Fuzzy Set Theoretical Analysis of Human Membership Values on the Color Triangle

Mapping from the Color Triangle (Antecedent) via the Color Triangle (Consequent) to the Tone Triangle

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Keywords: Fuzzy Set Theory, Three Additive Primary Colors, Membership Function, RGB System, Color Triangle, Vague Color, Membership Value, Center of Gravity, Tone Triangle.

Abstract: The present study considers a fuzzy color system in which three membership functions are constructed on a color triangle. This system can process a fuzzy input to a color triangle system and output the center of gravity of three weights associated with respective grades. Three fuzzy sets (red, green, and blue) are applied to the color triangle relationship. By treating the attributes of redness, greenness, and blueness on the color triangle, a target color can be easily obtained as the center of gravity of the output fuzzy set. In the present paper, 0% triangle is consisted of the lines of 0% redness, 0% greenness, and 0% blueness of the attributes. The colors on 0% triangle map into the right corner of tone triangle (on C or near C). As compare the inference results for fuzzy inputs with those for crisp inputs, move to W (white region). The input-output relationship is shown by redness and chromaticness. The inference outputs for crisp inputs and for fuzzy inputs are obviously different. Those for crisp inputs show vertically linear.

1 INTRODUCTION

Using the additive method of color mixing as reported in previous study (Sugano, 2007), the relationship between fuzzy sets on the color triangle and fuzzy conical membership functions is examined. A color triangle (planar region) represents the hue and saturation of various colors (Tilley, 1999). The six fundamental colors and white can be represented on a color triangle (see Fig. 1*a*). Vague colors on the color triangle and the chromaticity diagram are also clarified.

A recent study (Sugano, 2011) reported a technique that used a fuzzy set theoretical method and an additive color mixing method to obtain expressions for the tone triangle in the red-green-blue (RGB) system. The relationship between two or three fuzzy sets on the tone triangle (antecedent) in Fig. 1*b* and the conical fuzzy inputs was examined. The six fundamental colors and white can be represented on the color triangle (consequent) in Fig. 1*a*.

In the unpublished study, we reexamine a technique for obtaining expressions of the color triangle in the RGB system. The output of this system can be represented on a tone triangle, which clarifies colors that were vague on the color triangle. In the proposed system, the average color value is determined as the center of gravity of the attribute information for vague colors.

The motivation of this study is to better understand human-computer interaction with human subjectivity. The specific objective of this paper is to determine how fuzzy inputs (as the human membership values) are mapped from the color triangle (antecedent) via the color triangle (consequent) to the tone triangle. The applications for which this fuzzy set theoretical approach is useful include vague color information processing and color identification.

2 METHODS

2.1 Color Triangle and Additive Color Mixture

Additive color mixing occurs when two or three

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beams of differently colored light combine.

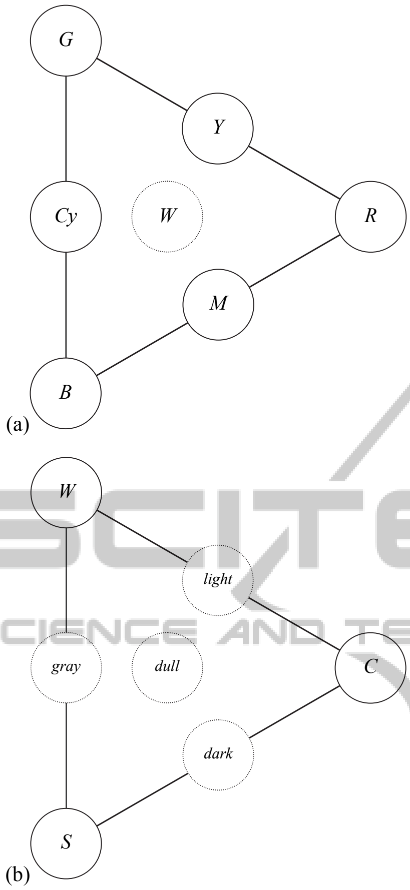

Figure 1: (*a*) Color triangle. A point in the plane of the triangle system represents the hue and saturation of a color. *Cy* is cyan. (*b*) Tone triangle. A point in the plane of the triangle system represents the lightness and saturation of a color. *C* is the maximal color of each hue. *S* is black.

