2D-0D Direct Capture of Carriers in Quantum Dot Lasers under Optical Feedback

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Abstract: In this paper the operation of quantum dot lasers under optical feedback is studied on the basis of calculated Lyapunov exponents of the dynamic multi-population rate-equations. Influence of the direct capture path for the wetting layer carriers on the sensitivity to initial conditions is discussed. Results show that positive exponents are achieved for different current injection scenarios, and that negative exponents are mainly due to carriers in the dot confined states. Furthermore, existence of hiperchaos is obtained with both cascade and direct capture models.

1 INTRODUCTION

Semiconductor quantum dot (QD) lasers have been intensively studied in the last years because of their potential compared to quantum well and bulk lasers, as well as due to the particular properties associated with the 3D confinement in the quantum dots, such as the high differential gain which should lead to reduced linewidth enhancement factor (LEF) and low chirp. Essentially, the LEF or alpha-factor is a parameter which measures the coupling between real (phase) and immaginary (gain) parts of the complex refractive index of the active material; any fluctuation of phase or amplitude of the laser field (due to spontaneous emission or even reflected back light, for instance) induces relaxation oscillations, changing the immaginary component of the refractive index and, consequently, the real part as well. This means that gain modulation (fluctuation of immaginary component) leads to phase modulation (real part) and, therefore spectral broadening of the laser linewidth (chirp) is observed.

Many papers have indeed addressed this issue and shown there is significant dependence of the alpha-factor on both internal (carrier scattering dynamics, wetting layer carrier population, etc.) and external factors (cavity length, temperature) (Carrol, 2005 and 2006, Melnik, 2006). From the above mentioned, the device becomes sensitive to out-of-phase optical field, and even the coupling to optical fiber could be a problem due to the delayed optical field reflected back to the cavity (Gioannini, 2008a). Literature has previously identified the 2D carriers present in the wetting layer carrier as responsible for most of the frequency fluctuation observed in quantum dot lasers when subject to optical feedback (Gioannini, 2008a and 2008b). Those works revealed different operating regimes of quantum dot lasers, ranging from stable to chaotic-like solutions as the injection current increased, but did not considered a more rigorous analysis to confirm the existence of chaos. On the other side, more recently, carriers of the wetting layer reservoir have also been shown to influence the steady state of solitary edge-emitting lasers when the scattering to 0D states has a non-negligible dynamics (The, 2012), thus justifying the inclusion of such direct capture of carriers in the rate-equations. This scenario naturally motivates one to investigate whether the existence of the 2D-0D direct capture channel may influence the laser regimes previously reported and how it differs to the case in which only the cascade scattering takes place. To accomplish with that, in the present work the quantum dot laser response to current injection with both cascade and direct capture models is analyzed from the calculated Lyapunov exponents of the dynamic system (Monteiro, 2002). Such exponents (each one associated to a given state variable) represent a measure of how the hypervolume of an n-dimensional sphere (thinking of the state space system representation) changes with time, being therefore essential to confirm chaotic dynamic.

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This paper is organized as follows: in the next section the rate-equations based model to take into account the delayed optical field and the direct capture path is presented, and a discussion of the Lyapunov exponents is added. In section III results are presented and discussed. Finally, we draw the conclusions.

2 MODELING

2.1 Rate-equations

The model here used is based on that of (The, 2012), which considers separate dynamics for electron and hole populations of InAs quantum dots inserted in InGaAs quantum well, with GaAs separate confinement heterostructure (SCH). In order to correctly include the inhomogeneous dot size distribution which takes place in the Stranski-Krastanow growth technique, a multi-population approach has been adopted (Rossetti, 2007); this means that quantum dots of an ensemble are separated into n small groups of dots similar in size, thus leading to n equations for carrier number of each confined state (GS, ES1 and ES2). Furthermore, it contains the scattering channel of carrier from the wetting layer to the confined states. To highlight this, for what concerns carriers of the conduction band, in the following only the population equations for wetting layer, first-excited and ground-state are respectively reported.

$$\dot{n}_w = \frac{n_s}{t_{sw}} + \frac{n_2}{t_{2w}} - \frac{n_w}{t_{ws}} - \frac{n_w \cdot (1-\rho_2)}{t_{w2}} - \frac{n_w \cdot (1-\rho_1)}{t_{w1}} - \frac{n_w \cdot (1-\rho_0)}{t_{w0}} \quad (1)$$

$$\dot{n}_1 = \frac{n_2 \cdot (1-\rho_1)}{t_{21}} + \frac{n_0 \cdot (1-\rho_1)}{t_{01}} + \frac{n_w \cdot (1-\rho_1)}{t_{w1}} - \frac{n_1 \cdot (1-\rho_2)}{t_{12}} - \frac{n_1 \cdot (1-\rho_0)}{t_{10}} \quad (2)$$

$$\dot{n}_0 = \frac{n_1 \cdot (1-\rho_0)}{t_{10}} + \frac{n_w \cdot (1-\rho_0)}{t_{w0}} - \frac{n_0 \cdot (1-\rho_1)}{t_{01}} \quad (3)$$

