

Experiments Assessing Learning of Agent Behavior using Genetic Programming with Multiple Trees

Takashi Ito, Kenichi Takahashi and Michimasa Inaba
Graduate School of Information Sciences, Hiroshima City University, Hiroshima, Japan

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Abstract: In this paper, experiments to assess agent behavior learning are conducted to demonstrate the performance of genetic programming (GP) with multiple trees. Using the methods, each has a chromosome representing agent behavior as several trees. We have proposed two variants using the conditional probability and the island model to improve the methods' performance. In GP using the conditional probability, individuals with high fitness values are used to produce conditional probability tables to generate individuals in the next generation. In GP using the island model, the population is divided into two islands of individuals: one island maintains diversity of individuals. The other emphasizes the accuracy of the solution. Moreover, this paper improves methods to seek the optimal number of executions of each tree in an individual. Those methods are applied to a garbage collection problem and a Santa Fe Trail problem. They are compared with traditional GP, GP with control nodes, and genetic network programming (GNP) with control nodes. Experimental results show that our methods are effective for improving the fitness.

1 INTRODUCTION

In the field of artificial intelligence, which aims at modeling human intelligence, many researchers have studied search algorithms to obtain agent decisions and action rules to reach a goal. Reinforcement learning and evolutionary learning are representative means to learn agent behavior. Evolutionary methods are known to be able to obtain optimum rules for agent action in a broad search space. Among evolutionary methods, genetic programming (GP) and genetic network programming (GNP) have been investigated eagerly and widely (Koza, 1992; Hirasawa et al., 2001; Iba, 2002; Mesot et al., 2002; Tanji and Iba, 2010). Genetic network programming (GNP) is also known to be able to find better solutions than genetic programming (GP) can (Hirasawa et al., 2001; Iba, 2002). As an extensional method of GNP, GNP with multi-start nodes and GNP with control nodes (GNP_{CN}) have been proposed (Murata and Nakamura, 2006; Eto et al., 2007). Although GNP_{CN} can search for better solutions than GNP can, GNP_{CN} has some shortcomings. For example, the readability of GNP_{CN} is low because obtained rules are expressed as a network. Moreover, the network

structure of GNP_{CN} corresponding to agent rules tends not to be fully used. As a method to improve readability, GP with control nodes (GP_{CN}) has been proposed. In GP_{CN}, an individual consists of several trees that express action rules (Minesaki, Ueda, and Takahashi, 2009). Each tree is constructed with a part of a network where the tree root node corresponds to a control node of GNP_{CN}. The GNP_{CN} network structure is divided into several trees. Therefore, the GP_{CN} readability becomes higher than that of GNP_{CN}. However, the GNP_{CN} ability is higher than that of GP_{CN}.

To improve the GP_{CN} ability, we have proposed GP_{CN} using conditional probabilities (GP_{CN_CP}) (Morioka, Ueda, and Takahashi, 2011) and the island model (GP_{CN_IL}) (Ito, Takahashi, and Inaba, 2013). We introduced conditional probabilities between nodes to use their relations in individuals with high fitness values. As a similar and more general idea, frequent trees, i.e., subtrees that frequently appear in the population, have been proposed. Chunks of strongly related nodes are regarded as frequent subtrees (Ono et al., 2012; Ono et al., 2013). In GP_{CN_CP}, individuals in the next generation are generated using either genetic operations or conditional probability tables, where the conditional probability tables are updated using

individuals with high fitness values. Thereby, GP_{CN_CP} can maintain the diversity of individuals and can inherit the structures of excellent individuals to the next generation with a high probability, where the excellent individuals are the individuals that have obtained appropriate rules for agent behavior in the environment.

