

# Activities Planning and Resource Assignment on Multi-place Hospital System

## *Exact and Approach Methods Adapted from the Bin Packing Problem*

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**Abstract:** In France, the Hospital Community of Territory (HCT) has been defined since the settlement of the pricing by activity (T2A) in 2004, and the new governance. This new community allows the pooling of the hospital resources between any places in the same territory. It aims at increasing the continuity of healthcare. A medical exam needs human resources and one material resource, both of them compatible with the exam. Human resources are supposed available all the time. In this paper, material resources are the critical resources. The objective is to create a decision support tool which plans the exams on each material resource. Some constraints have to be respected: compatibilities between exams and resources, resources opentime. We propose a mathematical model. It deals with an exams planning taking into account the assignment of material resources in the community over a given horizon planning. The problem is solved in an exact way and with approach methods applying bin packing heuristics and single solution based metaheuristics. Experimentation tackles with randomly generated instances.

## 1 INTRODUCTION

Nowadays, the main objective of hospital system is to improve its efficiency. Indeed, some hospitals can have patients waiting a few months or more to get an appointment. On the other hand, other hospitals have almost no waiting time. These two kinds of places can be very close geographically speaking. In France, a new organization was created a few years ago, the Hospital Community of Territory (HCT). It is a group of distinct places which aim at improving their efficiency by putting means from different places together. The idea developed in this paper is that by dispatching medical exams on several places, the global efficiency of all the places will be improved.

After the specification of our problem, the formulation of the mathematical model is detailed. This problem can be compared to the bin packing problem. This comparison is explained in the state of the art. This problem is first solved in an exact way, but because it is NP-hard, approach methods are also considered by applying some single solution based metaheuristics (kangaroo algorithm and simulated annealing algorithm) combined to a bin packing heuristic. The last section compares the results of the exact method to those of the approach methods. We will

conclude with some further works.

## 2 SPECIFICATION

First, the considered problem is detailed in a generic way and then the analogy with the field of medical imaging is done.

### 2.1 Problem Statement

The problem studied in this paper can be compared to a tasks assignment problem over a multi-place system.

Each task needs a resource to be treated and a time slot, a period when it will be done. We make the hypothesis that human resources are available all the time, only the material resources are discussed, they are the critical resources. Each task has a known process time. A task has to be completed before a due date. Each task has got a reference place where it has to be done, if possible.

The system is composed of several places. There is a known distance between these places. In each place, there are one to several resources. Every re-

sources cannot treat all the tasks, there is a list of compatibilities between tasks and resources. Resources are from one type. Resources have a planning defining their available time, we call it resources opentime.

The considered horizon planning is known. This horizon is divided into periods. Each task starts in one period and finishes in the same period.

The aim is to assign each task to one resource and one period, respecting the compatibilities and the different times. Tasks have to be done as soon as possible, so the assigned periods have to be the smallest possible.

The model has to respect some constraints which are the following:

- Resources have to be able to process their assigned tasks (resource compatibility).
- Tasks have to be assigned to a resource during the opentime of this resource.
- A task has to be assigned to one resource and one period.

## 2.2 Objective

The aim of this model is not to make a precise schedule of the tasks but to attribute a period to each task.

The criteria that can be studied are the following:

- The completion time of the last planned task. It is the biggest period assigned to the tasks. All the tasks are done before or during this period. By misnomer, we call it in the following makespan.
- The sum of the completion times of all the tasks planned in the system, that is to say the sum of each period assigned to all the tasks.
- The number of tasks done before their due date.
- The number of tasks done at their reference place, or the distance between the reference place and the effective one for each task.

The criteria which are the most relevant are about the economic aspect of the problem. The earlier the tasks will be planned, the earlier they will be completed. So resources will be available earlier to practice the next tasks. With the two first criteria, we can assume that the tasks will be done as soon as possible.