It has been found that mixing just three additive primary colors (red, green, and blue) can produce the majority of colors. In general, a color vector can be described by a scalar and a direction. These quantities are referred to as the tristimulus values, *R* for the red component, *G* for the green component, and *B* for the blue component, and are given as follows:

$$\vec{C} = \vec{R} + \vec{G} + \vec{B} \tag{1}$$

This is referred to as the RGB color model (Fig. 2) and allows colors to be represented by a planar diagram. The RGB color model can be used to identify colors as the red, green, and blue components (*R*, *G*, *B*) corresponding to the three axes of color space, as shown in Fig. 2. The coordinates (*r*, *g*, *b*) on the color triangle can specify one of a range of colors. These coordinates correspond to the relative amounts of *R*, *G*, and *B* that make up the color. For example, the coordinates specifying the center of the color triangle represent the case in which the three primary colors are mixed in equal proportion and thus indicate the color white. Such representations are referred to as chromaticity diagrams. A chromaticity diagram represents hue and saturation, but not lightness (Tilley, 1999). On the color triangle (the dotted area in Fig. 2) (Valberg, 2005), the percentages of redness, greenness, and blueness, where the total of the three attributes is equivalent to 100%, specify a color. In order to indicate only the direction of a color vector, i.e., the chromaticity, the redness *r*, greenness *g*, and blueness *b* are obtained as follows:

$$r = \frac{R}{R+G+B} \tag{2}$$

$$g = \frac{G}{R+G+B} \tag{3}$$

$$b = \frac{B}{R+G+B} \tag{4}$$

$$r + g + b = 1 \tag{5}$$

In other words, the direction is shown as the ratio of the tristimulus values *R*, *G*, and *B*. The total of these ratios is equal to 1, as shown in Eq. (5).

In Fig. 2, at red *R*, the components are (*R*, *G*, *B*) = (1, 0, 0) and the coordinates are (*r*, *g*, *b*) = (1, 0, 0). At red, green, and blue, the components are (*R*, *G*, *B*) = (*r*, *g*, *b*); however, at yellow, for instance, the components are (*R*, *G*, *B*) = (1, 1, 0) but the coordinates are (*r*, *g*, *b*) = (0.5, 0.5, 0). Colors on the three squares WMRY, WYGCy, and WCyBM in Fig. 3*a* (color space) correspond to those on the three diamonds WMRY, WYGCy, and WCyBM in Fig. 1*a* (color triangle). Thus, all of the colors on the upper three squares in a color space can be displayed in a color triangle.

2.2 Color Triangle and Tone Triangle Designs

Next we consider the color triangle and the tone triangle in RGB color space (Fig. 3). In the color space, the color triangle is fixed, as shown in Fig. 3*a*; for instance, when the hue of a color is red *R*, the tone triangle is fixed, as shown in Fig. 3*b*.

In Fig. 1*a*, *Cy* is cyan, midway between blue and green (No. 6: cyan *Cy* in Fig. 4*a*). In Fig. 1*b*, *C* is the maximal color (each hue at the maximum chromaticness) (Sivik, 1997) (No. 66: red *R* in Fig.

4*b*) as a hue.

The color triangle in Fig. 3*a* is an equilateral triangle, whereas if the hue is red *R*, the tone triangle is a right triangle. When the hue of a color is orange, i.e., midway between yellow and red, the tone triangle is nearly an isosceles triangle. By shaping another triangle into an equilateral triangle, it is possible to normalize the equilateral coordinates in the fuzzy system.