In the above equations, n_s is the SCH carrier number and n_2 is the carrier number of electrons in the second-excited state. Terms inside the brackets typed with greek ρ are the average occupation of confined states (2 for ES2, 1 for ES1 and 0 for GS). Terms in the denominator indicate the average time constants for the various scattering mechanisms between carriers of the conduction band; for example, t_{01} is the average escape time from level 0 (GS) to energy level 1 (ES1), whereas t_{w1} refers to direct scattering from WL energy states to ES1 energy state. These scattering phenomena are illustrated in Figure 1.

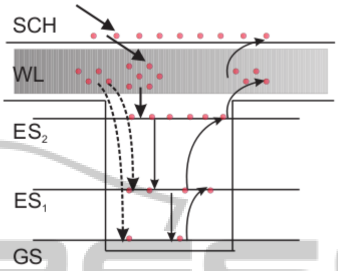

Figure 1: Schematic diagram of carrier dynamics in conduction band. In dashed arrows the 2D-0D capture channel.

In the valence band things are simpler, since hole dynamics is governed by two equations only: one for the dot states (index d) and one for holes in the SCH (index s):

$$\dot{n}_s = \frac{I}{e} - \frac{n_s}{t_{sd}} + \frac{n_d}{t_{ds}} \quad (4)$$

$$\dot{n}_d = \frac{n_s}{t_{sd}} - \frac{n_d}{t_{ds}} \quad (5)$$

Notation of equations (4) and (5) is similar to the previous ones, except that now they refer to holes, and n_d expresses the population of carriers in the quantum dot and wetting layer altogether (2D and 0D holes constitute one ensemble). Term *I/e* is the mean number of holes injected per unit time in the separate confinement heterostructure.

In the above equations non-radiative and radiative recombination terms are omitted for brevity.

The main difference between the model here used and that of (The, 2012) is that the photon equations for the cavity resonant modes have been replaced by one equation for the internal electrical field intensity, E_0 and one for its phase, Φ. This follows the classical Lang-Kobayashi model (Lang, 1980) used widely to study the effects of optical feedback in semiconductor single-mode lasers (Otto, 2010; O'Brien, 2004). Here are the remaining two equations:

$$\dot{E}_0 = \frac{-E_0}{2t_{ph}} + \frac{c}{2n_r} + B_{sp} - \sum\left(g_{nES2} + g_{nES1} + g_{nGS}\right)\cdot E_0 + k\cdot E_0\left(t-t_d\right)\cdot\cos\left(w_0 t_d + \Delta(t)\right) \quad (6)$$

$$\Phi \frac{-k\cdot E_0\left(t-t_0\right)}{E_0\left(t\right)}\cdot\sin\left(w_0 t_d + \Delta(t)\right) \quad (7)$$

In the above equations t_{ph} is the photon lifetime in the laser cavity, c is the free-space light velocity, n_r is the active material refractive index, k is the intensity of feedback light, w_0 is the angular frequency of the solitary laser, t_d is the external cavity roundtrip time and δf is the frequency chirp calculated according to (Gioannini, 2007). Finally, the terms inside the summation operator is the material gain coupling cavity photons and carriers of the ground-, first- and second-excited states.

2.2 Calculation of Lyapunov Exponents

According to the theory of dynamical systems, only nonlinear dissipative systems may experience chaotic behavior, being chaos related to sensitivity to initial conditions and characterized by a time evolution towards a strange attractor in the phase space (Monteiro, 2002). A widely used approach to test sensitivity to initial conditions of nonlinear systems and, therefore, conclude about the existence of chaos requires the calculation of Lyapunov exponents. Given a dynamic system with p velocity fields associated to state variables, there are two requirements to be satisfied before concluding if the process is chaotic:

a) at least one of the Lyapunov exponents associated to the velocity equations is positive: this is to guarantee divergence of adjacent trajectories (those starting at slightly different initial conditions);
b) the sum of all Lyapunov exponents associated to the whole set of velocity field equations must be negative: this is to ensure the system is dissipative (and therefore phase space evolution towards a strange attractor occurs).

In the present work the calculation of the Lyapunov exponents is based on the following formula:

$$\Lambda = \frac{1}{N}\cdot\sum\log_e\left(\frac{F\left(x_0+\delta_0\right)-F\left(x_0\right)}{\delta_0}\right) \quad (8)$$

In this expression, N is the size of the discrete time vector corresponding to the last time instants of every simulation (transient is discarded), δ_0 is the small deviation between two different initial conditions and F is the vector of state variables representing the dynamical system.