## 2.3 Medical Imaging Example

For the hospital system, a task is an exam. The considered system is the HCT, composed of several places. In each place, there are one to several material resources. A material resource is from one type: MRI, x-ray, or scanner. Each material resource has a planning with its available time per period. For example,

we can assume that an MRI engine is only available ninety minutes on Monday morning because it needs a maintenance operation or because an external physician reserved it. Its opentime will span over ninety minutes during this period. The considered horizon planning is divided into periods. Each period can represent one half-day, one day or one week, the horizon planning can represent one week, one month or one year.

Depending on the used horizon planning, the described model can be used at three levels:

- Strategic: to determine the system dimensional, the number of places needed in the system, how many hospitals are part of the HCT.
- Tactical: to determine the number of resources needed in each place, how many material resources of each type (MRI, x-ray, scanner, etc.) are needed in each hospital.
- Operational: to plan exams and assign them one resource and one period.

In this paper, we are dealing with instances in which the horizon planning is not greater than one week, we are focusing on the operational level.

In the following, we will use the word exam and not task.

## 3 STATE OF THE ART: BIN PACKING

The bin packing problem considers  $N$  objects and some bins. The aim is to put all the objects in bins by minimizing the number of bins, with respect of the size of objects and bins (Garey and Johnson, 1979). It is a NP-hard problem and has been widely studied (Beigel and Fu, 2012).

Our problem consists in the assignment of exams to a material resource and a period. If we consider that the horizon planning is composed of couples  $(l, s)$  with material resource  $l \in (1, L)$  and period  $s \in (1, T)$ , the aim of the problem is to assign exams to these couples  $(l, s)$ . Exams have to be planned as soon as possible, the aim is to minimize the number of different couples  $(l, s)$ , that is to say to minimize the number of bins. Table 1 summarizes the links between the bin packing problem and the problem described in this paper.

Several heuristics have been developed to solve this problem (Coffman Jr et al., 1996).

- The typical one is *NextFit*. Objects are put in the current bin. If the current bin is full, objects are put in the next one. We cannot come back to the current bin anymore.

Table 1: Analogies between the bin packing and the considered problem.

| Bin packing problem                    | Considered problem                                  |
|--|---|
| Object                                 | Exam  |
| Bin                                    | Couple $(l, s)$                                     |
| Size of an object                      | Process time  |
| $\emptyset$                            | Due date  |
| $\emptyset$                            | Reference place                                     |
| To assign objects in bins              | To assign exams to a period and a material resource |
| Constraint of the capacity of the bins | Constraint of material resources opentime           |
| $\emptyset$                            | Compatibility constraint                            |
| To minimize the number of used bins    | To minimize the number of used couples $(l, s)$     |
| $\emptyset$                            | To minimize the last assigned period                |
| $\emptyset$                            | To minimize the sum of the number of bins           |

- *FirstFit* is different from *NextFit* because objects are put in the first available bin. A new bin is taken into account if and only if all the previous bins do not have enough space to accept the current object. Bins are not definitively closed. If a smaller object fits in a previous bin, it will be put in this one. In the worst case, the quadratic complexity in time of this algorithm is  $O(N \log(N))$ , with  $N$  the number of objects (Corcoran and Wainwright, 1995).

- *BestFit* consists in putting objects in the best bin which has some available space. The best bin is the one which will have the least available space after this object will be put in it.

*FirstFit* heuristic is adapted to our problem. This will be explained in Section 5.2.

## 4 MATHEMATICAL MODEL

The used data in this problem are the following. Let be:

- $N$  the number of exams to plan.
- $K$  the number of distinct places.
- $L$  the number of available material resources.
- $T$  the horizon planning. The planning will be done for  $s(s = 1, T)$  periods.
- $t_i(i = 1, N)$  the process time of exam  $i$ .
- $d_i(i = 1, N)$  the due date of exam  $i$ .
- $r_{i,k}(i = 1, N$  and  $k = 1, K) = 1$  if place  $k$  is the reference place of exam  $i$ , 0 otherwise.

- $c_{i,l}(i = 1, N$  and  $l = 1, L) = 1$  if exam  $i$  can be handled by material resource  $l$ , 0 otherwise.
- $s_{l,k}(l = 1, L$  and  $k = 1, K) = 1$  if material resource  $l$  is located on place  $k$ , 0 otherwise.
- $p_{l,s}(l = 1, L$  and  $s = 1, T)$  the opentime of material resource  $l$  during period  $s$ .