Additionally, we have used the island model to promote diversity for overcoming local optima and for improving the fitness because GP_{CN} has a shortcoming: it tends to be trapped by local optima (Iwashita and Iba, 2002). We designate GP_{CN} using the island model GP_{CN_IL} . In GP_{CN_IL} , the population is divided into two islands of individuals: One island emphasizes maintenance of the diversity of individuals. The other emphasizes improving fitness. We designate the former as the diversity-oriented island. The latter is the performance-oriented island. The island model can be expected to prevent the solution by GP_{CN_IL} from reaching a local optimum because GP_{CN_IL} can emphasize two points such as maintaining diversity and improving the fitness.

In this paper, we improve the method to seek the optimal number of processing nodes activated per tree (P) by gradually updating the value of P as evolution proceeds. We designate GP_{CN_CP} and GP_{CN_IL} with the improved search method for P $GP_{CN_CP}(e)$ and $GP_{CN_IL}(e)$. We apply traditional GNP_{CN} and GP_{CN} and our methods, GP_{CN_CP} , GP_{CN_IL} , $GP_{CN_CP}(e)$, and $GP_{CN_IL}(e)$ to a garbage collection problem and Santa Fe Trail problem to compare the performance. We use these problems because the garbage collection problem and the Santa Fe Trail are used to show the ability of GNP and GP (Koza, 1992; Mesot et al., 2002; Eto et al., 2007; Ono et al., 2013; Iwashita and Iba, 2002). Although the symbolic regression problem exists as another type of benchmark problem for GP and GNP , we chose the garbage collection problem and the Santa Fe Trail problem because the objective of this paper is to obtain rules for agent actions. The former is relatively easy, but the latter is difficult. Experimentally obtained results are presented to confirm the effectiveness of those methods.

2 GNP WITH CONTROL NODES (GNP_{CN})

GNP with control nodes (GNP_{CN}) has been proposed as an extensional method of GNP ; GNP_{CN} can search for better solutions than GNP . Each individual of GNP_{CN} has a network structure in

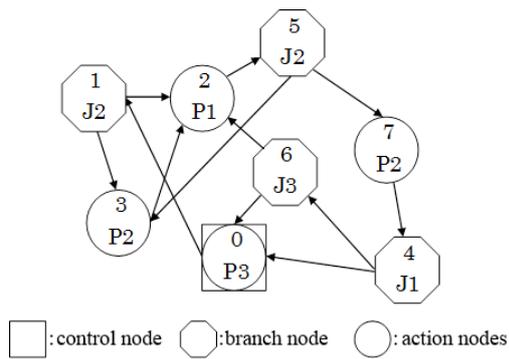
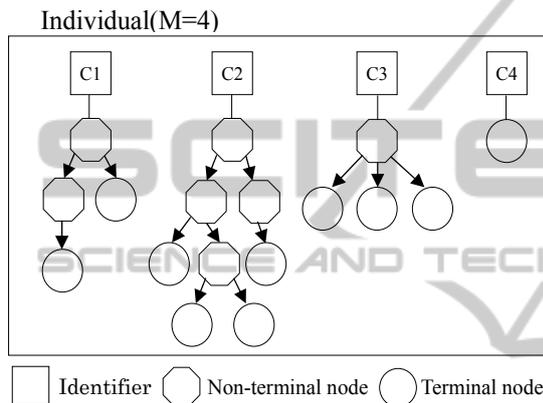
which nodes of three kinds are connected: control nodes, branch nodes, and action nodes. Nodes of the latter two kinds are also used in GNP . An example of an individual of GNP_{CN} is presented in Figure 1. The control node in GNP_{CN} controls the transition of nodes that an agent refers to. Each control node has a number representing the order of its execution. An agent starts to refer to the node indicated by the control node 1 and continues referring to nodes according to the network connection until the designated number of action nodes is executed. Subsequently the agent refers to the control node with the next number and carries out the node designated by the control node. The agent refers to the control node with the smallest number after the control node with the largest number is processed. Genetic operations of GNP_{CN} are only crossover and change of node connection. The connection of control nodes is not changed by genetic operations.