3 RESULTS AND DISCUSSION

The model has been used to study the laser response when it is subject to optical feedback, under different current injection conditions. The device considered is a 0.6 mm long edge-emitting device emitting from the ground-state lasing line, at 1285 nm. This device has been chosen to allow for better comprehension of already reported results, especially the chaotic-like solutions which had not been explained in terms of Lyapunov exponents (Gioannini, 2008a and 2008b). Another goal of the present analysis is to check whether the direct capture carrier scattering phenomenon may influence the conditions for chaos in the laser device. Other parameters used in the simulations are listed in Table 1.

Table 1: Quantum-dot material and laser parameters used.

Laser width: 4 microns	QD density: $4.10^{14}m^{-2}$
Number of QD layers: 10	Energy separation (meV): WL-ES2: 13 meV ES2-ES1: 36 meV ES1-GS: 37.3 meV
Internal loss: 1.5 cm^{-1}	
Left/Right end-reflectivity: 0.3/0.3	

To accomplish with that, the following results focus on the time-domain evolution of the Lyapunov exponents. Figures 2 to 4 report for 150 mA, 300 mA and 600mA, respectively, in the top part the sum of the Lyapunov exponents (for a total of 142 dynamic equations describing the laser under feedback), and in the bottom the largest Lyapunov exponent. First result is that at any of these injection conditions there is always at least one positive exponent and, besides, the whole sum of exponents is always negative. This formally confirms, therefore, that quantum dot lasers, as suggested by previous theoretical and experimental works are sensitive to feedback oscillations, entering chaotic regime at different operating conditions.

Figures 2, 3 and 4 solid lines refer to results obtained with a model including the direct capture process, whereas dotted lines refer to a cascade scattering based model. Simulation revealed two interesting point about this: first, by comparison of the largest Lyapunov exponent in Figures 2 and 4, we see that at lower current injections higher positive Lyapunov exponents are obtained when the direct capture path is present, whereas at higher electrical current levels the cascade model got higher positive exponents. Since the positive Lyapunov exponent is a measure of how fast two slightly different initial conditions evolves to a divergence of the velocity field, this result can be explained in terms of the higher carrier density in the wetting layer achieved with the cascade model (this is because the direct capture represents an additional drain for wetting layer carriers down to the dot confined states).

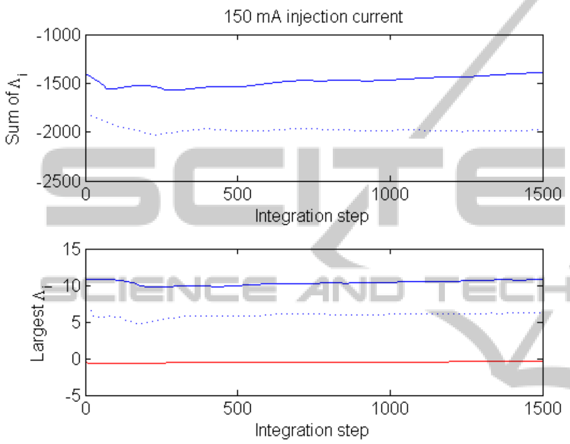

Figure 2: Time evolution of Lyapunov exponents for 150 mA injection condition. Solid lines refer to direct capture model and dotted lines to cascade only model.

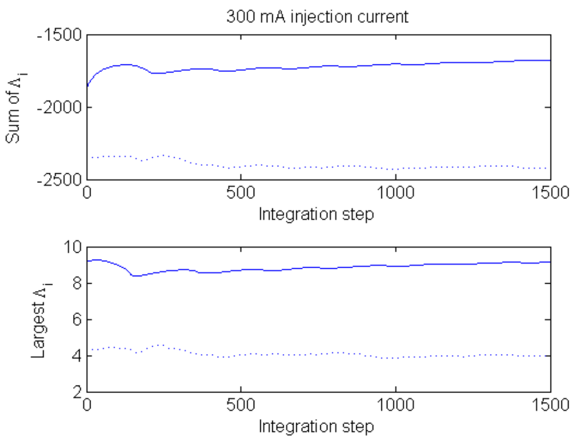

Figure 3: Time evolution of Lyapunov exponents for 300 mA injection condition. Solid lines refer to direct capture model and dotted lines to cascade only model.

Another important result in these figures is the indication of hiperchaos in both direct capture and cascade models, suggested by the existence, in steady-state, of two positive Lyapunov exponents in Figures 2 (solid lines, direct capture model) and 4 (dotted lines, cascade model).

To illustrate the sensitivity to initial conditions, in Figure 5 it is shown the laser response in time-domain, for a 150 mA driving current, when the direct capture scattering process is included. Solid line refers to the optical power evolution after a certain initial condition, and the dotted line is the same, except for a slight change in the initial condition (from 10^{-6} to 1.1×10^{-6}) of some of the state variables. Notice how the responses differ after 11000 integration steps.