The considered variables are the following:

- $x_{i,l,s}(i = 1, N; l = 1, L$  and  $s = 1, T) = 1$  if exam  $i$  is assigned to material resource  $l$  during period  $s$ , 0 otherwise.
- $C_i$  is the period assigned to exam  $i$ .

The constraints are written as follows:

- Material resources opentimes have to be respected (1).

$$\sum_{i=1}^N t_i \cdot x_{i,l,s} \leq p_{l,s}, \quad \forall l \in \{1, L\}, \forall s \in \{1, T\} \quad (1)$$

- An exam is assigned to one material resource and one period (2).

$$\sum_{l=1}^L \sum_{s=1}^T x_{i,l,s} = 1, \quad \forall i \in \{1, N\} \quad (2)$$

- An exam is assigned to a compatible material resource (3).

$$\sum_{l=1}^L (1 - c_{i,l}) \sum_{s=1}^T x_{i,l,s} = 0, \quad \forall i \in \{1, N\} \quad (3)$$

- (4) defines  $C_i$ , which is the period assigned to exam  $i$ .

$$C_i = \sum_{s=1}^T \sum_{l=1}^L s \cdot x_{i,l,s}, \quad \forall i \in \{1, N\} \quad (4)$$

- (5) and (6) integrity and non negativity constraints.

$$x_{i,l,s} \in \{0, 1\}, \forall i \in \{1, N\}, \forall l \in \{1, L\}, \forall s \in \{1, T\} \quad (5)$$

$$C_i \geq 0, \quad \forall i \in \{1, N\} \quad (6)$$

Four criteria can be studied:

- The sum of the completion times, i.e. of the assigned periods to the planned exams (7).

$$H_1 = \sum_{i=1}^N C_i \quad (7)$$

- The makespan, i.e. the period of the last planned exam (8).

$$H_2 = \max_i C_i \quad (8)$$

- The number of exams done after their due date (9).

$$H_3 = N - \sum_{i=1}^N \sum_{l=1}^L \sum_{s=1}^{d_i} x_{i,l,s} \quad (9)$$

- The number of exams performed out of their reference place (10).

$$H_4 = N - \sum_{i=1}^N \sum_{k=1}^K \sum_{l=1}^L \sum_{s=1}^T x_{i,l,s} \cdot s_{l,k} \cdot r_{i,k} \quad (10)$$

In the following, we are taking into consideration only the economics aspects, that is to say to assign the maximum number of exams during the smaller number of periods and material resources. The studied objective function is the minimization of the first two criteria.

## 5 APPROACH METHOD

### 5.1 Coding of the Set of Solutions

A solution  $X$  is the assignment of exam  $i$  to material resource  $l$  during period  $s$ . It is coded by a three-dimensional matrix, each dimension defines each set of subscript. The dimension of the matrix is  $N * L * T$ .

$$X = (x_{i,l,s}), \quad \forall i \in \{1, N\}, l \in \{1, L\}, s \in \{1, T\}$$

Let be  $\Omega$  the set of all the solutions. Let be  $C \in \Omega$  the set of admissible solutions.

Size of  $\Omega$  is given in (11).

$$\text{card}(\Omega) = L^{N^T} \quad (11)$$

For each solution  $X$ , all the exams are assigned to one period and one material resource (2). If the material resources opentimes (1) or the compatibilities between exams and material resources (3) are not respected, the solution is called non admissible solution.

### 5.2 Initial Solution

Two ways are used to compute an initial solution.

An admissible initial solution can be built by a heuristic based on bin packing: *FirstFit* heuristic. It is described in Algorithm 1. The maximum number of exams is assigned to the first material resource during the first period. The exam is assigned to the material resource if they are compatible. When there is no free time anymore on this material resource during this period, exams are assigned during the same

period to the second material resource, then the third one, etc. When there is no free time anymore on all the material resources during the first period, exams are assigned to the second period in the same way.