3 GENETIC PROGRAMMING WITH CONTROL NODES (GP_{CN})

3.1 Genetic Programming with Control Nodes (GP_{CN})

To improve the readability of GNP_{CN} and to maintain its efficiency, GP_{CN} has been proposed. An example of an individual of GP_{CN} is depicted in Figure 2. Individuals of GP_{CN} comprise several trees which correspond to rules. The trees have numbers corresponding to numbers of control nodes. The number indicates the order in which an agent refers to a tree. The trees correspond to networks starting from control nodes of GNP_{CN} . The number of trees in one individual, M (i.e. the number of control nodes), is determined in advance. In the action phase of autonomous agents, agents receive perceptual information from the environment and determine actions by referring to trees according to the tree numbers.

A tree comprises terminal nodes and non-terminal nodes. A terminal node denotes an action that an agent can execute. A non-terminal node denotes branch information by the perceptual information. An agent refers to a tree with the smallest number and carries out an action according to the tree. When the number of actions that an agent carries out using the tree exceeds a designated number P , the agent refers to a tree with the next number. The number P represents the number of

Figure 1: Example of an individual of GNP_{CN} .Figure 2: Example of an individual of GP_{CN} where the number of trees is 4.

executions. After the tree with the largest number is processed, the agent refers to the tree with the smallest number.

The algorithm of GP_{CN} is the same as that of the traditional GP. First, GP_{CN} performs the initial generation of the population of individuals. Then, it evaluates the fitness of each that has been generated. If the condition to terminate processing is not met, then it performs reproduction of the population of individuals and genetic operations. It then generates a population of individuals in the next generation. Here, the condition to terminate processing is that the number of generation becomes the designated number of generations. Although the GP_{CN} individuals have several trees, the fitness is evaluated for each individual, not for each tree. Details of the genetic operations for GP_{CN} are described in the next subsection.

3.2 Genetic Operations

Because one individual has several trees unlike normal GP in GP with control nodes (GP_{CN}), for each genetic operation an individual is selected at

random. then one tree is selected at random from the selected individual. Each genetic operation is applied to the selected tree.

3.2.1 Crossover

Crossover is the operation that exchanges subtrees in trees of two parent individuals. First, two trees are selected from two parent individuals respectively, and nodes are selected at random for crossover from all nodes of each tree. Second, subtrees whose root nodes are the selected nodes are exchanged. However, no crossover is executed when a tree consists only of a root node.

3.2.2 Mutation

We use mutation of two kinds: a mutation-tree and a mutation-node. A mutation-tree is an operation that randomly selects one node from all nodes in a tree of a parent individual and then replaces the subtree subsequent to the selected node with a randomly generated subtree. The mutation-node is the operation that changes the content of the selected node after selecting a node in a tree of a parent individual. In mutation-node, if the selected node is a non-terminal (terminal) node, then the node content is replaced with another content of a non-terminal (terminal) node. When any content of a non-terminal node is changed, the edge number might change. If the number of edges of a new content becomes smaller, then the extra edges and the succeeding subtrees are removed. However, if the number of edges becomes larger, then randomly generated subtrees are connected to the increased edges.

3.2.3 Inversion

The inversion operation selects only a non-terminal node at random from all nodes of a tree in a selected individual, selects at random two child nodes. Then it exchanges the subtrees that have the two child nodes as the root nodes.

3.3 GP_{CN} using Conditional Probabilities (GP_{CN_CP})

In GP_{CN} , a problem that the search ability is insufficient exists. In order to utilize connections among nodes of trees in individuals with high fitness values, GP_{CN} using conditional probabilities (GP_{CN_CP}) has been proposed. The GP_{CN_CP} algorithm is the following.

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1: BEGIN
2: Generate initial population;
3: WHILE(i < Ng) DO
4: Evaluate fitness;
5: WHILE(j < Ni) DO
6: IF(Fit[j] >= Tf) THEN
7: Add node counts to frequency
  tables;
8: END IF
9: j++;
10: END DO
11: Build conditional probability
  tables;
12: j=0;
13: WHILE(j < Ni) DO
14: IF(j < Ncp) THEN
15: Generate an individual using
  conditional probability tables;
16: ELSE
17: Generate an individual with
  genetic operations;
18: END IF
19: j++;
20: END DO
21: i++;
22: END DO
23: END.