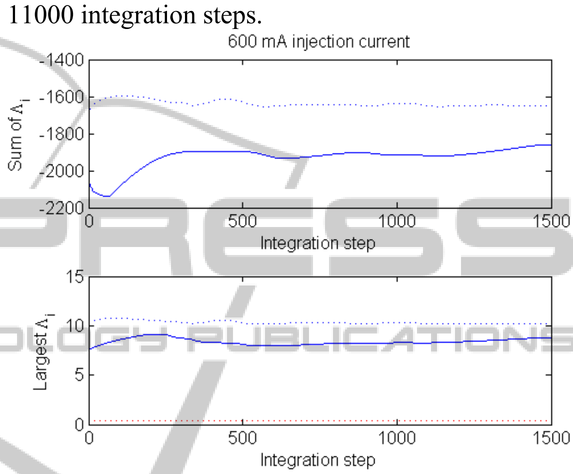

Figure 4: Time evolution of Lyapunov exponents for 600 mA injection condition. Solid lines refer to direct capture model and dotted lines to cascade only model.

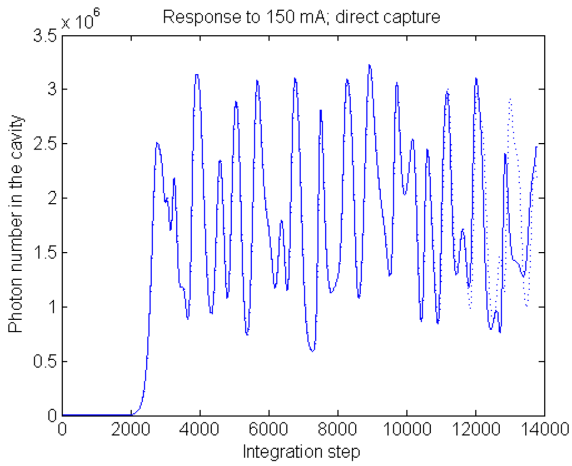

Figure 5: Time-domain evolution of the photon number in the laser cavity when the direct capture model is considered. Dotted-line and solid-lines differ slightly in the initial values for the state variables.

Finally, to complete this look at the Lyapunov exponents, in Figure 6 the exponents associated to the equations for carriers in the dot confined states (ground-, first-excited and second-excited states) are shown in color map. It refers again to the 150 mA driving condition.

According to this figure we can see that the exponents associated to the confined states are all negative (thus being essential for the dissipative feature of the system), and that there is a clear dependence of the exponents on the quantum dot grouping (carriers from the quantum dots more likely in the ensemble lead to less negative exponents). Additionally, if ES2 carriers are compared to ES1 and GS ones, it can be pointed out that carriers resonant with the spectral window of lower material gain (that containing ES2 states) contribute to less negative values of Lyapunov exponents. This suggests that the way to get other positive Lyapunov exponents is to operate with shallow dots, favoring carrier escape up to wetting layer, a scenario in which higher energy confined states are less populated.

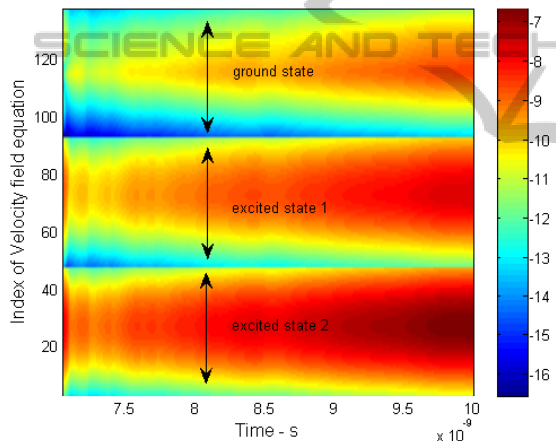

Figure 6: Time-domain evolution of the Lyapunov exponents associated to the carrier number equations of confined states. Color indicates magnitude of the Lyapunov exponents.

4 CONCLUSIONS

In this work a numerical model for the direct capture scattering process in quantum dot lasers under optical feedback has been developed, and an analysis of the possibility of chaotic operation has been done after numerical calculation of the Lyapunov exponents of the system.

Simulation results revealed that positive Lyapunov exponents are achieved at different driving conditions, for both direct capture and cascade only models. Comparison between these models showed different sensitivity to initial conditions at different electrical driving levels: with the direct capture included in the model, higher positive exponents are obtained at lower currents, and an inverse trend is obtained for the cascade model.

Finally, results of the Lyapunov exponents associated to the carriers in the dot confined states showed that there is major tendency to negative values and that the threshold of positive values may be related to the spectrum window of lower material gain.

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