling operations are carried out. It consists of a given number of timetable minutes in which a given number of trains are scheduled on the railway line. Movements after the time horizon are not taken into account in the rescheduling process, even if they belong to trains of which first movements belong to the time horizon.

Of course, depending on the density of traffic at the time of the disturbance, and its duration, the number of disturbed movements can vary considerably. In this paper, we consider all the scheduled movements of the day but one of our objective functions can be designed to minimize the number of rescheduled trains.

In the following section, we present the Mixed Integer Linear Programming Model that allows to solve the rescheduling problem, i.e. to give a value to each effective scheduled time while respecting the safety constraints and optimizing practical criteria.

3 MATHEMATICAL MODELING

3.1 Decision Variables

We introduce additional variables and constants used to express the problem in a linear way.

- $X_{\mu,v} \in \{0,1\}^{\mathbb{M}\times\mathbb{V}}$ is the variable that identifies the track v on which a movement μ occurs. $X_{\mu,v} = \varphi(v = v_\mu)$ where the function $\varphi(C)$ is the indicator $\varphi(C) = 1$ if the condition C is verified, 0 otherwise.
- $X^{\text{before}}_{\mu_i,\mu_j} \in \{0,1\}^{\mathbb{M}\times\mathbb{M}}$ is the variable that characterizes the chronological order of two movements μ_i, μ_j if they use the same segment. $X^{\text{before}}_{\mu_i,\mu_j} = \varphi(\mu_i$ is scheduled before $\mu_j)$.
- $X^{\text{delay}}_t \in \{0,1\}^{\mathbb{T}}$ is the variable that specify if a train t deviates from its original schedule and is therefore delayed. $X^{\text{delay}}_t = \varphi(\beta^{eff} > \beta^{ref})$.
- $X^{\text{delay}}_\mu \in \{0,1\}^{\mathbb{T}}$ is the variable that specify if a movement μ deviates from its original schedule and is therefore delayed. $X^{\text{delay}}_\mu = \varphi(\beta^{eff} > \beta^{ref})$.
- $B \in \mathbb{N}$ is a sufficiently large positive constant.
- $H \in \mathbb{N}$ is a parameter that defines the size of the time horizon.

By definition, the previous decisions variables are subject to constraints characterizing their *physical sense*.

- Any movement can only be scheduled on one track of its segment, consequently:

$$\forall \mu \in \mathbb{M}, \sum_{v \in \mathbb{V}^{b_\mu}} X_{\mu,v} = 1 \quad (12)$$

- Two movements scheduled on the same segment must be ordered:

$$\forall b \in \mathbb{B}, \forall \mu_i, \mu_j \in \mathbb{M}^b, \quad X^{\text{before}}_{\mu_i,\mu_j} + X^{\text{before}}_{\mu_j,\mu_i} = 1 \tag{13}$$

Safety constraints presented above must be expressed using those variables in linear way.

3.2 Linearization of Safety Constraints

Security constraints (6), (7) and (8) are expressed through the use of additional variables and constraints in order to obtain a linear formulation.

3.2.1 Tracks Occupation Constraints

Constraint (6) specifies that a track v cannot be occupied by several movements at the same time. Constraints (7) and (8) express a separation delay must elapse between two movements occupying the same track. These conditions can be expressed by the following equations:

$$\forall b \in \mathbb{B}, \forall \mu_i, \mu_j \in \mathbb{M}^b s.t.\ d_{\mu_i} = d_{\mu_j}, \forall v \in \mathbb{V}^b, \tag{14}$$
$$\beta^{\text{eff}}_{\mu_i} - \alpha^{\text{eff}}_{\mu_j} + \Delta_f \leq B \cdot \left(3 - X^{\text{before}}_{\mu_i,\mu_j} - X_{\mu_i,v} - X_{\mu_j,v}\right)$$

$$\forall b \in \mathbb{B}, \forall \mu_i, \mu_j \in \mathbb{M}^b s.t.\ d_{\mu_i} \neq d_{\mu_j}, \forall v \in \mathbb{V}^b, \tag{15}$$
$$\beta^{\text{eff}}_{\mu_i} - \alpha^{\text{eff}}_{\mu_j} + \Delta_m \leq B \cdot \left(3 - X^{\text{before}}_{\mu_i,\mu_j} - X_{\mu_i,v} - X_{\mu_j,v}\right)$$

Note for any pair of movements μ_i, μ_j, two instances of the previous equations (14) and (15) are considered in the mathematical model depending of the the order of movements: (μ_i, μ_j) or (μ_j, μ_i).

The previous equation expresses that if two movements μ_i, μ_j occurs on the same track, and if $X^{\text{before}}_{\mu_i,\mu_j} = 1$, then μ_i must end before the start of μ_j. If $X^{\text{before}}_{\mu_i,\mu_j} = 0$ or μ_i and μ_j do not occur in the same track, equation (14) and (15) are trivially verified.

The disjunction operator $\vee$ in equations (7) and (8) is taken into account by the boolean variable $X^{\text{before}}_{\mu_i,\mu_j}$ that denotes the two possible alternatives.

3.3 Objective Functions

The optimization problem compares four alternative objective functions defined as follows:

$$Obj_1\text{: } \min \sum_{\mu_i \in \mathbb{M}} (\beta^{\text{eff}}_{\mu_i} - \beta^{\text{ref}}_{\mu_i}) \tag{16}$$

$$Obj_2\text{: } \min \sum_{\mu_i \in \mathbb{M}^{last}} (\beta^{\text{eff}}_{\mu_i} - \beta^{\text{ref}}_{\mu_i}) \tag{17}$$

$$Obj_3\text{: } \min \sum_{t \in \mathbb{T}} {X_t}^{\text{delay}} \tag{18}$$

$$Obj_4\text{: } \min \sum_{\mu \in \mathbb{M}} {X_\mu}^{\text{delay}} \tag{19}$$

- Obj_1 minimizes the delay of the traffic (i.e. the sum of the delays of all the movements trains in the railway line).
- Obj_2 minimizes the total final delay of the traffic (i.e. the final delays when trains arrive at their final destination, or rather the last stop considered within the rescheduling time horizon).
- Obj_3 minimizes the number of delayed trains.
- Obj_4 minimizes the number of delayed movements for each train.