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In the GP_{CN_CP} algorithm, i and j respectively denote the generation number and the individual number. N_g and N_i respectively represent the maximum generation number and the maximum individual number. Moreover, $Fit[j]$ is the fitness value of individual j . T_f stands for the fitness threshold to build conditional probability tables, and N_{cp} signifies the maximum number of individuals generated using conditional probability tables.

GP_{CN_CP} differs from GP_{CN} in the way of generating individuals for the next generation. In GP_{CN_CP} , trees of individuals in the next generation are generated using conditional probabilities in addition to genetic operations. The conditional probabilities are calculated from individuals with high fitness values and are stored into conditional probability tables for nodes.

Conditional probability tables are produced per tree number using frequency tables that are produced using individuals with high fitness values. We expect to obtain action rules corresponding to roles. First, frequency tables are made by counting the frequency of child nodes attached to branching edges for perceptual information of each nonterminal node in trees of which the numbers are the same over individuals. Additionally, we maintain the diversity of individuals by inheriting frequency tables of the previous generation at a constant rate to the next generation. We update the values in the frequency tables as follows.

$$F_t(i) = F_{tp}(i) \times (1 - \alpha) + F_{tc}(i) \times \alpha \quad (1)$$

Therein, $F_t(i)$ is the next frequency table for tree i , $F_{tp}(i)$ is the frequency table used in the previous generation, and $F_{tc}(i)$ is the frequency table evaluated using only individuals of the current generation. We designate $(1 - \alpha)$ as the inheritance probability, where $\alpha \in [0,1]$. The frequency table for tree i in the current generation is calculated from trees whose number is i in the individuals with high fitness values in the current generation. We designate the individuals with high fitness values as elite individuals. We produce conditional probability tables from the frequency tables.

We generate individuals using conditional probabilities. In generating individuals using conditional probabilities, the root node of a tree is determined with the occurrence probability of each node. Then, we determine child nodes of the root node using the conditional probability table for the root node. The decisions of child nodes using conditional probabilities are repeated until a terminal node is selected for the child node or until the depth of the child node reaches the maximum depth determined in advance. When the child node depth is the maximum depth, a terminal node is selected for the child node.

3.4 GP_{CN} using the Island Model (GP_{CN_IL})

A shortcoming of traditional GP_{CN} is that it tends to end the search with local optima when evolution proceeds. We propose GP_{CN} using the island model (GP_{CN_IL}) as a method to improve the performance. The objective of GP_{CN_IL} is to increase the fitness while maintaining the diversity. The island model is a parallel distributed processing method which has been proposed as an extension of the genetic algorithm. In GP_{CN_IL} in this study, individuals are divided into two islands of individuals: the diversity-oriented island and the performance-oriented island.

A flowchart of GP_{CN_IL} is presented in Figure 3, where N_g represents the maximum generation number. First, we generate an initial population. Second, we evaluate the population of individuals and perform migration, which is one feature of the island model. Subsequently if the termination condition is not satisfied, then we perform selection and generate individuals of the next generation. The termination condition is the same as GP_{CN} : when the number of generations reaches the designated number, the algorithm stops.

We preserve elite individuals and generate individuals of the next generation by crossover in

the performance-oriented island. However, in the diversity-oriented island, we generate individuals of the next generation by replacing some individuals with randomly generated individuals and using genetic operations. Crossover used in the performance-oriented island is the depth-dependent crossover (Ono et al., 2012). Unlike the normal crossover, the depth-dependent crossover determines a depth in the selected tree at random and then selects a node for the crossover from nodes at that depth. Consequently, a destructive crossover is unlikely to occur. Individuals with high fitness values are preserved (Iwashita and Iba, 2002).

