Iterative Bit- and Power Allocation in Correlated MIMO Systems

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Abstract: In this contribution a novel iterative bit- and power allocation (IBPA) approach has been developed when transmitting a given bit/s/Hz data rate over a correlated frequency non-selective (4×4) Multiple-Input Multiple-Output (MIMO) channel. The iterative resources allocation algorithm developed in this investigation is aimed at the achievement of the minimum bit-error rate (BER) in a correlated MIMO communication system. In order to achieve this goal, the available bits are iteratively allocated in the MIMO active layers which present the minimum transmit power requirement per time slot.

1 INTRODUCTION

The use of multiple antennas at the transmitter and multiple antennas at the receiver side, which is well-known MIMO system, improves channel performance and link reliability of wireless communications. MIMO systems can considerably increase data rate through spatial multiplexing and significantly improve robustness and coverage through diversity combining (Yang et al., 2011).

Adaptive Modulation (AM) is a promising technique able to increase the spectral efficiency of wireless transmission systems by adapting the signal parameters, such as modulation constellation or transmit power, based on the uncertain channel conditions (Zhou et al., 2005). In order to achieve a better system performance given a fixed data rate an adaptive spatial modulation transmission scheme was proposed in (Yang et al., 2011). The performance of MIMO systems using spatial multiplexing is analysed under bit- and power allocation techniques. Existing bit loading and transmit power allocation techniques are often optimized for maintaining both a fixed power and a fixed target bit-error rate while attempting to maximize the overall data-rate. However, delay-critical real-time interactive applications, such as voice or video transmission, may require a fixed data rate (Ahrens et al., 2008). Provided perfect channel state information (PCSI) is available at the transmitter side two major optimization problems are considered to be solved. First, the optimal bit loading. Second, the power allocation optimization problem (Weng et al., 2010). Given PCSI at the transmitter, power and bits can be allocated to different layers. Adaptive bit and power allocation algorithms, which can significantly improve the MIMO system performance, have received a huge research activity lately, and can be divided into two groups according to their performance: optimal and suboptimal algorithms. Optimal allocation algorithms usually have high computational complexity, making them difficult to apply to practical communication systems (Zheng et al., 2013). In order to implement bit and power allocation in practical communication systems many computationally efficient suboptimal allocation algorithms have been proposed where most of them are iterative. Krongold proposed a Lagrange-multiplier-based integer-bits power allocation algorithm (Krongold et al., 2000). The algorithm in (Goldfeld, 2002) minimizes the BER subject to a requested data rate and total transmit power by using adaptive power loading and uniform bit allocation over all subchannels. Zheng proposed a dynamic bound restriction iterative algorithm framework to reduce the computational complexity (Zheng et al., 2013).

In this paper an optimized scheme with fixed transmission modes per time slot are firstly analysed. Furthermore, the proposed algorithm performance has been compared to already developed strategies. The novel contribution of this research is the proposal

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of a new iterative bit and power allocation (IBPA) approach per MIMO layer based on unequal power distribution per MIMO active layer. The remaining part of this paper is structured as follows: Section 2 introduces the system model and the quality criteria considerations. The proposed solutions of bit and power allocation are discussed in section 3, while the associated performance results are presented and interpreted in section 4. Finally, section 5 provides some concluding remarks.

2 MIMO SYSTEM MODEL

According to (Ahrens et al., 2008) a frequency non-selective MIMO communication link with n_{T} antennas in transmission and n_{R} in reception can be described as

$$\mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{n} \ , \tag{1}$$

where $\mathbf{u}$ corresponds to the $(n_{\mathrm{R}} \times 1)$ received data vector, $\mathbf{H}$ is the $(n_{\mathrm{R}} \times n_{\mathrm{T}})$ channel matrix, $\mathbf{c}$ is the $(n_{\mathrm{T}} \times 1)$ transmitted data vector and $\mathbf{n}$ is the Additive White Gaussian Noise (AWGN) vector. Furthermore, it is assumed that the coefficients of the channel matrix $\mathbf{H}$ are independently Rayleigh distributed with equal variance and that the number of transmit antennas n_{T} equals the number of receive antennas n_{R}. In order to avoid the inter-antenna interferences, Singular Value Decomposition (SVD) is used to transform the MIMO channel into independent layers. By applying the SVD technique to the MIMO channel matrix $\mathbf{H}$, it is possible to rewrite the channel matrix as $\mathbf{H} = \mathbf{S} \cdot \mathbf{V} \cdot \mathbf{D}^{\mathrm{H}}$, where $\mathbf{S}$ and $\mathbf{D}^{\mathrm{H}}$ are unitary matrices and $\mathbf{V}$ is a real-valued diagonal matrix of the positive square roots of the eigenvalues of the matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ sorted in descending order. By using the SVD technique in pre- and post-processing, the MIMO channel can be described as multiple independent SISO (single-input single-output) channels (so called layers) with different gains given by the singular values in $\mathbf{V}$,