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**Algorithm 1:** Heuristic for an initial solution.

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**Require:**

$temp_{l,s} = 0, \forall l \in \{1, L\}, \forall s \in \{1, T\}$

**begin**

**for** each exam  $i = 1, N$  **do**

$OK = false, l = 1, s = 1$

**while**  $s \leq T$  **and**  $OK = false$  **do**

**while**  $l \leq L$  **and**  $OK = false$  **do**

**if** exam  $i$  is compatible with resource  $l$  **then**

**if**  $p_{l,s} - temp_{l,s} \geq t_i$  **then**

$x_{i,l,s} = 1$

$temp_{l,s} = temp_{l,s} + t_i$

$OK = true$

**end if**

**end if**

$l = l + 1$

**end while**

$s = s + 1$

**end while**

**end for**

**end**

---

We can also randomly build an initial solution. It consists in assigning a random material resource to each exam during a random period. In this case, the initial solution is admissible or not, compatibilities between exams and material resources and resources opentimes are not taken into account.

### 5.3 Metaheuristics

The used metaheuristics are single solution based. Two stochastic algorithms are detailed and used: simulated annealing and kangaroo algorithm (iterated local search). Under some hypotheses, inhomogeneous algorithm for simulated annealing and kangaroo algorithm converge in probability to an optimal solution (Aarts and van Laarhoven, 1987) and (Fleury, 1993). Principle algorithms are given, using the following writing:

- $X$  current solution,
- $Y$  candidate solution (or neighbour),
- $V, W$  two neighbourhood systems,
- $H$  cost function.

### 5.3.1 Simulated Annealing

Originally, the inhomogeneous simulated annealing was used by Metropolis (Metropolis et al., 1953) to simulate the physical annealing in metallurgy. Simulated annealing converges in probability to the set of optimal solutions if neighbourhood system  $V$  satisfies the accessibility and reversibility property. The algorithm of simulated annealing is described in Algorithm 2.

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**Algorithm 2:** Simulated Annealing.

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**Require:** temperature  $T_0$ , initial solution  $X$ , record solution  $RX := X$ , decreasing factor  $\alpha$ , maximum number of iterations IterMax

**begin**

iter := 0,  $T := T_0$

**while** iter < IterMax **do**

    Choose randomly and uniformly  $Y \in V(X)$

**if**  $H(Y) < H(RX)$  **then**

$RX := Y$

**end if**

**if**  $H(Y) \leq H(X)$  **then**

$X := Y$

**else**

$X := Y$  with probability  $e^{-\frac{H(Y)-H(X)}{T}}$

**end if**

    iter = iter + 1

    Generate a new temperature  $T = \alpha * T$

**end while**

**end**

---

Two parameters have to be chosen: the initial temperature and the decreasing factor. The initial temperature  $T_0$  is chosen so as all the transitions are authorized at the beginning i.e.  $e^{-\frac{H(Y)-H(X)}{T_0}} \simeq 1, \forall (X, Y)$ . (Aarts and van Laarhoven, 1987) proposes an algorithm to compute the initial temperature.

The used formula to compute the decreasing factor  $\alpha$  is defined in Equation (12).  $T_a$  is the latest temperature computed in the simulated annealing, close to 0.

$$\alpha = \frac{\text{IterMax} \sqrt{\frac{T_a}{T_0}}}{T_0} \quad (12)$$

### 5.3.2 Kangaroo Algorithm

Kangaroo algorithm consists in a stochastic descent, but if there is no improvement of the current solution after a number  $A$  of iterations, a jump is made. To make this jump, a solution is chosen in a neighbourhood system  $W$  different from  $V$ . Kangaroo algorithm

converges in probability to the set of optimal solutions if neighbourhood system  $W$  satisfies the accessibility property. Algorithm 3 describes kangaroo algorithm.

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**Algorithm 3:** Kangaroo Algorithm.