The present study considers a system of the three primary colors (red, green, and blue: RGB), presented on a color triangle. When the hue of a color (e.g., red, green, or blue) is fixed on the color triangle in Fig. 1*a*, the color exists on the tone triangle in Fig. 1*b*. The colors *C*, *W*, and *S* in Fig. 1*b* are the maximal color (Sivik, 1997), white, and black, respectively, and dark (or deep), light (or pale), and dull are modifiers.

The color triangle and tone triangle in Fig. 4 correspond to the schematic diagrams shown in Fig. 1.

The colors *B*, *Cy*, *G*, *Y*, *R*, *M*, and *W* in Figs. 1*a*, 2, and 3*a* are No. 1: blue, No. 6: cyan, No. 11: green, No. 51: yellow, No. 66: red, No. 46: magenta, and No. 104: white in Fig. 4*a*. White (No. 104) is surrounded by six neighboring colors, as shown in the detail inset, and these seven colors (Nos. 101–107) are surrounded by No. 34, No. 35, and No. 42. The details of the extension of the fundamental type from 66 colors (excluding white) to 496 colors (including white) are examined in Appendix (figure).

The color names or modifiers in Fig. 1*b* are No. 1: black, No. 6: gray, No. 11: white, No. 46: dark (or deep), No. 51: light (or pale), and No. 66: the maximal color (Sivik, 1997) (e.g., vivid red) in Fig. 4*b*.

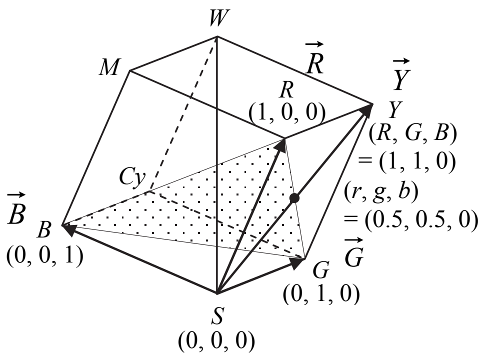

Figure 2: Color triangle and additive color mixture. The origin *S* is black.

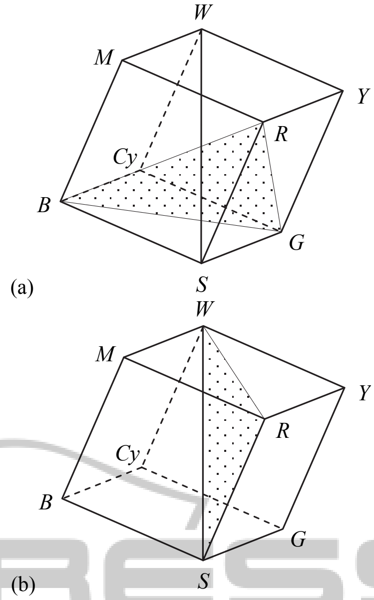

Figure 3: (*a*) Color triangle and (*b*) tone triangle, in the same color space. *S* is black. *Cy* is cyan.

2.3 Fuzzy Rules

Figure 5 illustrates a fuzzy input set, fuzzy input on the color triangle (antecedent), a crisp output set, fuzzy output on the color triangle (consequent), and crisp output on the graphic plane. The fuzzy rules are as follows (see Fig. 5):

$$R^k : if\ U\ is\ A_k\ then\ V\ is\ O_k \tag{6}$$

where k is the rule number (k = 1, 2, 3) corresponding to the components of *R*, *G*, and *B*, A_k is a fuzzy set of inputs, O_k is a crisp set of outputs, $U = (r_i, g_i, b_i)$ is the input parameter (variable), and $V = (r_o, g_o, b_o)$ is the output parameter. Here, *U* and *V* are fixed to these RGB parameters. A fuzzy set A_k of inputs shows a triangular pyramid-like shape with a plateau at the corner points R_i, G_i, and B_i, and a crisp set O_k of outputs of rule R^k is shown at corner points R_o, G_o, or B_o (a fuzzy set O_k', indicated by vertical arrows in Fig. 5*b*) on the color triangle, and the output is O_k if the input is A_k.