When Obj_3 or Obj_4 is used, we introduced five additional constraints (20) to (24) as follows:

$$\forall t \in \mathbb{T}, \beta^{\text{eff}}_{\mu^t_{last}} - \beta^{\text{ref}}_{\mu^t_{last}} \leq B \cdot {X_t}^{\text{delay}} \tag{20}$$

$$\forall t \in \mathbb{T}, \beta^{\text{eff}}_{\mu^t_{last}} - \beta^{\text{ref}}_{\mu^t_{last}} > 1 + B \cdot \left({X_t}^{\text{delay}} - 1\right) \tag{21}$$

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t \Rightarrow \alpha^{\text{eff}}_\mu \leq H \tag{22}$$

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t \Rightarrow \delta^{\text{eff}}_\mu \leq H \tag{23}$$

$$\forall t \in \mathbb{T}, \forall \mu \in \mathbb{M}^t \Rightarrow \gamma^{\text{eff}}_\mu \leq H \tag{24}$$

Constraints (20) and (21) specify that if the rescheduled ending time of the train coincides with its reference time, the train is not delayed and ${X_t}^{\text{delay}} = 0$. Conversly, if one movement of a train is delayed, the last movement is necessarily delayed according to equations (10), (11) and equation (21) implies that ${X_t}^{\text{delay}} = 1$.

Constraints (22), (23) and (24) mean that movements scheduled in the analyzed time horizon, after the rescheduling process must start within the same time horizon. This constraint prevents that the trains are postponed for a long time or even suppressed by moving outside the considered time horizon.

When Obj_4 is used, constraints (20) (resp. (21)) are replaced by constraints (25) (respectively (26)) that refers to all movements on the line.

$$\forall \mu \in \mathbb{M}, \beta^{\text{eff}}_\mu - \beta^{\text{ref}}_\mu \leq B \cdot {X_t}^{\text{delay}} \tag{25}$$

$$\forall \mu \in \mathbb{M}, \beta^{\text{eff}}_\mu - \beta^{\text{ref}}_\mu > 1 + B \cdot \left({X_t}^{\text{delay}} - 1\right) \tag{26}$$

The full model is given in Figure 3.

Let $(\mathbb{B},\mathbb{V})$ be a railway network, $\mathbb{T}$ a set of trains with their schedules and (μ_d,Δ_d,T_d) the characteristics of a disturbance occuring on the system. The mixed integer linear programming model *MILP* is defined by:

$$Obj_1 \quad Minimize \quad \sum_{\mu\in\mathbb{M}} \beta_\mu^{\text{eff}} - \beta_\mu^{\text{ref}} \tag{27}$$

$$Obj_2 \quad Minimize \quad \sum_{\mu\in\mathbb{M}^{last}} \beta_\mu^{\text{eff}} - \beta_\mu^{\text{ref}} \tag{28}$$

$$Obj_3 \quad Minimize \quad \sum_{t\in\mathbb{T}} X_t^{delay} \tag{29}$$

$$Obj_4 \quad Minimize \quad \sum_{\mu\in\mathbb{M}} X_\mu^{delay} \tag{30}$$

Subject to:

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t \ s.t.\ b_\mu\in\mathbb{B}^s,\quad \delta_\mu^{\text{eff}} = 0 \tag{31}$$

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t \ s.t.\ b_\mu\in\mathbb{B}^c\quad \gamma_\mu^{\text{eff}} = 0 \tag{32}$$

$$\forall t\in\mathbb{T},\quad \forall p\in[\![1,\mathbb{M}^t[\![,\quad \alpha_{\mu_{p+1}}^{\text{eff}} - \alpha_{\mu_p}^{\text{eff}} - \delta_{\mu_p}^{\text{eff}} - \gamma_{\mu_p}^{\text{eff}} = 0 \tag{33}$$

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} \le T_d,\quad \alpha_\mu^{\text{eff}} = \alpha_\mu^{\text{ref}} \tag{34}$$

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} \le T_d,\quad \delta_\mu^{\text{eff}} = \delta_\mu^{\text{ref}} \tag{35}$$

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} \le T_d,\quad \gamma_\mu^{\text{eff}} = \gamma_\mu^{\text{ref}} \tag{36}$$

$$\forall t\in\mathbb{T},\ \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} > T_d \ and \ b_\mu\in\mathbb{B}^s,\quad \alpha_\mu^{\text{eff}} \ge \alpha_\mu^{\text{ref}} \tag{37}$$

$$\forall t\in\mathbb{T},\ \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} > T_d \ and \ b_\mu\in\mathbb{B}^s,\quad \delta_\mu^{\text{eff}} = \delta_\mu^{\text{ref}} (=0) \tag{38}$$

$$\forall t\in\mathbb{T},\ \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} > T_d \ and \ b_\mu\in\mathbb{B}^s,\quad \gamma_\mu^{\text{eff}} \ge \gamma_\mu^{\text{ref}} \tag{39}$$

$$\forall t\in\mathbb{T},\ \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} > T_d \ and \ b_\mu\in\mathbb{B}^c,\quad \alpha_\mu^{\text{eff}} \ge \alpha_\mu^{\text{ref}} \tag{40}$$

$$\forall t\in\mathbb{T},\ \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} > T_d \ and \ b_\mu\in\mathbb{B}^c,\quad \delta_\mu^{\text{eff}} \ge \delta_\mu^{\text{ref}} \tag{41}$$