$$\mathbf{y} = \mathbf{S}^{\mathrm{H}} \cdot \mathbf{u} = \mathbf{S}^{\mathrm{H}}(\mathbf{H} \cdot \mathbf{D} \cdot \mathbf{c} + \mathbf{n}) = \mathbf{V} \cdot \mathbf{c} + \mathbf{w} \ . \tag{2}$$

where $\mathbf{H} \cdot \mathbf{D} \cdot \mathbf{c}$ is the pre-processed transmit data vector and $\mathbf{S}^{\mathrm{H}} \cdot \mathbf{u}$ is the post-processed data vector at the receiver side.

The considered quality criteria for end-to-end wireless communication system performance is given in terms of the bit-error-rate (BER), which quantifies the reliability of the entire wireless system from input to output. The signal-to-noise ratio (SNR) per quadrature component is defined by

$$\rho = \frac{(\text{Half vertical eye opening})^2}{\text{Noise Power}} = \frac{(U_{\mathrm{A}})^2}{(U_{\mathrm{R}})^2} \ , \tag{3}$$

where U_{A} is the half vertical eye opening and U_{R}^2 is the noise power per quadrature component taken at the detector input. The relationship between the signal-to-noise ratio ρ and the bit-error probability evaluated for AWGN channels and M-ary Quadrature Amplitude Modulation (QAM) is given by

$$P_{\mathrm{b}} = \frac{2}{\log_2(M)} \cdot \left(1 - \frac{1}{\sqrt{M}}\right) \cdot \operatorname{erfc}\left(\sqrt{\frac{\rho}{2}}\right) \ . \tag{4}$$

The application of the aforementioned SVD pre- and

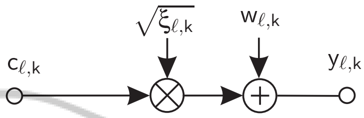

Figure 1: System model per MIMO layer ℓ and transmitted data block k after SVD pre- and post-processing.

post-processing leads to the diagram in Fig. 1 with different eye openings per activated MIMO layer ℓ and per transmitted symbol block k according to

$$U_{\mathrm{A}}^{(\ell,k)} = \sqrt{\xi_{\ell,k}} \cdot U_{\mathrm{s}\ \ell} \ . \tag{5}$$

Considering QAM constellations, the average transmit power $P_{\mathrm{s}\ \ell}$ per MIMO layer may be expressed as

$$P_{\mathrm{s}\ \ell} = \frac{2}{3}\, U_{\mathrm{s}\ \ell}^2\, (M_\ell - 1) \ . \tag{6}$$

By taking $L \leq \min(n_{\mathrm{T}}, n_{\mathrm{R}})$ MIMO activated layers into account, the overall transmit power results in

$$P_{\mathrm{s}} = \sum_{\ell=1}^{L} P_{\mathrm{s}\ \ell} \ . \tag{7}$$

The layer-specific bit-error probability at the time slot k is obtained from combining (3), (4), and (5) as

$$P_{\mathrm{b}}^{(\ell,k)} = \frac{2}{\log_2(M_\ell)} \left(1 - \frac{1}{\sqrt{M_\ell}}\right) \operatorname{erfc}\left(\frac{U_{\mathrm{A}}^{(\ell,k)}}{\sqrt{2}\, U_{\mathrm{R}}}\right) \ . \tag{8}$$

The aggregate bit-error probability at the time slot k, taking L activated MIMO-layers into account, results in

$$P_{\mathrm{b}}^{(k)} = \frac{1}{\sum\limits_{\nu=1}^{L} \log_2(M_\nu)} \sum_{\ell=1}^{L} \log_2(M_\ell)\, P_{\mathrm{b}}^{(\ell,k)} \ . \tag{9}$$

Finally, the BER of the whole system is obtained by considering the different transmission block SNRs.