The fuzzy inference method is as follows. Let the inputs be $r_i = r_i$', $g_i = g_i$', and $b_i = b_i$'.

1) In the input of rule R^k, grade $\alpha_k = A_k(U')$, where k = 1, 2, 3.
2) The output of rule R^k, a crisp output set, is shown as a vertical post.
3) $O_k' = \alpha_k O_k$, where O_k' is a fuzzy set (vertical arrow), and O_k is a crisp set (vertical post) in Fig. 5*b*.

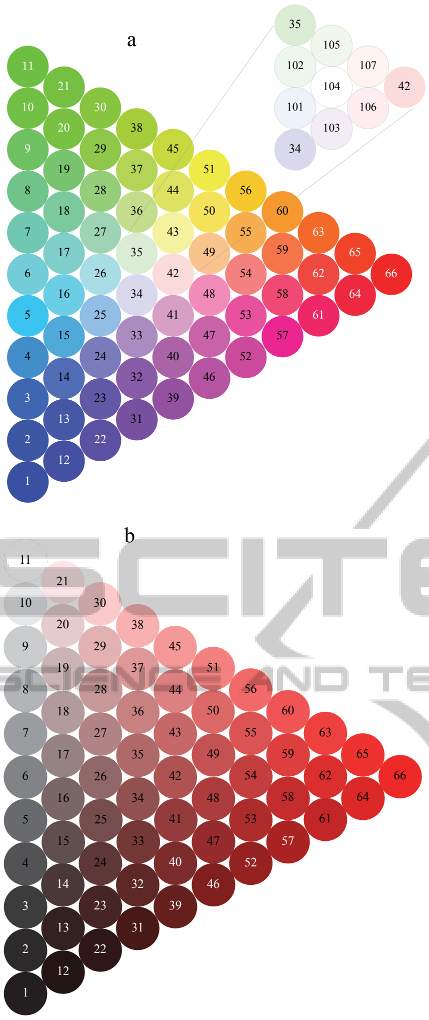

Figure 4: (*a*) Sixty-six crisp color inputs and white, with six neighboring colors (detail) on the color triangle. (*b*) Sixty-six crisp color inputs on the tone triangle.

The complete inference is the results *O*' of rules R^1, R^2, and R^3:

$$O' = \alpha_1 O_1 \cup \alpha_2 O_2 \cup \alpha_3 O_3 = O_1' \cup O_2' \cup O_3' \quad (7)$$

The output parameter, *V*' = (r_o', g_o', b_o'), is equivalent to the coordinates of the center of gravity of the output fuzzy set of *O*'. In addition, in Fig. 5*c*, *V*' = (r_o', *uk*') corresponds to the coordinates of the graphical system, where *uk*' (on the vertical axis) is calculated from g_o' and b_o', and *uk*' shows the value (as the distance from *B*) on the line *B*-*G*.

A fuzzy input set A_1 of *red* can be characterized by the following membership function:

$$\mu_1(r_i, uk) = r_i s; \quad r_i < \frac{1}{s} \quad (8)$$

$$\mu_1(r_i, uk) = 1; \quad r_i \geq \frac{1}{s} \quad (9)$$

where *s* is the slope of projection, and *s* ranges from 0.02 to 0.03.

The membership functions of *green* and *blue* are described by similar equations.