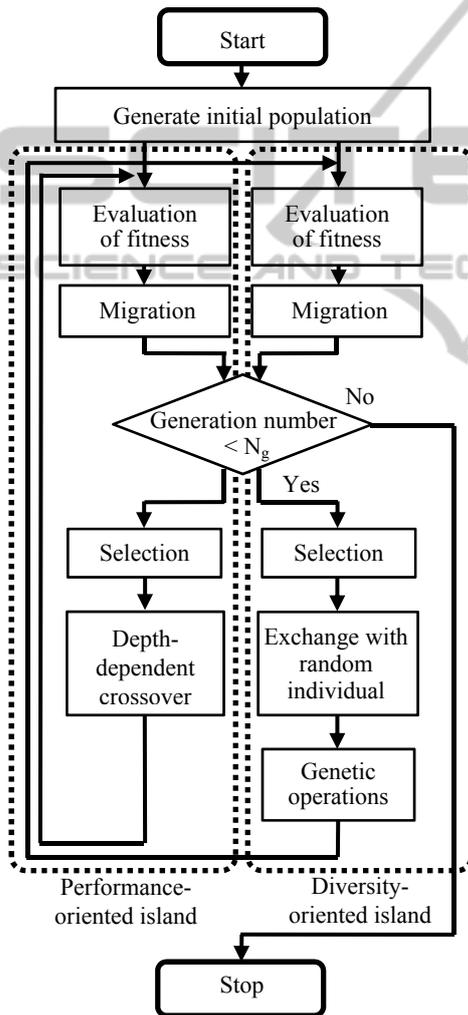


Figure 3: Flowchart of GP_{CN_IL} .

3.5 Search for the Optimal Value of P

We think that optimal values of the number of control nodes (C) and the number of executions (P), which is the number of action nodes repeatedly

processed in a tree differ according to the problem to be solved and the size of action rules. Therefore, we propose $GP_{CN}(e)$, $GP_{CN_CP}(e)$, and $GP_{CN_IL}(e)$, which are extensions of GP_{CN} , GP_{CN_CP} , and GP_{CN_IL} respectively, to obtain optimal values of P by evolution. Let P_i denote the value of P at generation i . Then P is updated as follows.

$$P_i = P_{i-1} + \alpha \quad (2)$$

In (2), α is the range for updating the value of P , and $P_0 = TotalSteps/C$, where $TotalSteps$ and C respectively represent the maximum simulation steps and the number of trees. For example, when $TotalSteps$ is 250, and an individual has two trees, i.e. $C=2$, then the value of P_0 is selected as 125. The value of α is chosen randomly between $[-\beta, +\beta]$. The initial value of β is 12 for a garbage collection problem and 16 for the Santa Fe Trail problem. The values of β are decreased respectively to 1 at every 100 and 1,000 generations for the garbage collection problem and the Santa Fe Trail problem. The value is selected independently for each tree.

4 EXPERIMENTS

4.1 Garbage Collection Problem

The objective of a garbage collection problem is that an agent picks up all pieces of trash scattered in the field and carries them to a garbage dump site. An example of the field of the garbage collection problem is depicted in Figure 4. Field comprises a two-dimensional lattice plane of the size 11×11 cells, and the outermost cells are walls. The garbage collection problem has one agent, ten pieces of trash, and one dump site on the field. The agent can move forward, turn left or right, or stay at each step. The agent can also pick up a piece of trash by reaching the cell where it exists and then can carry it to the dump site. The maximum number of pieces of trash that the agent can carry is assumed as two. We prepare 10 environments generated by placing the agent, trash, and the dump site at randomly selected cells in advance. We define the fitness as the number of total pieces of trash carried to the dump sites in the 10 environments in 250 steps per environment. Let N_i denote the number of collected pieces of trash in environment i . Then, the fitness value is calculated as shown below.