$$\forall t\in\mathbb{T},\ \forall \mu\in\mathbb{M}^t \ s.t.\ \beta_\mu^{\text{ref}} > T_d \ and \ b_\mu\in\mathbb{B}^c,\quad \gamma_\mu^{\text{eff}} = \gamma_\mu^{\text{ref}} (=0) \tag{42}$$

$$\gamma_{\mu_d}^{\text{eff}} = \gamma_{\mu_d}^{\text{ref}} + \Delta_d \tag{43}$$

$$\delta_{\mu_d}^{\text{eff}} = \delta_{\mu_d}^{\text{ref}} + \Delta_d \tag{44}$$

$$\forall \mu\in\mathbb{M},\quad \sum_{v\in\mathbb{V}^{b_\mu}} X_{\mu,v} = 1 \tag{45}$$

$$\forall b\in\mathbb{B},\quad \forall \mu_i,\mu_j\in\mathbb{M}^b,\quad X_{\mu_i,\mu_j}^{\text{before}} + X_{\mu_j,\mu_i}^{\text{before}} = 1 \tag{46}$$

$$\forall b\in\mathbb{B}, \forall \mu_i,\mu_j\in\mathbb{M}^b,\ \text{s.t. } d^{\mu_i} = d^{\mu_j}, \forall v\in\mathbb{V}^b,\quad \beta_{\mu_i}^{\text{eff}} - \alpha_{\mu_j}^{\text{eff}} + B\cdot\left(X_{\mu_i,\mu_j}^{\text{before}} + X_{\mu_i,v} + X_{\mu_j,v}\right) \le 3\cdot B - \Delta_f \tag{47}$$

$$\forall b\in\mathbb{B}, \forall \mu_i,\mu_j\in\mathbb{M}^b,\ \text{s.t. } d^{\mu_i} \ne d^{\mu_j}, \forall v\in\mathbb{V}^b,\quad \beta_{\mu_i}^{\text{eff}} - \alpha_{\mu_j}^{\text{eff}} + B\cdot\left(X_{\mu_i,\mu_j}^{\text{before}} + X_{\mu_i,v} + X_{\mu_j,v}\right) \le 3\cdot B - \Delta_m \tag{48}$$

$$\forall t\in\mathbb{T},\quad \beta_{\mu_{\text{last}}^t}^{\text{eff}} - B\cdot X_t^{\text{delay}} \le \beta_{\mu_{\text{last}}^t}^{\text{ref}} \tag{49}$$

$$\forall t\in\mathbb{T},\quad \beta_{\mu_{\text{last}}^t}^{\text{eff}} - B\cdot X_t^{\text{delay}} > \beta_{\mu_{\text{last}}^t}^{\text{ref}} - B \tag{50}$$

$$\forall \mu\in\mathbb{M},\quad \beta_\mu^{\text{eff}} - B\cdot X_t^{\text{delay}} \le \beta_\mu^{\text{ref}} \tag{51}$$

$$\forall \mu\in\mathbb{M},\quad \beta_\mu^{\text{eff}} - B\cdot X_t^{\text{delay}} > \beta_\mu^{\text{ref}} - B \tag{52}$$

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t,\quad \alpha_\mu^{\text{eff}} \le H \tag{53}$$

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t,\quad \delta_\mu^{\text{eff}} \le H \tag{54}$$

$$\forall t\in\mathbb{T},\quad \forall \mu\in\mathbb{M}^t,\quad \gamma_\mu^{\text{eff}} \le H \tag{55}$$

Figure 3: Mixed Integer Linear Programming Model.

Constraints (31) - (32) express that if a movement occurs in a station (resp. in a rail connaction) the effective running time (res. the effective stopping time) is null.

Constraint (33) specifies that each train movement is directly succeeded by the next one, that means that when a train leaves a track, it instantly begins to occupy the next one.

Constraints (34), (35) and (36) ensure that movements scheduled completely before the occurrence of the disturbance remain unchanged.

Constraints (37) to (39) (resp.(40) to (42)) enforce the restrictions related to planned stops (respectively to planned running times) and the consequent earliest possible departure time.

Constraint (43) (resp. (44)) means that the running time of perturbed movements (respectively the stopping time) is increased according to disturbance duration.

Constraint (45) means that each movement has to use exactly one track of its segment.

Constraint (46) implies that two movements scheduled on the same segment must be ordered.

Constraints (47) - (48) mean that if several movements have to use the same track of a segment, a safety time (Δ_f or Δ_m) must elapse between the end of the first movement and the beginning of the second one.

Constraints (49)-(50) (resp. (51)- (52)) denote if a train (respectively a movement) is delayed. Constraints (53) to (55) enforce the beginning and the duration of a movement within the time horizon. These constraints are active only when Obj_3 (respectively Obj_4) is applied.

4 NUMERICAL EXPERIMENTS

The presented model is applied to Ferrovie del Sud Est (FSE), the largest public transport company operating in the Apulia region of Southern Italy. We analyze the railway ring connecting Mungivacca and Putignano stations (Figure 4), where a CTC system is installed. The operation system is installed in Mungivacca station that is independent, not controlled by the CTC, as is Putignano, whereby these stations are not studied here. In particular, we refer to the line 1 of the railway ring – passing through Conversano – that is single tracked except for the line connecting Noicattaro to Rutigliano, that is double tracked. Moreover, Grotte di Castellana is a single track station, not chaired by an operator. 24 even trains and 22 odd trains run on the railway line during a day. We assume a safety time $\Delta_m = 3$ *min* for two trains traveling in opposite directions and a time $\Delta_f = 1$ *min* for trains in the same direction. In attempt to evaluate the optimality of the algorithm, we used IBM CPLEX 12.5 installed and run on an Intel Core 2 Duo 1.83 GHz CPU and 3 GB RAM, under Windows with the model formulated in AMPL.