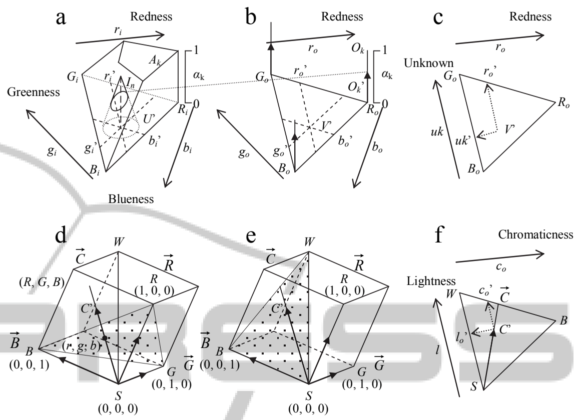

Figure 5: Fuzzy system using the membership function of fuzzy sets A_j (red) with (*a*) conical fuzzy input I_n on the color triangle (antecedent), (*b*) output crisp sets O_k, three weights (vertical arrows) on the color triangle (consequent), and (*c*) a color coordinate on the graphic plane, (*d*) color vectors and a color triangle, (*e*) a compound vector with a tone triangle in the color space, and (*f*) a tone coordinate on the graphic plane.

2.4 Extraction of Tone Triangles

Three weights as grades, α_1, α_2, and α_3 (vertical arrows) in Fig. 5*b*, are scalar values in the RGB components. A color vector $\vec{C}$ (a compound vector with these values) can be drawn in the color spaces in Fig. 5*d* (and 5*e*). The intersection of color vector $\vec{C}$ and color triangle (dotted area) in 5*d*, is equal to *V*' in 5*b*. In 5*e*, the tone triangle (dotted area) includes the color vector $\vec{C}$. This triangle CWS is normalized to equilateral coordinates. In this case, the maximal color *C* (Fig. 1*b*) is *B* (blue) in 5*f*. The coordinate *C*' (α_1, α_2, α_3) of vector $\vec{C}$ in 5*d* is equal to *C*' (c_o', l_o') in 5*f*.

2.5 Fuzzy Sets

The membership values μ_k (r_i', g_i', b_i') of the fuzzy input set A_k on the color triangle; μ_k (r_i', g_i', b_i') is equal to μ_k (r_i', *uk*'). The membership function μ_k is based on the values (RGB components) of the seven colors (*R*, *Y*, *G*, *C*, *B*, *M*, and *W*).

In Fig. 6, the shape of the fuzzy set is shown by including W_i (white). The top of the plateau is shown to have a diamond-like shape.

The intersection of a fuzzy input set A_k with a fuzzy input I_n is $A_k \cap I_n$. Grade α_k' = height ($A_k \cap I_n$). If the input is crisp, α_k' becomes α_k. R_o is the new red output. Proj(O_k) is the projection of an output crisp set at the corner point R_o (see Fig. 5*b*).

The system considered in the present study can translate the input data U of a vague color to output data V of a simple color on the color triangle (consequent). The fuzzy input is transformed using the center U' = (r_i', g_i', b_i').

Table 1: Number of subjects in the experiment. Semantic differential method is used. 66 is fundamental type.

Type	No. of subjects	Male	Female	Age
66	86	47	39	10-70

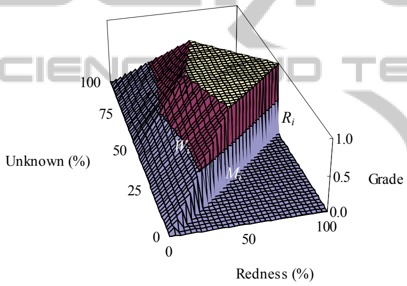

Figure 6: Membership function μ_1 (r_i, uk) of fuzzy set A_1 (red) on the color triangle. The basal triangle corresponds to Fig. 1*a*.

3 EXPERIMENTAL METHODS

For the experiment, 86 (in Table 1) undergraduate students, graduate students, and participants in a university festival volunteered to participate as subjects for this study. The subjects sat in a chair and were requested to watch a display continuously.

Using the ensemble average of the fuzzy sets obtained from the experiment result then, the normalized membership values of subjects are computed (Sugano and Chiba, 2007).