In order to balance the bit error probability along the MIMO system activated layers bit and power loading can be helpful. The bit error probability at a given time k is influenced by both the chosen QAM constellation and the layer-specific weighting factors. In particular the layer-specific weighting factors influence the overall performance. Therein, the ratio between the largest and the smallest weighting factor is an unique indicator of the unequal weighting of the MIMO layers at a given time k. Furthermore, the ratio between the largest and the smallest singular values is significantly influenced by the antennas correlation effect due to the proximity of the multiple antennas available at the transmitter and receiver sides. In consequence, the transmit-to-receive antenna paths become similar affecting the channel behaviour by increasing the overall BER in the wireless communication link (Cano-Broncano et al., 2012).

In this work we focus on the influence of transmitter-side antennas correlation on the MIMO system performance. The correlation effect is described by the correlation coefficients. Transmitter-side antennas correlation coefficients describe the similitude between paths corresponding to a pair of antennas (at the transmitter side) with respect to a reference antenna (at the receiver side). Fig. 2 describes the basic set-up for obtaining the correlation coefficient, where d is the transmitter-side antennas spacing, d_1 is the distance between transmit antenna #1 and the receiver-side antenna (taken as reference) and d_2 is the distance from transmit antenna #2 to the reference receive antenna (it is assumed $d << d_1, d_2$); ϕ is the departure angle. In consequence two paths are established and the correlation coefficient describes how like they are.

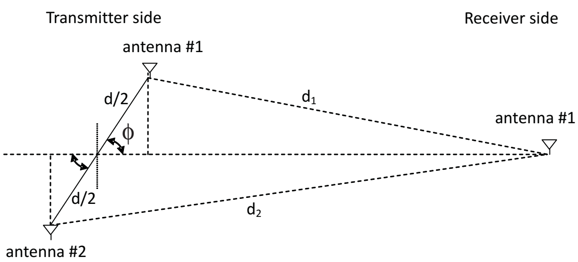

Figure 2: Antennas' physical disposition: two transmit and one receive antennas.

As shown in (Ahrens et al., 2013), the transmitter-side correlation coefficient is given by

$$\rho_{1,2}^{(\mathrm{TX})} = \mathrm{e}^{\mathrm{j}\,2\,\pi\,d_\lambda\,\cos(\phi)} \quad . \tag{10}$$

The antennas path correlation coefficient for line of sight (LOS) trajectories depends on the antennas separation d_λ and the transmit antennas reference axis rotation angle ϕ (or *signals angle of departure*). By taking the scattered environment of wireless channels into consideration, the transmit antennas reference axis rotation angle ϕ becomes time-variant and (10) results in:

$$\rho_{1,2}^{(\mathrm{TX})}(\phi,\sigma_\xi) = \mathrm{e}^{\mathrm{j}\,2\,\pi\,d_\lambda\,\cos(\phi)}\,\mathrm{e}^{-\frac{1}{2}(2\,\pi\,d_\lambda\,\sin(\phi)\,\sigma_\xi)^2} \quad , \tag{11}$$

where σ_ξ corresponds to the standard deviation of the scatters' angle ξ. The parameters of the investigated channel constellations are shown in Tab. 1. According

Table 1: Parameters of the investigated channel constellations.

Description	ϕ	σ_ξ	d_λ
Weak correlation	30°	1	1
Strong correlation	30°	1	0,25

to (Ahrens et al., 2013) the $(n_\mathrm{T}, n_\mathrm{R})$ correlated matrix MIMO system model $\mathbf{H}_\mathrm{c}$ is given by

$$\mathrm{vec}\,(\mathbf{H}_\mathrm{c}) = \mathbf{R}_{\mathrm{HH}}^{\frac{1}{2}} \cdot \mathrm{vec}\,(\mathbf{H}) \quad , \tag{12}$$

where $\mathbf{H}$ is a $(n_\mathrm{T}, n_\mathrm{R})$ uncorrelated channel matrix with independent, identically distributed complex valued Rayleigh elements and $\mathrm{vec}\,(\cdot)$ is the operator stacking the matrix $\mathbf{H}$ into a vector column-wise. Following the quite common assumption that the correlation between the antenna elements at the transmitter side is independent from the correlation between the antenna elements at the receiver side, the correlation matrix $\mathbf{R}_{\mathrm{HH}}$ can be decomposed into a transmitter side correlation matrix $\mathbf{R}_{\mathrm{TX}}$ and a receiver side correlation matrix $\mathbf{R}_{\mathrm{RX}}$ by using the Kronecker product.