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**Require:** number of iterations  $A$  before a jump, initial solution  $X$ , record solution  $RX := X$ , maximum number of iterations IterMax

**begin**

iter := 0,  $c := 0$

**while** iter < IterMax **do**

**if**  $c < A$  **then**

        Choose randomly and uniformly  $Y \in V(X)$

**if**  $H(Y) \leq H(X)$  **then**

**if**  $H(Y) < H(X)$  **then**

$c := 0$

**if**  $H(Y) < H(RX)$  **then**

$RX := Y$

**end if**

**end if**

$X := Y$

**end if**

**else**

        Choose randomly and uniformly  $Y \in W(X)$

**if**  $H(Y) \neq H(X)$  **then**

**if**  $H(Y) < H(RX)$  **then**

$RX := Y$

**end if**

$c := 0$

**end if**

$X := Y$

**end if**

$c := c + 1$

    iter = iter + 1

**end while**

**end**

---

The used formula to compute  $A$  is given in (13) (Fleury, 1993).

$$A \geq \text{card}(V) \cdot \ln(2) \quad (13)$$

### 5.3.3 Neighbourhood System

The used neighbourhood system  $V$  is defined in Algorithm 4. An exam  $i$  is randomly chosen. Its current assignment is deleted. A material resource is randomly chosen among the compatible ones:  $l \in \{1, L\}$  such as  $c_{i,l} = 1$ . A period is randomly chosen:  $s \in \{1, T\}$ . Exam  $i$  is assigned to material resource  $l$  and period  $s$ . Equation (14) gives the size of  $V$ .