The experiments were performed in an isolated area in order to restrict visual cues with regard to the display.

In this study, using a graphical user interface (GUI) for the questionnaire, 86 subjects compared the differences between a target color (e.g. red) and neighboring colors of 65 colors (Fig. 4*a*) using semantic differential (SD) method for color words.

4 RESULTS AND DISCUSSION

Figures 7-12 show the ensemble average of the experimental results for six colors (red, green, blue, cyan, yellow, magenta) on the coordinates (r_i', uk') in fundamental type (66 colors) using SD method.

Figure 13 illustrates a relationship between the redness value r_i and the unknown value uk. Filled circles indicate the centers of gravity for fuzzy inputs. A center (average) of fuzzy input is shown as a trend. Open circles indicate crisp inputs of colors (as target colors). Target color means the center of cone as vagueness in Figs. 5*a*. The differences between target colors (open circles) and the outputs (filled circles) are not so large.

Figure 14 illustrates a relationship between the redness value r_o and the unknown value uk. In *a*, filled circles indicate the inference outputs for crisp inputs as the centers of gravity of fuzzy inputs. Open circles also indicate crisp inputs of colors (as target colors). In *b*, filled circles indicate the inference outputs for fuzzy inputs of colors, open circles also indicate crisp inputs of colors (as target colors). The outputs (filled circles) for fuzzy inputs are grouped at the center of the color triangle. The open and filled circles are clearly different in this case. The differences between target colors (open circles) and the outputs (filled circles) are so large. The differences in *a* are smaller than those in *b*. The experimental results for GUI are also shown in Sugano and collaborators (2009).

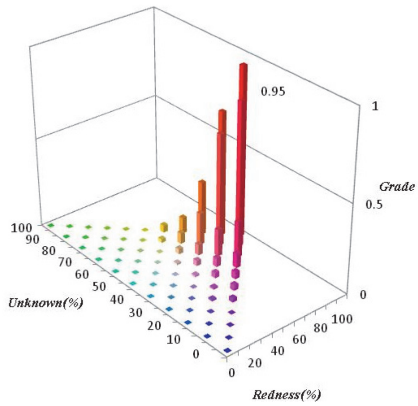

Figure 7: Membership values of red-relevant colors as fuzzy input.

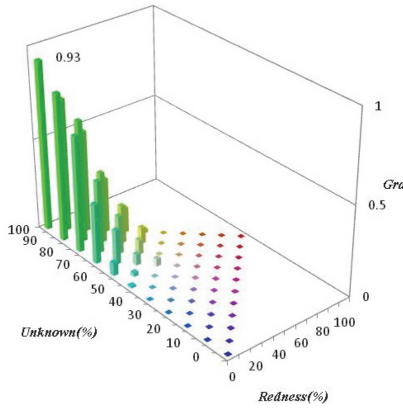

Figure 8: Membership values of green-relevant colors as fuzzy input.

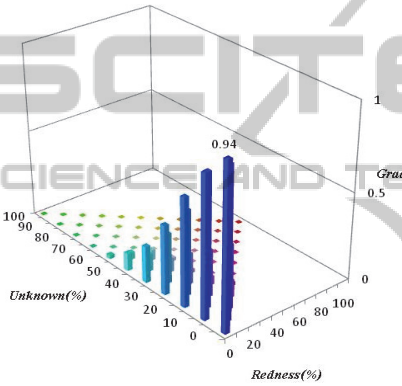

Figure 9: Membership values of blue-relevant colors as fuzzy input.

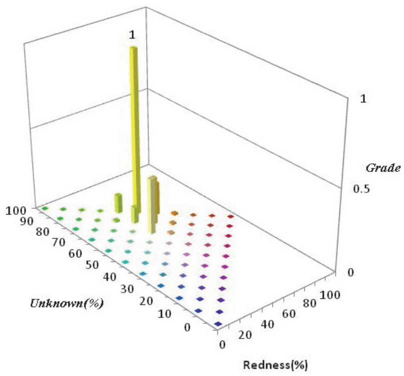

Figure 10: Membership values of yellow-relevant colors as fuzzy input.