4.1 One Disturbance on the Line

We consider a real data set referring to a train going from Putignano to Mungivacca that stops along the line that connects Castellana G. and Conversano due to a disturbance occurring at 7:50 am. That same disturbance event is used for all the experiments but with different disturbance sizes Δ_d and solved with different time horizons H. Various disturbance times are been considered, ranging from 10 to 50 minutes. A time horizon of H minutes means that movements that should have started (according to the initial timetable) H minutes or more after the instant at which the disturbance occurs are not considered in the computation. Time horizons are expressed in minutes and take values equal to 30, 60, 90, 120, 180, 240, 300, 360, 420, 480, 540, 600 and 1440. The main aspects considered to present results are Obj_1, Obj_2, Obj_3, Obj_4 that correspond respectively to sum of delays for all train movements, delay of the last movement of each train, number of rescheduled trains and number of rescheduled movements. All operational times are given in minutes. CT refers to computational time given in seconds. N and V refers to number of variables and constraints before and after the presolve phase.

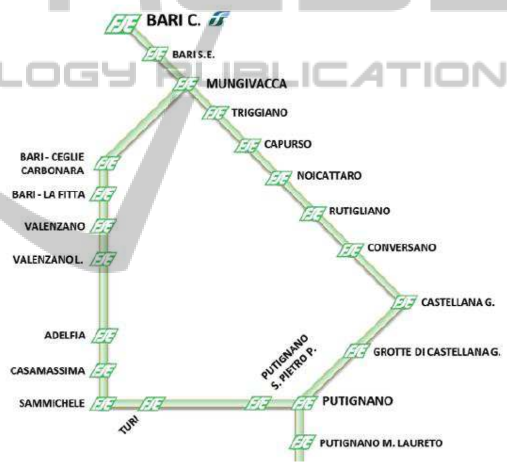

Figure 4: Ther railway ring used for the scenarios.

4.1.1 Overall Analysis

When only one disturbance occurs on the railway line, results from experiments using the four objective functions for all different disturbance size (Δ_d) and time horizons (H) are presented in Table 5.

The methodology is applied to a real case study in which the occurence of short-term disturbance on the railway line is frequent, this is not a trivial problem. The model provides a proactive approach to solve, in real time, problems that occur on the railway line.

The computational time (CT) for all time horizons and disturbance sizes is of the order of a few second. Comparison between different time horizons is done in order to demonstrate that the model is able to quickly solve even considerables problems that take into account all trains on the railway line. In fact, the highest value of CT (equal to 69.77 seconds) is obtained using Obj_1 for $H = 1440$ and $\Delta_d = 45$ minutes. This means that if a disturbance lasting 45 minutes occurs along the line, in just over a minute the train dispatcher can obtain the rescheduled timetable for the 24 hours following the occurrence of the fault. The speed of resolution is a very important factor for the presented problem, since the main objective of the real-time traffic management is to quickly establish a new timetable, in order to minimize the inconvenience for passengers.

Compared to the previous methodology used in (Dotoli et al., 2013) there is an improvement due to a reduction of total delay, number of rescheduled trains and computational time. The objective of this model is the minimization of delay of all movements. In Table 1 we compare results of the rescheduling process obtained with the application of the actual and the previous methodology (AM, PM) when $\Delta_d = 30$ minutes and $H = 180$ minutes. We noticed that Obj_1 is unchanged for all subsequent values of time horizon. This means that the delay caused by the disturbance is absorbed within the 180 minutes after its occurrence. For the previous methodology, we present values obtained for $H = 30$ minutes in addition to values obtained with the application of the heuristic algorithm after the time horizon. CT refers only at the optimization procedure.

By analyzing values we observe that with the actual methodology the new timetable is computed in 0.72 seconds. Three trains are involved in the rescheduling process for a total delay of 608 minutes. Previous methodology required 10.74 seconds to obtain the new timetable within a 30 minutes window after the occurrence of the fault. 10 trains are involved in the rescheduling process and total delay is equal to 665 minutes. Actual methodology allows to obtain the optimal solution with an exact approach, without the application of the heuristic algorithm which does not always provide optimal results. We should also take into account that solvers used by the two methodology are different. MATLAB with GLPK used by the previous methodology is replaced by IBM CPLEX in the actual one. Resolution methods used by the two solvers are different as well as their performance. The previous methodology has obtained an improvement compared to the current practice used by the train dispatcher; the actual methodology provides a further amelioration. This is in line with objectives of the real-time traffic management.

4.1.2 Comparison between Objective Functions

We compare values obtained using the four objective functions for $H = 1440$ and $\Delta_d = 50$ minutes, shown in Table 2. We analyze the time horizon of 1440 minutes because the complexity of the problem is high due to the presence, in the rescheduling process, of all trains movements on the railway line until the end of the day.

Lowest values in terms of total delay are obtained using Obj_1. Comparing results obtained with the first and the second objective function we notice that although the value of Obj_2 is the same, Obj_1 changes. In particular, Obj_1 obtained while Obj_2 is greater. The reason is simple: in this case, minimizing the delay of the last movement of a train, the second objective function increases the number of its delayed movements. This means that although values of Obj_3 are unchanged using the first and the second objective function, values of Obj_4 varies. In general, Obj_2 increases the arrival time at intermediate stations, in order to minimize the delay at the last station of trains path. In this case, there are no differences between values of Obj_1 and Obj_2 obtained using Obj_3 and Obj_4. Extending the analysis to all time horizons, we notice that in some cases (e.g. $H = 180$ and $\Delta_d = 50$ minutes) there is a difference between the two values, due to the fact that these objective functions does not take into account the exact dealy of trains. Thus, any solution showing the same number of delayed trains is optimal, whatever the value of Obj_1 and Obj_2. Multicriteria objective functions should be used to obtain an unique optimal solution. The minimization of the number of delayed trains (respectively movements) may imply an increase of the total delay of rescheduled trains.

Extending the analysis to values obtained in all time horizons (H) and for all disturbance size (Δ_d) we notice that minimizing Obj_1 provides better results in terms of Obj_1 and Obj_2. Minimizing Obj_4 provides better values in terms of Obj_3 and Obj_4.