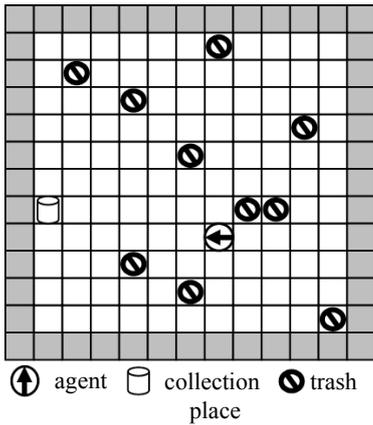


Figure 4: Example of the field of a garbage collection problem.

$$Fitness = \sum_{i=1}^{10} N_i \quad (3)$$

The maximum value of $Fitness$ is 100. In experiments, we measure the highest fitness value obtained in a simulation run at each generation and calculate the average of those fitness values obtained through 30 simulation runs.

Table 1 presents the functions of non-terminal nodes and terminal nodes in the garbage collection problem. We have nodes of two kinds: 0 denotes non-terminal nodes (branch nodes), and 1 denotes terminal nodes (action nodes). Table 2, Table 3, and Table 4 respectively show parameters of GNP_{CN} , GP_{CN} , GP_{CN_CP} , and GP_{CN_IL} used for experiments. Figure 5 shows the change of the average fitness of the garbage collection problem obtained for 1,000 generations. Therein, the vertical axis expresses the fitness values; the horizontal axis expresses the generation number. Figure 5 shows that $GP_{CN_CP}(e)$ and $GP_{CN_IL}(e)$ show good performance in both the maximum fitness and the evolution rate. $GP_{CN_CP}(e)$ and $GP_{CN_IL}(e)$ are methods in which the optimal value of P is sought. Consequently, the method to obtain the optimal value of P is effective for improving the performance.

Comparison of GP with GNP_{CN} shows that GNP_{CN} has better capability than GP_{CN} . Comparison of GP_{CN} with GP_{CN_CP} shows the effectiveness of the conditional probabilities. Moreover, comparison of GP_{CN} with GP_{CN_IL} shows that the island model is effective for improving the fitness, and that the island model gives higher improvement than the conditional probability tables do.

The experiments of the garbage collection problem confirmed that using conditional

Table 1: Function of non-terminal nodes and terminal nodes.

kind	function (number of edges)
0	check the distance from the agent to the dump site (3)
0	how many pieces of trash the agent has (3)
0	check the direction of the agent to the dump site (8)
0	check the direction of the agent to the nearest trash (9)
0	check the direction of the agent to the second nearest trash (9)
1	move forward (1)
1	turn right (1)
1	turn left (1)
1	stay (1)

Table 2: Parameters of GNP_{CN} .

Maximum number of generations	1,000
Population size	300
Number of nodes	18
Crossover probability of nodes, P_c	0.1
Probability of changing connection of nodes, P_m	0.01
Number of control nodes	10

Table 3: Parameters of GP_{CN} and GP_{CN_CP} .

Maximum number of generations	1,000
Population size	300
Tournament size	2
Elite number	1
Probability of mutation-node, P_{mn}	0.05
Probability of mutation-tree, P_{mt}	0.1
Crossover probability, P_c	0.8
Inversion probability, P_i	0.2
Number of population generated by conditional probability	75
Maximum depth of trees	6
Probability of changing value of P , P_p	0.05

probabilities and the island models is effective. We also ascertained that the readability of GP_{CN} is higher than GNP_{CN} by examining the obtained trees and the network.

4.2 Santa Fe Trail Problem

In the Santa Fe Trail problem, an agent must obtain action rules to pick up all pieces of food in the field

Table 4: Parameters of GP_{CN_IL} .