$$\mathbf{R}_{\mathrm{HH}} = \mathbf{R}_{\mathrm{TX}} \otimes \mathbf{R}_{\mathrm{RX}} \quad . \tag{13}$$

In order to investigate the potential of bit- and power loading, two different correlated channel profiles (weak and strong) are depicted in Fig. 3 and Fig. 4. Therein, the probability density function (PDF) of the ratio ϑ between the smallest $\sqrt{\xi_{4,k}}$ and the largest $\sqrt{\xi_{1,k}}$ layer specific singular value for weakly and strongly correlated frequency non-selective (4×4) MIMO systems are shown. An uncorrelated one is considered for comparison reasons.

From Fig. 3 it is observed that the difference between layers increases (i.e. the unequal weighting) as the correlation does. This means that the ratio between the largest and the smallest singular value increases, and due to that, the probability of having predominant layers increases. A similar conclusion can be obtained by analysing the PDF curves from Fig. 4 where it is shown that the unequal weighting ratio between the largest and the smallest layer is even higher.

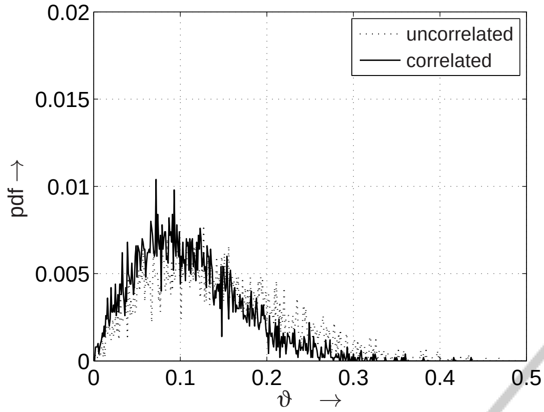

Figure 3: PDF (probability density function) of the ratio ϑ between the smallest and the largest singular value for weakly correlated (solid line) as well as uncorrelated (dotted line) frequency non-selective (4×4) MIMO channel.

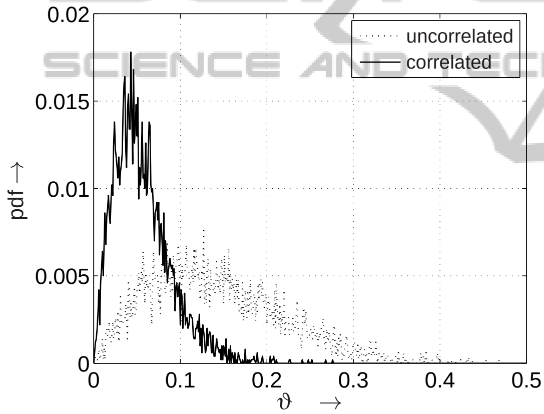

Figure 4: PDF (probability density function) of the ratio ϑ between the smallest and the largest singular value for strongly correlated (solid line) as well as uncorrelated (dotted line) frequency non-selective (4×4) MIMO channel.

3 BIT AND POWER ALLOCATION

Thanks to bit and power allocation techniques it is possible to use wireless channel in an optimised way, e.g. minimizing the BER performance at a fixed data rate under the constraint of a limited total MIMO transmit power. In general, regarding the channel quality, the BER performance is affected by both the layer-specific weighting factors $\sqrt{\xi_{\ell,k}}$ and the QAM-constellation sizes M_ℓ. Assuming a fixed data rate regardless the channel quality the resulting layer-specific QAM constellations for a fixed data throughput are highlighted in Tab. 2.