4.1.3 Impact of Constraints (22), (23) and (24)

In order to prove the necessity of constraints (22), (23) and (24) when using Obj_3 and Obj_4, we present a simple railway line made by 7 segments on which circulate 5 trains, represented in Figure 5. Railway line is made by 4 single-tracked connection segment (b_1,b_3,b_5 and b_7) between 3 double-tracked stations (b_2, b_4 and b_6). Trains t_1 and t_3 are directed to the South, while t_2, t_4 and t_6 travel in the opposite direction.

Table 1: Comparison between actual and previous methodology.

	Obj_1	Obj_3	CT
Our Methodology	608	3	0.72
Dotoli et al.	665	10	10.74

Table 4: Two independent disturbances.

	FirstDisturbance					*SecondDisturbance*					*TwoDisturbances*					*Sum*				
	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT
Obj_1	516	61	1	9	13.43	308	61	3	26	17.98	824	122	4	35	31.41	824	122	4	36	17.95
Obj_2	516	61	1	9	11.95	308	61	3	26	17.63	824	122	4	35	29.58	826	122	4	36	19.83
Obj_3	516	61	1	9	9.08	336	71	1	5	9.81	852	132	2	14	18.89	871	132	2	14	9.16
Obj_4	516	61	1	9	10.42	336	71	1	5	12.21	852	132	2	14	22.63	871	132	2	14	10.12

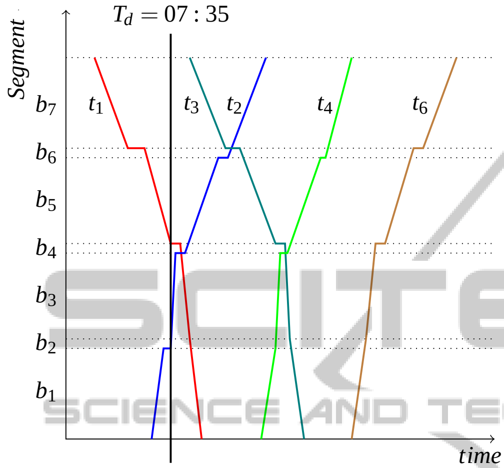

Figure 5: Disturbance of t_1.

Table 2: Comparison between the four objective functions with $H = 1440$ and $\Delta_d = 50$.

	Obj_1	Obj_2	Obj_3	Obj_4
Obj_1	829	854	878	878
Obj_2	81	81	91	91
Obj_3	3	3	1	1
Obj_4	33	39	11	11
CT	30.07	16.86	6.56	26.18

Table 3: Results from experiments using Obj_3 without and with additional constraints.

	Obj_1	Obj_2	Obj_3	Obj_4	CT
WT	10040	10000	1	5	0.01
W	50	10	1	5	0.01

A disturbance occurs in the segment b_5 at 07:35 am and affects the train t_1. We analyze a problem with time horizon $H = 25$ minutes and a disturbance size $\Delta_d = 10$ minutes. The same scenarios have been solved without (WT) and with (W) the additional constraints using Obj_3. Results are presented in Table 3.

By analyzing values of Obj_1 and Obj_2 we notice that despite the number of rescheduled trains is unchanged, without the additional constraints the departure or the duration of some movements is delayed for a long time. This means that minimizing Obj_3 tends to postpone movements of trains involved in the rescheduling process at the end of the time horizon, in order to affect the lowest number of trains on the line.

In the example, t_1 is the only delayed train. By introducing additional constraints, values of Obj_1 (and consequently Obj_2) are lower because the system is forced to reschedule movements within the time horizon. The number of rescheduled trains remains unchanged.

4.2 Two Disturbances on the Line

When two independent disturbances occur on the railway line, the rescheduling process of the first disturbance does not influence the rescheduling process of the second one. We consider the same railway line presented in Section 4 and we suppose that two independent disturbances occur at $T_d = 09:00$ a.m. respectively in the line that connects Rutigliano to Conversano stations (segment b_9) and Conversano to Castellana G. (segment b_{11}). We consider a time horizon $H = 1440$ minutes and a disturbance size $\Delta_d = 50$ minutes.

First, we solve the problem considering only the disturbance that occurs in the segment b_9 and that affects a train directed from Putignano to Mungivacca station.

Then, we solve the problem considering only the disturbance that occurs in the segment b_{11} and that affects a train directed from Mungivacca to Putignano station.

Finally, we solve the problem considering the two disturbances at the same time. We compare results with those given by the sum of values obtained solving the two problems separately.

Table 4 presents values of Obj_1, Obj_2, Obj_3, Obj_4 and CT for the four scenarios.

By comparing values obtained from simultaneous resolution with those obtained from the sum of individual resolutions of disturbances, i.e. values presented in the third and the fourth block of Table 4, we notice that the simultaneous resolution provides a better result in terms of computational time for all objective functions. However, the the sum of individual resolutions provides an improvement in terms of Obj_1 and Obj_4 using the four objective functions. In particular by applying Obj_1, there is a reduction of the number of rescheduled movements and by applying Obj_2, there is also a reduction of total delay. By

Table 5: Results of analysis using Obj_1, Obj_2, Obj_3 and Obj_4 for all H and Δ_d.