	Performance-oriented island	Diversity-oriented island
Maximum number of generations	1,000	
Population size	150	150
Tournament size	4	2
Number of elites	1	0
Probability of mutation-node, P_{mn}	0	0.2
Probability of mutation-tree, P_{mt}	0	0.1
Crossover probability, P_c	1.0	0.8
Inversion probability, P_i	0	0.1
Migration size	100	
Maximum depth of trees	6	
Probability of changing value of P, P_p	0.05	

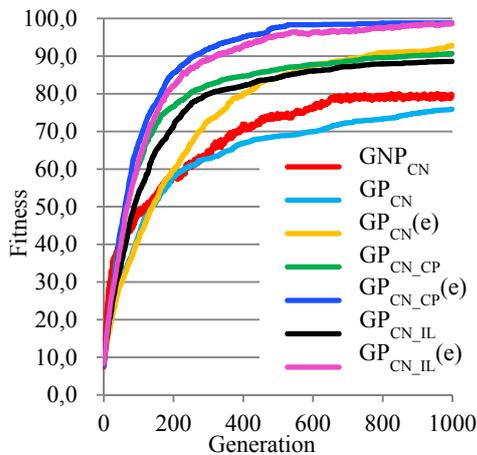


Figure 5: Change of fitness of the garbage collection problem obtained for 1,000 generations.

efficiently. The Santa Fe Trail problem field is depicted in Figure 6. A two-dimensional lattice plane comprises 32×32 cells. The Santa Fe Trail problem has one agent and 89 pieces of food in the field. The agent and the food are placed in determined cells. The agent can move forward, turn left or right, and stay in each step. Additionally, the agent can pick up a piece of food by reaching the

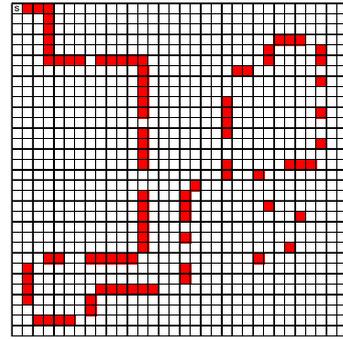


Figure 6: Field of Santa Fe Trail problem.

Table 5: Function of non-terminal nodes and terminal nodes.

kind	function (number of edges)
0	if there is food ahead (2)
0	act X; then Y (2)
0	act X; then Y; then Z (3)
1	move forward (1)
1	turn right (1)
1	turn left (1)

cell in which it exists. We define the fitness as the total number of pieces of food picked up in 400 steps. The maximum number of fitness is 89. The population size is 500, and the maximum generation number is 10,000. In experiments, we measure the highest fitness value obtained in a simulation run at each generation and calculate the average of those fitness values obtained through 30 simulation runs.

Table 5 presents the functions of non-terminal nodes and terminal nodes in the Santa Fe Trail problem. We have nodes of two kinds: 0 denotes non-terminal nodes (branch nodes), and 1 denotes terminal nodes (action nodes).

Figure 7 shows the change of the average fitness of the Santa Fe Trail problem obtained for 10,000 generations. Therein, the vertical axis expresses the fitness values; the horizontal axis expresses the generation number. Figure 7 shows that GP_{CN_CP} and GP_{CN_IL} show better performance than GP_{CN} in terms of the maximum fitness and the evolution rate, which indicates that the conditional probability tables and the island model are effective to improve the fitness. However, the fitness values of $GP_{CN_CP}(e)$ and $GP_{CN_IL}(e)$ are lower than those of GP_{CN_CP} and GP_{CN_IL} . The search method for the optimum number of execution P does not work well in this problem. We must examine the cause further.

In the experiment, the GNP_{CN} performance is low because the PROG function is not implemented.

For the Santa Fe Trail problem, introducing the conditional probability for generation of individuals

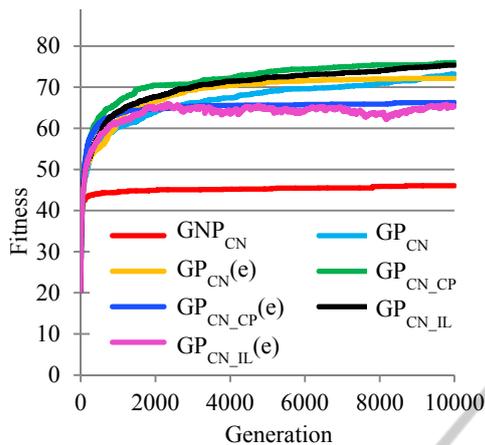


Figure 7: Change of fitness of Santa Fe Trail obtained for 10,000 generations.

and the island model also shows their effectiveness to improve the fitness.