Following the allocation of the bits per layer,

Table 2: Investigated QAM transmission modes.

throughput	layer 1	layer 2	layer 3	layer 4
8 bit/s/Hz	256	0	0	0
8 bit/s/Hz	**64**	**4**	**0**	**0**
8 bit/s/Hz	**16**	**16**	**0**	**0**
8 bit/s/Hz	**16**	**4**	**4**	**0**
8 bit/s/Hz	4	4	4	4

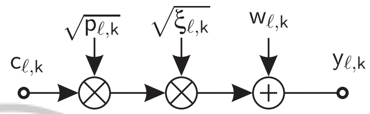

Figure 5: Resulting layer-specific system model including MIMO-layer PA.

Power Allocation (PA) can be added to optimize the overall BER. The layer-specific power allocation factors $\sqrt{p_{\ell,k}}$ adjust the half-vertical eye opening per symbol block to

$$U_{\mathrm{A\,PA}}^{(\ell,k)} = \sqrt{p_{\ell,k}} \cdot \sqrt{\xi_{\ell,k}} \cdot U_{\mathrm{s}\,\ell} \quad . \tag{14}$$

This results in the layer-specific transmit power per symbol block k

$$P_{\mathrm{s\,PA}}^{(\ell,k)} = p_{\ell,k} \cdot P_{\mathrm{s}\,\ell} \quad , \tag{15}$$

where $P_{\mathrm{s}\,\ell}$ denotes the allocated power per MIMO layer without PA e.g. $P_{\mathrm{s}\,\ell} = P_\mathrm{s}/L$. Therein the parameter L describes the number of activated MIMO layers. Taking all activated MIMO layers into account, the overall transmit power per symbol block k is obtained as

$$P_{\mathrm{s\,PA}}^{(k)} = \sum_{\ell=1}^{L} P_{\mathrm{s\,PA}}^{(\ell,k)} \quad . \tag{16}$$

With (14) the layer-specific bit-error probability at the time k changed to

$$P_{\mathrm{b\,PA}}^{(\ell,k)} = \frac{2}{\log_2(M_\ell)} \left(1 - \frac{1}{\sqrt{M_\ell}}\right) \operatorname{erfc}\left(\frac{U_{\mathrm{A\,PA}}^{(\ell,k)}}{\sqrt{2}\,U_\mathrm{R}}\right) \quad . \tag{17}$$

In order to find the optimal set of PA parameters minimizing the overall BER, i. e., $\sqrt{p_{\ell,k}}$, the Lagrange multiplier method is used. Using the Lagrange multiplier method, the cost function $J(p_0, \ldots, p_{N_\mathrm{b}-1})$ may be expressed as

$$J(\cdots) = \frac{1}{\sum_{\nu=1}^{L} \log_2(M_\nu)} \sum_{\ell=1}^{L} \log_2(M_\ell) P_\mathrm{b}^{(\ell,k)} + \lambda \cdot B \quad , \tag{18}$$

where λ denotes the Lagrange multiplier. The parameter B in (18) describes the boundary condition

$$B = \sum_{\ell=1}^{L} \left(P_{\mathrm{s}\,\ell} - P_{\mathrm{s\,PA}}^{(\ell,k)} \right) = 0 \quad (19)$$

$$= \sum_{\ell=1}^{L} P_{\mathrm{s}\,\ell} \left(1 - p_{\ell,k}\right) = 0 \ . \quad (20)$$

A natural choice is again to opt for a scheme that uniformly distributes the overall transmit power along the number of activated MIMO layers, i. e. $P_{\mathrm{s}\,\ell} = P_{\mathrm{s}}/L$. In this case, the boundary condition simplifies to

$$B = \frac{P_{\mathrm{s}}}{L} \sum_{\ell=1}^{L} \left(1 - p_{\ell,k}\right) = 0 \ . \quad (21)$$

Following this equation the transmit power coefficients have to fulfill the following equation $\sum_{\ell=1}^{L} p_{\ell,k} = L$. Differentiating the Lagrangian cost function $J(p_{1,k}, p_{2,k}, \ldots, p_{L,k})$ with respect to the $p_{\ell,k}$ and setting it to zero, leads to the optimal set of PA parameters. In order to study the effect of PA thoroughly, two different fixed channel profiles as shown in Tab. 3 are investigated. For comparison reason, the channel profile CM-1 describes a MIMO channel with low degree of correlation with $\vartheta = 0{,}125$ whereas the channel CM-2 introduces a high degree of correlation, with $\vartheta = 0{,}037$. Since the optimal PA solution is notably computationally complex to implement, a suboptimal solution is investigated, which concentrates on the argument of the complementary error function. In this particular case the argument of the complementary error function

Table 3: Investigated channel profiles assuming a (4×4) MIMO system.