H	Tot.		PrSlv		Obj	$\Delta_d = 10$					$\Delta_d = 15$					$\Delta_d = 20$					$\Delta_d = 25$					$\Delta_d = 30$					$\Delta_d = 35$					$\Delta_d = 40$					$\Delta_d = 45$					$\Delta_d = 50$				
	#V	#C	#V	#C		Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT	Obj_1	Obj_2	Obj_3	Obj_4	CT
30	6316	6555	1957	1897	1	70	10	1	7	0.04	105	15	1	7	0.04	140	20	1	7	0.03	175	25	1	7	0.06	210	30	1	7	0.03	245	35	1	7	0.04	280	40	1	7	0.03	315	45	1	7	0.03	350	50	1	7	0.10
30	6316	6555	1172	1897	2	70	10	1	7	0.05	105	15	1	7	0.05	140	20	1	10	0.08	175	25	1	10	0.01	210	30	1	10	0.05	245	35	1	10	0.03	280	40	1	10	0.16	315	45	1	10	0.03	350	50	1	10	0.06
30	6316	7047	1960	1897	3	70	10	1	7	0.14	105	15	1	7	0.47	140	20	1	7	0.05	175	25	1	7	0.05	210	30	1	7	0.03	245	35	1	7	0.05	280	40	1	7	0.05	315	45	1	7	0.05	350	50	1	7	0.05
30	6472	7359	1972	1921	4	70	10	1	7	0.06	105	15	1	7	0.03	140	20	1	7	0.05	175	25	1	7	0.03	210	30	1	7	0.05	245	35	1	7	0.05	280	40	1	7	0.03	315	45	1	7	0.05	350	50	1	7	0.20
60	7636	8190	2675	2818	1	110	10	1	11	0.07	211	23	3	25	0.09	296	31	2	19	0.09	367	36	2	21	0.09	428	43	2	13	0.08	433	43	2	13	0.07	450	45	2	13	0.09	515	55	2	13	0.06	580	65	2	13	0.06
60	7636	8190	2675	2818	2	110	10	1	11	0.06	211	23	3	19	0.08	300	31	2	21	0.09	367	38	3	21	0.12	428	43	2	21	0.18	437	43	2	23	0.09	450	45	2	23	0.09	470	55	2	26	0.09	610	56	1	26	0.08
60	7636	8758	2678	2814	3	110	10	1	11	0.06	304	38	2	19	0.09	300	31	2	19	0.08	531	61	2	13	0.08	667	75	2	19	0.08	595	56	1	11	0.06	600	56	1	11	0.06	605	56	1	11	0.08	610	56	1	11	0.09
60	7816	9118	2690	2831	4	110	10	1	11	0.07	304	38	2	19	0.12	502	69	2	13	0.09	567	79	2	13	0.08	428	43	2	13	0.11	595	56	1	11	0.08	600	56	1	11	0.08	605	56	1	11	0.09	610	56	1	11	0.08
90	8920	10074	3401	3995	1	110	10	1	11	0.12	266	31	2	21	0.30	310	31	2	21	0.20	431	44	6	44	0.23	512	51	5	34	0.21	517	55	5	34	0.20	554	58	4	32	0.30	697	76	3	28	0.28	751	69	1	11	0.45
90	8920	10074	3401	3995	2	110	10	1	11	0.12	289	31	2	21	0.12	310	31	2	23	0.05	431	44	6	23	0.90	540	55	4	23	0.20	517	55	5	34	0.16	572	58	5	34	0.22	752	69	1	35	0.23	751	69	1	35	0.20
90	8920	10712	3380	3937	3	110	10	1	11	0.06	913	83	1	11	0.17	891	83	1	11	0.18	913	83	1	11	0.18	891	83	1	11	0.18	892	83	1	11	0.17	913	83	1	11	0.12	878	83	1	11	0.18	913	83	1	11	0.20
90	9122	11916	3436	4848	4	110	10	1	11	0.08	913	83	1	11	0.20	891	83	1	11	0.22	913	83	1	11	0.20	913	83	1	11	0.23	892	83	1	11	0.22	891	83	1	11	0.22	751	69	1	11	0.18	913	83	1	11	0.23
120	10519	12275	4345	5762	1	110	10	1	11	0.20	266	31	2	21	0.42	310	31	2	21	0.28	477	46	7	67	0.51	586	68	5	39	0.36	591	68	5	39	0.36	616	69	5	39	0.56	753	82	5	45	0.73	814	80	2	18	0.88
120	10519	12275	4345	5762	2	110	10	1	11	0.17	289	31	2	21	0.50	310	31	2	23	0.37	477	46	7	23	0.40	604	59	6	23	0.45	609	59	6	35	0.40	672	65	6	35	0.55	839	80	2	36	1.18	838	80	2	39	0.62
120	10519	12996	4192	5290	3	110	10	1	11	0.14	864	82	2	18	0.45	406	47	2	18	0.40	864	82	2	18	0.40	857	82	2	18	0.40	864	83	2	18	0.28	847	80	2	18	0.28	878	82	2	18	0.30	830	81	2	18	0.26
120	10748	13454	4279	5467	4	110	10	1	11	0.18	855	82	2	18	0.44	864	83	2	18	0.72	856	82	2	18	0.50	833	80	2	18	0.50	863	82	2	18	0.36	861	82	2	18	0.33	863	82	2	18	0.30	830	81	2	18	0.31
180	13051	16644	5925	9048	1	110	10	1	11	0.3	266	31	2	21	0.95	310	31	2	21	0.50	523	48	8	90	0.83	608	76	3	29	0.72	613	76	3	29	0.62	638	77	3	29	1.17	829	81	3	33	0.51	829	81	3	33	1.40
180	13051	16644	5925	9048	2	110	10	1	11	0.31	267	31	2	21	1.12	310	31	2	23	0.60	523	48	8	23	0.73	714	65	8	23	0.68	719	65	8	35	0.80	825	73	8	35	1.30	853	81	3	36	1.68	853	81	3	39	0.99
180	13051	17485	5916	9008	3	110	10	1	11	0.17	875	91	1	11	0.64	875	91	1	11	0.65	875	91	1	11	0.64	875	91	1	11	0.61	875	91	1	11	0.61	875	91	1	11	0.67	875	91	1	11	0.64	948	91	1	11	0.48
180	13318	18019	6037	9249	4	110	10	1	11	0.20	875	91	1	11	0.70	875	91	1	11	0.72	875	91	1	11	0.75	875	91	1	11	0.81	875	91	1	11	0.69	875	91	1	11	0.80	875	91	1	11	0.75	875	91	1	11	0.72
240	18149	21612	7721	13225	1	110	10	1	11	0.31	266	31	2	21	3.01	310	31	2	21	0.98	523	48	8	90	1.87	608	76	3	29	1.99	613	76	3	29	1.64	638	77	3	29	2.20	829	81	3	33	8.16	829	81	3	33	8.35
240	15804	43004	5376	24173	2	110	10	1	11	0.30	267	31	2	21	2.24	314	31	2	23	1.46	523	48	8	23	1.31	716	65	8	23	1.56	721	65	8	35	1.50	820	73	8	35	2.11	855	81	3	36	2.84	853	81	3	39	1.98
240	15804	22574	7684	13117	3	110	10	1	11	0.31	875	91	1	11	0.70	875	91	1	11	0.78	875	91	1	11	0.15	875	91	1	11	0.84	875	91	1	11	0.78	875	91	1	11	0.90	875	91	1	11	0.81	875	91	1	11	0.96
240	16110	23186	7844	13436	4	110	10	1	11	0.36	875	91	1	11	0.95	875	91	1	11	0.90	875	91	1	11	0.95	875	91	1	11	0.90	875	91	1	11	1.01	875	91	1	11	0.99	875	91	1	11	0.90	875	91	1	11	1.01
300	17528	25234	8889	16141	1	110	10	1	11	0.57	266	31	2	21	3.91	310	31	2	21	2.31	523	48	8	90	2.88	608	76	3	29	2.77	613	76	3	29	2.56	638	77	3	29	2.96	829	81	3	33	7.05	829	81	3	33	5.04
300	17528	25234	8889	16141	2	110	10	1	11	0.37	289	31	2	21	2.04	318	31	2	26	1.23	523	48	8	23	1.90	716	65	8	23	2.42	721	65	8	35	1.95	820	73	8	35	2.43	854	81	3	36	3.62	853	81	3	39	2.01