5 CONCLUSIONS

We applied our methods, GP_{CN_CP}, GP_{CN_IL}, GP_{CN_CP}(e), and GP_{CN_IL}(e) to a garbage collection problem and the Santa Fe Trail problem, to assess their performance. In those problems, our methods show good performance in both the maximum fitness and the evolution rate. The authors consider that using conditional probabilities and the island model prevented the solution from reaching a local optimum. Additionally, results show that the method to obtain the optimal value of P improves the fitness.

To improve the fitness of the sub-population of GP_{CN_IL}, our future work will integrate the conditional probability shown to be effective into GP_{CN_IL}.

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REFERENCES

Koza, J. R., 1992. *Genetic Programming: On the Programming of Computers by Natural Selection*, Cambridge, MA: MIT Press.

Hirasawa, K., Okubo, M., Katagiri, H., Hu, J., and Murata, J., 2001. *Comparison between Genetic Network Programming and Genetic Programming Using Evolution of Ant's Behaviors*, IEEJ Transactions on Electronics, Information and System, Vol.121, No.6, pp.1001-1009.

Iba, H., 2002. *Genetic Algorithm*, Igaku Shuppan, Japan.

Mesot, B., Sanchez, E., Pena, C.-A., and Perez-Uribe, A., 2002. *SOS++: Finding Smart Behaviors Using Learning and Evolution*, Eighth International Conference on the Simulation and Synthesis of Living Systems (Alife 8), Artificial Life 8, pp.264-273.

Tanji, M., and Iba, H., 2010. *A New GP Recombination Method Using Random Tree Sampling*, IEEJ Transactions on Electronics, Information and Systems, Vol.130, No.5, pp.775-781.

Iba, H., 2002. *Genetic Programming*, University of Tokyo Press.

Murata, T., and Nakamura, T., 2006. *Multi-Start Node Genetic Network Programming for Controlling Multiple Agents*, 2006 IEEE International Conference on Systems, Man, and Cybernetics, Vol.3, pp.1927-1932.

Eto, S., Mabu, S., Hirasawa, K., Huruzuki, T., 2007. *Genetic Network Programming with Control Nodes*, 2007 IEEE Congress on Evolutionary Computation (CEC2007), pp.1023-1028.

Minesaki, T., Ueda, H., and Takahashi, K., 2009. *Comparison experiment using Genetic Network Programming*, The Conference Program of the 2009 (60th) Chugoku-branch Joint Convention of Institutes of Electrical and Information Engineers, p.546.

Morioka, T., Ueda, H., and Takahashi, K., 2011. *Efficient Evolutionary Learning of Agent Behavior by Genetic Programming Using the Conditional Probabilities*, Proc. of 12th International Symposium on Advanced Intelligent System 2011 (ISIS2011), pp.342-345.

Ito, T., Takahashi, K., Inaba, M., 2013. *Improvement of Genetic Programming with multiple trees*, 2013 IEEE SMC Hiroshima Chapter Young Researchers' Workshop Proceedings, pp.9-12.

Ono, K., Hanada, Y., Shirakawa, K., Kumano, M., Kimura, M., 2012. *Depth-dependent crossover in genetic programming with frequent trees*, 2012 IEEE International Conference on Systems, Man, and Cybernetics, pp.359-363.

Ono, K., Hanada, Y., Kumano, M., Kimura, M., 2013. *Island model genetic programming based on frequent trees*, 2013 IEEE Conference on Evolutionary Computation (CEC2013), pp.2988-2995.

Iwashita, M., and Iba, H., 2002. *Parallel Distributed GP with Immigrants Aging and Depth-dependent Crossover*, Transactions of Information Processing Society of Japan, Vol.43, No.SIG10, pp.146-156.