$$\text{card}(V) \leq N \cdot L \cdot T \quad (14)$$

**Algorithm 4:** Neighbourhood System.

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```

Choose randomly and uniformly  $i \in \{1, N\}$ 
begin
for  $l$  and  $s$  such as  $x_{i,l,s} = 1$  do
     $x_{i,l,s} = 0$ 
end for
Choose randomly and uniformly  $l \in \{1, L\}$  such as
 $l$  is compatible with  $i$ 
Choose randomly and uniformly  $s \in \{1, T\}$ 
 $x_{i,l,s} = 1$ 
end

```

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In the kangaroo algorithm, a second neighbourhood system  $W$  is used. Its principle is to choose randomly and uniformly an exam and to assign it to another period and another material resource randomly and uniformly chosen.  $W$  is used eight times each time it is called.  $W$  satisfies the accessibility and reversibility property.

### 5.3.4 Studied Criteria

According to the mathematical model, two constraints have to be respected:

- Material resources opentimes (1),
- Compatibility between assigned material resource and the considered exam (3).

To check these constraints, overruns constraints are computed. So a new criterion  $Cont$  is defined. It is equal to the sum of differences between the sum of the process times of the assigned exams to a material resource during period and the opentime resource over this period, plus the number of exams which are assigned to non compatible material resources. It is computed thanks to the constraints (1) and (3). If  $Cont$  is equal to 0, all the constraints are respected.

$$Cont(X) = \sum_{l,s} \max(0, \sum_i t_i * x_{i,l,s} - pl_{l,s}) + \sum_{i,l,s} x_{i,l,s} * (1 - c_{i,l}) \quad (15)$$

A weight sum of the three criteria,  $H_1, H_2$  and  $Cont$  can be used. The general form of the objective function is given in Equation (16), with  $\alpha_1, \alpha_2, \alpha_3$  the weight coefficient.

$$H(X) = \alpha_1.H_1(X) + \alpha_2.H_2(X) + \alpha_3.Cont(X) \quad (16)$$

## 6 EXPERIMENTS AND RESULTS

The HCT is composed of three places. There are four or eight material resources, each one located on one place. The horizon planning is composed of several periods, each period represents one half-day. The aim is to plan  $N$  exams in the HCT, that is to say to assign one period and one compatible material resource to each exam. Exams have to be planned during material resources opentimes. The objective is to plan all the exams at the beginning of the horizon planning, the last exam must be done over the smallest period.

### 6.1 Dataset

The data are randomly generated but the size of the data represents real instances. Two kinds of data are created:

- For instances 50A to 500A, each process time is between 5 and 45, with 5 minutes steps. Each material resource opentime is equal to 300 minutes.
- For instances 50B to 500B, each process time is between 1 and 100, and each material resources opentime is equal to 100.

Table 2 summarizes the data.

### 6.2 Exact Method

To solve this model, we use the version 12.4.0 of CPLEX. The host machine is powered by an Intel Xeon X5687 quad-core CPU running at 3.6 GHz. For each size of problem, results about makespans and sums of the different periods assigned to each exam are summarized in Table 3. Computation times are given in seconds.

The computations which are stopped after thirty minutes are written with a \*. In this case, results about lower and upper bounds are given. The bigger the problem is, the longer the computation times are.

$H_1$  is the sum of the completion times, of the assigned periods to all the planned exams in the HCT.  $H_2$  is the makespan, the period of the last planned exam. The used objective function is given in Equation (17).  $\alpha_2$  equals 10.000 because in all the instances,  $H_1$  is always smaller than 10.000. Thus, each criterion is easily readable.

$$H(X) = H_1(X) + 10.000.H_2(X) \quad (17)$$

For the smallest size of problem, fifty exams have to be planned. The makespan is equal to two, that means that at least two periods are used to plan all of them. The available time in one period is equal to

Table 2: HCT Data.

| Exams $N$     | 50A | 50B | 100A | 100B | 200A | 200B | 300A | 400A | 500A | 500B |
|---------------|-----|-----|------|------|------|------|------|------|------|------|
| Resources $L$ | 4   | 4   | 4    | 4    | 8    | 8    | 8    | 8    | 8    | 8    |
| Periods $T$   | 8   | 8   | 8    | 15   | 10   | 20   | 15   | 20   | 25   | 40   |

Table 3: Results with the exact method.

| Exams       | 50A  | 50B*  | 100A | 100B*    | 200A | 200B*   | 300A | 400A*   | 500A*   | 500B*    |
|-------------|------|-------|------|----------|------|---------|------|---------|---------|----------|
| $H_1$       | 51   | -     | 128  | -        | 262  | -       | 486  | -       | -       | -        |
| $H_2$       | 2    | -     | 2    | -        | 2    | -       | 3    | -       | -       | -        |
| Time        | 0.22 | -     | 0.16 | -        | 0.98 | -       | 4.41 | -       | -       | -        |
| Lower bound | -    | 70135 | -    | 120483.1 | -    | 91056.3 | -    | 40774.2 | 51114.8 | 162329.9 |
| Upper bound | -    | 70137 | -    | 120520   | -    | 141074  | -    | 40776   | 51118   | 327231   |

material resources opentime during this period, three hundreds minutes in this case. So one period is more or less equal to half-a-day. At least, one day is used to plan fifty exams over the four material resources in the HCT.

### 6.3 Approach Methods

Results of the approach methods are compared to the exact method's ones. Two cases are possible:

- $[X \in \Omega]$ : we can accept non admissible solutions. Initial solution is randomly and uniformly generated. The criteria  $Cont$  is used to compute overruns constraints. In the objective function,  $\alpha_3$  is set to  $10^6$ . Thus  $Cont$  is the first criterion to minimize. At the beginning of the computation, it is greater than zero but at the end, it is always equal to zero. So the best solution will be admissible.