300	17528	26266	8892	16115	3	110	10	1	11	0.40	875	91	1	11	1.01	875	91	1	11	0.89	875	91	1	11	0.98	875	91	1	11	0.94	875	91	1	11	1.01	875	91	1	11	1.07	875	91	1	11	0.93	875	91	1	11	1.06
300	17856	26922	9074	16478	4	110	10	1	11	0.44	875	91	1	11	1.25	875	91	1	11	1.17	875	91	1	11	1.20	875	91	1	11	1.34	875	91	1	11	1.25	875	91	1	11	1.23	875	91	1	11	1.12	875	91	1	11	1.22
360	20240	30791	10762	20933	1	110	10	1	11	0.78	266	31	2	21	3.04	310	31	2	21	1.77	523	48	8	90	3.44	608	76	3	29	3.33	613	76	3	29	2.85	638	77	3	29	4.29	829	81	3	33	12.48	829	81	3	33	5.55
360	20240	30791	10762	20933	2	110	10	1	11	0.51	319	31	2	21	3.20	324	31	2	23	2.07	523	48	8	23	2.48	716	65	8	23	2.92	721	65	8	35	2.84	820	73	8	36	4.47	855	81	3	36	4.37	854	81	3	39	4.70
360	20240	31929	10575	20491	3	110	10	1	11	0.87	875	91	1	11	1.34	875	91	1	11	1.22	875	91	1	11	1.30	875	91	1	11	1.18	875	91	1	11	1.38	875	91	1	11	1.26	875	91	1	11	1.34	875	91	1	11	1.32
360	20602	32653	10791	20922	4	110	10	1	11	1.12	875	91	1	11	1.48	875	91	1	11	1.57	875	91	1	11	1.51	875	91	1	11	1.48	875	91	1	11	1.64	875	91	1	11	1.46	875	91	1	11	1.40	875	91	1	11	1.64
420	23226	37541	12889	26619	1	110	10	1	11	0.98	266	31	2	21	3.52	310	31	2	21	2.29	523	48	8	90	5.66	608	76	3	29	4.76	613	76	3	29	3.66	638	77	3	29	6.50	829	81	3	33	19.40	829	81	3	33	10.28
420	23226	37541	12889	26619	2	110	10	1	11	0.68	289	31	2	21	3.80	314	31	2	23	3.20	523	48	8	23	3.43	716	65	2	23	3.57	721	65	8	35	3.40	840	73	8	35	5.27	853	81	3	36	6.32	853	81	3	39	4.15
420	23226	38787	12892	26585	3	110	10	1	11	0.69	875	91	1	11	1.73	875	91	1	11	1.62	875	91	1	11	1.68	875	91	1	11	1.64	875	91	1	11	1.73	875	91	1	11	1.80	875	91	1	11	1.65	875	91	1	11	1.81
420	23622	39579	13142	27084	4	110	10	1	11	0.76	875	91	1	11	2.03	875	91	1	11	2.06	875	91	1	11	2.07	875	91	1	11	2.12	875	91	1	11	2.07	875	91	1	11	1.60	875	91	1	11	1.88	875	91	1	11	2.12
480	26152	44235	14868	32237	1	110	10	1	11	1.74	266	31	2	21	5.52	310	31	2	21	2.71	523	48	8	90	5.21	608	76	3	29	4.66	613	76	3	29	4.62	638	77	3	29	8.70	829	81	3	33	31.04	829	81	3	33	9.70
480	26152	44235	14868	32237	2	110	10	1	11	1.36	267	31	2	21	5.63	324	31	2	23	4.08	523	48	8	23	4.93	716	65	8	23	4.85	721	65	8	35	4.77	820	73	8	35	7.63	853	81	3	36	8.05	853	81	3	39	6.47
480	26152	45601	14729	31903	3	110	10	1	11	1.35	876	91	1	11	2.54	876	91	1	11	2.21	876	91	1	11	2.18	876	91	1	11	2.18	876	91	1	11	2.14	876	91	1	11	2.18	876	91	1	11	2.26	876	91	1	11	2.21
480	26586	46469	15017	32478	4	110	10	1	11	1.57	876	91	1	11	2.40	876	91	1	11	2.37	876	91	1	11	2.43	876	91	1	11	2.37	876	91	1	11	2.40	876	91	1	11	2.30	876	91	1	11	2.42	876	91	1	11	2.42
540	29808	53122	17727	40361	1	110	10	1	11	1.66	266	31	2	21	12.53	310	31	2	21	4.30	523	48	8	90	7.72	608	76	3	29	6.75	613	76	3	29	8.83	638	77	3	29	12.96	829	81	3	33	30.15	829	81	3	33	18.48
540	29808	53122	17727	40361	2	110	10	1	11	1.72	267	31	2	21	7.58	314	31	2	24	5.55	523	48	8	24	2.14	716	65	8	24	5.60	721	65	8	36	6.16	820	73	8	36	9.36	853	81	3	36	10.98	853	81	3	39	7.59
540	29808	54588	17314	39441	3	110	10	1	11	1.74	875	91	1	11	2.71	875	91	1	11	2.80	875	91	1	11	2.73	875	91	1	11	2.93	875	91	1	11	2.70	875	91	1	11	2.79	875	91	1	11	3.85	875	91	1	11	2.87
540	30274	55520	17634	40080	4	110	10	1	11	1.87	875	91	1	11	2.87	875	91	1	11	2.99	875	91	1	11	2.92	875	91	1	11	3.00	875	91	1	11	2.94	875	91	1	11	3.32	875	91	1	11	3.07	875	91	1	11	3.20
600	33335	61819	20398	48293	1	110	10	1	11	2.07	266	31	2	21	12.87	310	31	2	21	4.60	523	48	8	90	10.12	608	76	3	29	8.52	613	76	3	29	8.13	638	77	3	29	10.15	829	81	3	33	38.12	829	81	3	33	15.33
600	33335	61819	20398	48293	2	110	10	1	11	2.15	267	31	2	21	8.92	321	31	2	24	7.84	523	48	8	24	6.86	716	65	8	24	6.58	721	65	8	36	7.06	834	73	8	36	10.75	867	81	3	36	12.45	853	81	3	39	9.41
600	33335	63394	20131	47675	3	110	10	1	11	2.23	877	91	1	11	3.26	877	91	1	11	3.66	878	91	1	11	3.54	877	91	1	11	3.43	877	91	1	11	3.60	877	91	1	11	3.45	877	91	1	11	3.54	877	91	1	11	3.62
600	33836	64396	20486	48384	4	110	10	1	11	2.12	877	91	1	11	3.46	877	91	1	11	3.54	878	91	1	11	3.66	877	91	1	11	3.60	877	91	1	11	3.62	877	91	1	11	3.70	877	91	1	11	3.52	877	91	1	11	3.73
1440	48247	101160	32059	84305	1	110	10	1	11	3.85	266	31	2	21	22.40	310	31	2	21	9.17	523	48	8	90	19.81	608	76	3	29	17.08	613	76	3	29	16.55	638	77	3	29	20.23	829	81	3	33	69.77	829	81	3	33	30.07
1440	48247	101160	32059	84305	2	110	10	1	11	3.96	266	31	2	21	21.60	332	31	2	23	17.94	523	48	8	24	14.43	716	65	8	24	14.24	721	65	8	36	14.22	820	73	8	34	22.37	853	81	3	36	24.27	854	81	3	39	16.86
1440	48247	103146	31766	83615	3	110	10	1	11	4.40	878	91	1	11	6.60	878	91	1	11	7.08	878	91	1	11	6.32	878	91	1	11	6.58	878	91	1	11	6.97	878	91	1	11	6.95	878	91	1	11	8.47	878	91	1	11	6.56
1440	48879	104428	32252	84586	4	110	10	1	11	2.99	878	91	1	11	5.63	878	91	1	11	5.27	878	91	1	11	6.64	878	91	1	11	9.64	878	91	1	11	14.21	878	91	1	11	14.28	878	91	1	11	11.23	878	91	1	11	26.18