Vague color inputs to the fuzzy system, the system outputs crisp color in the RGB, and also outputs crisp color on the graphical plane (Fig. 14). These inference results for fuzzy 496 colors (Sugano et al., 2009) and fuzzy 66 colors (Sugano and Shinomiya, 2010) are similar. These inference results for partition method (Sugano and Chiba, 2007) and semantic differential method are also similar.

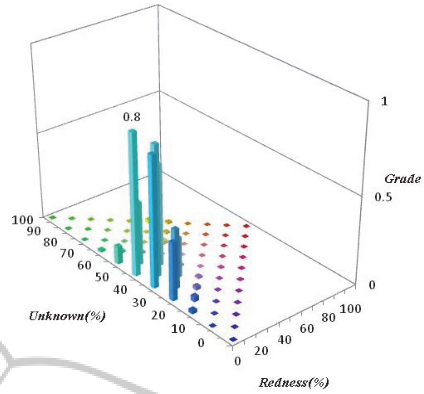

Figure 11: Membership values of cyan-relevant colors as fuzzy input.

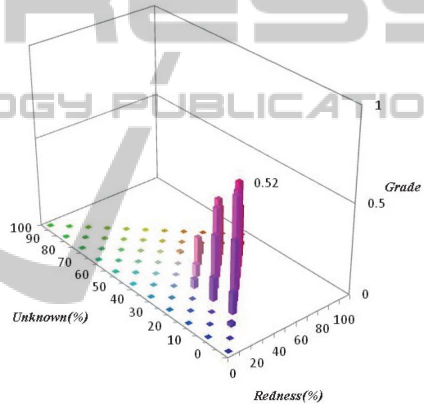

Figure 12: Membership values of magenta-relevant colors as fuzzy input.

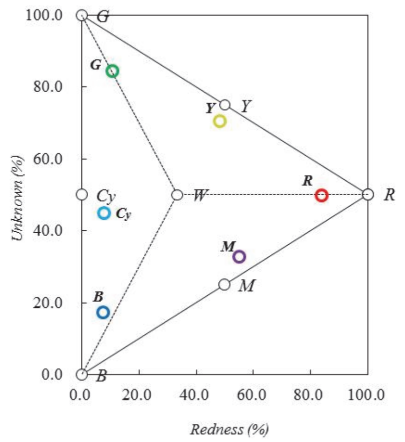

Figure 13: Centers of gravity of fuzzy inputs (filled circles). *W* is white (open circle).

Figure 15*a* illustrates the relationship between the lightness l_o and the chromaticness c_o obtained from data (c_o’, l_o’). The points (open circles) indicate the outputs for crisp inputs of colors. The inference outputs for crisp inputs are grouped at the upper side of the tone triangle. This effect is caused by the fuzzy inference output.

Figure 15*b* also illustrates the relationship between the lightness l_o and the chromaticness c_o obtained from data (c_o’, l_o’). The points (filled circles) indicate the outputs for fuzzy color input. The inference outputs for fuzzy inputs are also grouped at the upper side of the tone triangle. The inference outputs for crisp inputs in Fig. 15*a* are different from those for fuzzy inputs in Fig. 15*b*.

For example, the output is far from the corner *C* of the triangle (Fig. 15*b*). That is, vagueness increases. However, the input-output (three attributes-chromaticness) relationship cannot be seen in Fig. 15.

If the height of the human membership values (the indicated possibility) in Figs. 7-12 has grade less than 1.0, the points in Fig. 15 move to the inside of the tone triangles.