- $[X \in C]$ : we only accept admissible solutions. Initial solution is created by the proposed heuristic. Results about initial solution are given in table 4. The generated neighbour has to be admissible, that is to say  $Cont = 0$ .

Table 4 summarizes results about the makespan ( $H_1$ ) and the sum of the assigned periods to each exam ( $H_2$ ).

For each replication, the number of iterations is equal to 1.000.000. The gold standard are the results of the exact method given by CPLEX in the previous section. CPLEX optimal solution is written if found. If not, upper bounds are written because they are the best found integer solutions (\*). Mostly, methaheuristics find the optimal makespan.

For big instances (\*\*), the makespan is not the same in approach and exact methods. In these cases,  $H_2$  in the approach way can be better than in the exact one.

Moreover, the heuristic can find a better makespan than the metaheuristics for instances B. In these instances, size of a "bin" is equal to 100 and some ex-

ams have got a process time which is equal to 100. With the heuristic, exams are sorted by decreasing processing times. Thus, the biggest exams are processed in the first periods. With the metaheuristics, once a big exam is put in one of the last period, it cannot be moved to another "bin" otherwise it will not respect resources opentimes. But every time, metaheuristics give better results than the heuristic about  $H_2$ . Most of the times, simulated annealing gives the best results.

Results about  $H_2$  are optimal if the number of exams is small. For large instances, there is no optimal solution. The exact method gives an approximation after thirty minutes of computation. With the approach methods, optimal makespan, if existing, is found in some seconds and a good result of  $H_2$  in some minutes. Almost all the exams are assigned to the optimal period, only some of them are assigned during the next period of the optimal one.

## 7 CONCLUSIONS

In the current economic context, solutions have to be found to improve hospital efficiency. By dividing up exams over an Hospital Community of Territory, exams could be done more quickly. In a same horizon planning, more exams could be planned.

In this paper, a formulation of the problem is given. Exact and approach methods are compared. For large instances, exact method does not return the optimal solution while approach method find a good solution in some minutes. Optimal makespan, if existing, is found in a few seconds and a good result for all assignments is found in a few minutes. Other heuristics and other neighbourhood systems have to be tested to improve the best solution.

At the operational level, in this paper, assignment are made over seven days. In real life, planner needs to anticipate the planning one month in advance. By

Table 4: Results with the metaheuristics.

| Exams                               | 50A | 50B* | 100A | 100B** | 200A | 200B* | 300A | 400A | 500A | 500B** |
|-------------------------------------|-----|------|------|--------|------|-------|------|------|------|--------|
| CPLEX (upper bound)                 |     |      |      |        |      |       |      |      |      |        |
| $H_1$                               | 2   | 7    | 2    | 12     | 2    | 14    | 3    | 4    | 5    | 32     |
| $H_2$                               | 51  | 137  | 128  | 520    | 262  | 1074  | 486  | 776  | 1118 | 7231   |
| Solution generated by the heuristic |     |      |      |        |      |       |      |      |      |        |
| $H_1$                               | 2   | 7    | 3    | 12     | 3    | 14    | 4    | 4    | 6    | 31     |
| $H_2$                               | 54  | 203  | 142  | 684    | 295  | 1435  | 579  | 957  | 1408 | 8228   |
| Kangaroo algorithm, $x \in \Omega$  |     |      |      |        |      |       |      |      |      |        |
| $H_1$                               | 2   | 7    | 2    | 14     | 2    | 16    | 3    | 4    | 5    | 36     |
| $H_2$                               | 51  | 154  | 129  | 554    | 266  | 1220  | 504  | 815  | 1185 | 6520   |
| Kangaroo algorithm, $x \in C$       |     |      |      |        |      |       |      |      |      |        |
| $H_1$                               | 2   | 7    | 2    | 13     | 2    | 15    | 3    | 4    | 5    | 33     |
| $H_2$                               | 51  | 140  | 128  | 508    | 264  | 1126  | 496  | 800  | 1161 | 6465   |
| Simulated annealing, $x \in \Omega$ |     |      |      |        |      |       |      |      |      |        |
| $H_1$                               | 2   | 7    | 2    | 13     | 2    | 15    | 3    | 4    | 5    | 35     |
| $H_2$                               | 51  | 138  | 128  | 503    | 263  | 1154  | 491  | 796  | 1164 | 6326   |
| Simulated annealing, $x \in C$      |     |      |      |        |      |       |      |      |      |        |
| $H_1$                               | 2   | 7    | 2    | 13     | 2    | 15    | 3    | 4    | 5    | 34     |
| $H_2$                               | 51  | 139  | 128  | 488    | 262  | 1127  | 494  | 799  | 1172 | 6228   |

increasing the number of periods, or by increasing the size of the periods, we could apply this study to the tactical level or the strategic one. This could be used to determine the number of resources or places needed in a HCT.

The made hypothesis here is that human resources are always available. In further research, we could include them to our model. Human resources can work or not overtime and they can be allowed to move over the HCT. So the load between the places can be balanced.

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