using Obj_3 and Obj_4, there is a decrease of values of Obj_1. However, it is interesting to note that when multiple independent disturbances occur on the line, it is possible to decompose the problem in independent subproblems. In this way, the train dispatcher can give priority to the rescheduling of trains which paths include stations where a higher level of service is required or that have to comply connections with other trains. One could expect that two disturbances would be more difficult to solve but, according to the first experiments, this is not the case. More particularly, the time needed to solve the first disturbance is greater than the time needed to solve both, perhaps due to number of embedded variables. More explicit, we are in progress to verify that.

5 CONCLUSIONS AND PERSPECTIVES

In this paper, we propose a formalization of the rescheduling real-time problem for a regional single-tracked railway network in which a CTC control system is installed. We propose a mathematical model that operates as a decision support system for the train dispatcher. The main goal is to find a decision support system for the train dispatcher that is able to restore normal traffic conditions after the occurrence of a disturbance and to provide an adequate level of service to passengers. We analyze four alternative objective functions in order to find the optimal solution that is a good compromise between total delay, number of rescheduled trains and computational time.

There are many perspectives for this work:

- increase the complexity of the analysis, considering a greater number of disturbances on the line that occur at different times and have different size.
- introduce robustness in the rescheduling process. A robust rescheduled timetable is less subject to change if a new disturbance occurs on the railway line.
- perform a structural analysis of the railway line in order to verify if there are independent sectors in which it is possible to predetermine an optimal solution to applied when a disturbance occours.
- introduce indicators of the complexity of the problem in order to assess the sensibility of the computational time with these parameters.
- include a resolution strategy that allow the cancellation of a train when the delay that it would accumulate along the line exceeds a certain threshold.
- study other resolution methods most suitable to the complexity of the problem, such as constraints programming, able toproduce a setof possible solutions.

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