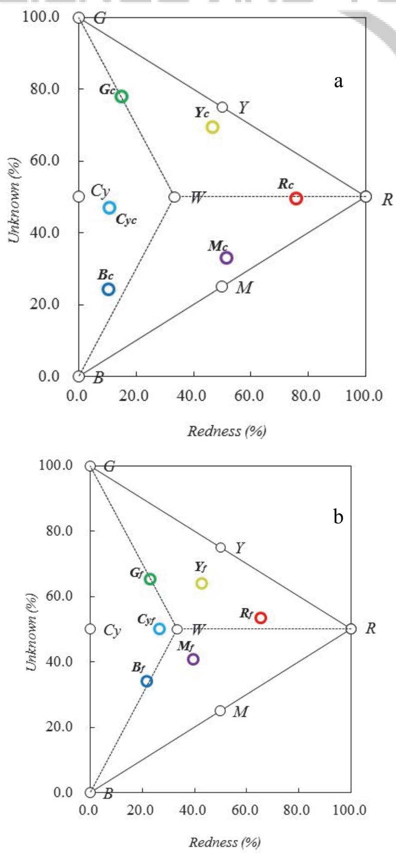

Figure 14: Inference outputs for (*a*) crisp inputs and (*b*) fuzzy inputs, on the graphic plane (Fig. 5*c*). Suffix *f* shows fuzzy inference output. *W* (open circles) as a landmark exists in the coordinate (33.3%, 50.0%).

That is, the points that are far away from corner *C* in Fig. 15 are also far away from the upper side of a tone triangle. As a result, vagueness increases but the possibility decreases. This can be explained in terms of Fig. 5*f*. As shown, the arrow (as a compound vector) is inside the tone triangle and does not reach to the upper side, but if the vagueness decreases to the minimum and the possibility increases to the maximum, then the arrow will reach to the corner *B* at the maximum chromaticness. Alternately, if vagueness increases to the maximum, then the arrow reaches to the corner *W*. On the other hand, if the possibility decreases to the minimum, then the arrow disappears into the corner *S*. However, the complete relationship between vagueness and possibility is not considered in this study.

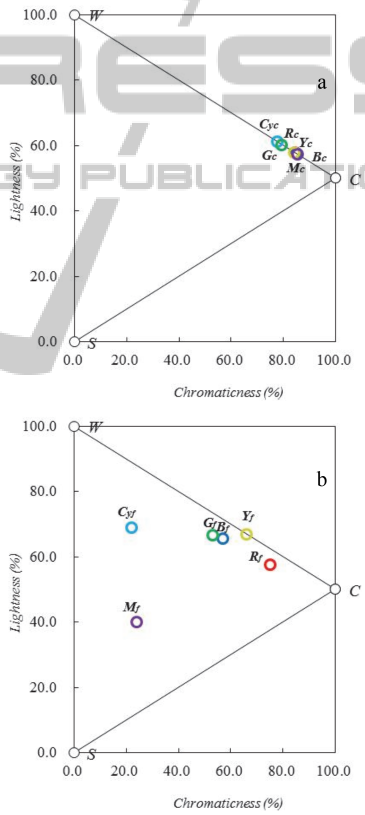

Figure 15: Inference outputs for (*a*) crisp inputs and (*b*) fuzzy inputs. Suffix *f* shows fuzzy inference output.

5 CONCLUSIONS

The present paper examined how vagueness is presented on the color triangle using semantic differential methods and performed fuzzy set theoretical analysis. The subjects are asked the difference between fundamental color (as a target color) and neighboring colors (as a sample color) using semantic differential method. Each data and the ensemble average of those data are fuzzy sets. The results of experiments show a similar trend to that for the tone triangle (Sugano et al., 2008). Using the fuzzy inference for RGB data (as a fuzzy set), it is found that these results move to white direction as a center of color triangle. On the tone triangle, the inference results of the cyan-relevant colors show large vagueness and these of the magenta-relevant colors show low possibility.

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