Profile	layer 1	layer 2	layer 3	layer 4
CM-1	1,7500	0,8750	0,4375	0,2188
CM-2	1,9000	0,6333	0,2111	0,0704

$$\rho_{\mathrm{PA}}^{(\ell,k)} = \frac{\left(U_{\mathrm{A\ PA}}^{(\ell,k)}\right)^2}{U_{\mathrm{R}}^2} \quad (22)$$

is assumed to be equal for all activated MIMO layers per data block k, i. e., $\rho_{\mathrm{PA}}^{(\ell,k)} = \text{constant} \quad \ell = 1, 2, \ldots, L$. Assuming that the transmit power per layer is uniformly distributed, the power to be allocated to each activated MIMO layer ℓ and transmitted data block k can be simplified as follows:

$$p_{\ell,k} = \frac{(M_\ell - 1)}{\xi_{\ell,k}} \cdot \frac{L}{\sum\limits_{\nu=1}^{L} \frac{(M_\nu - 1)}{\xi_{\nu,k}}} \ . \quad (23)$$

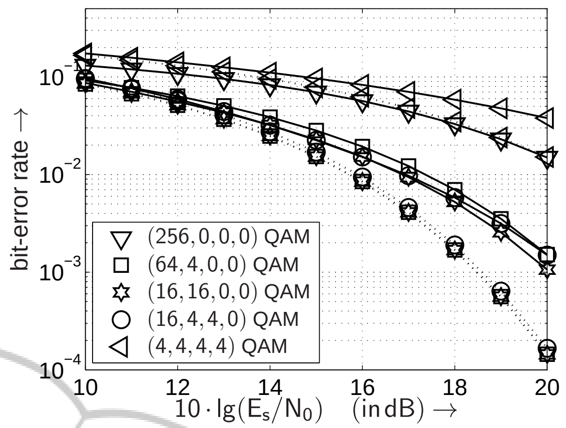

Figure 6: BER with optimal PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over channel CM-1.

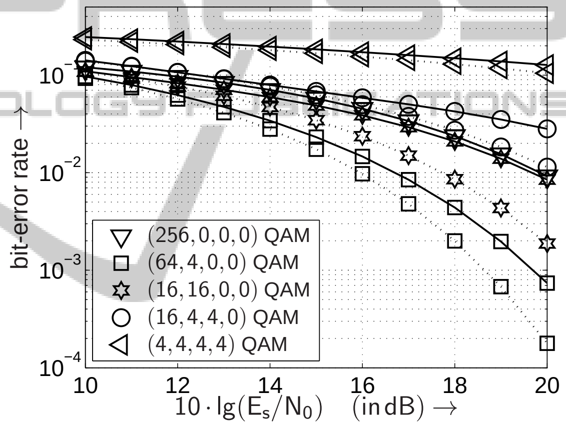

Figure 7: BER with optimal PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over channel CM-2.

Here, for each symbol of the transmitted MIMO symbol vector the same half vertical eye opening of

$$U_{\mathrm{A\ PA}}^{(\ell,k)} = \sqrt{p_{\ell,k}} \cdot \sqrt{\xi_{\ell,k}} \cdot U_{\mathrm{s}\,\ell} \quad (24)$$

can be guaranteed ($\ell = 1, \ldots, L$), i. e.,

$$U_{\mathrm{A\ PA}}^{(\ell,k)} = \text{constant} \quad \ell = 1, 2, \ldots, L \ . \quad (25)$$

When assuming an identical detector input noise variance for each channel output symbol, the above-mentioned equal quality scenario is encountered. The BER curves for channel profiles CM-1 and CM-2 are shown in Fig. 6 and Fig. 7. In order to use the MIMO channel in an optimized way not all the MIMO layers should be necessarily activated. Furthermore, PA in combination with an appropriate number of activated MIMO layers guarantee the best BER performance when transmitting a fixed data rate of 8 bit/s/Hz over uncorrelated non-frequency selective MIMO channel.

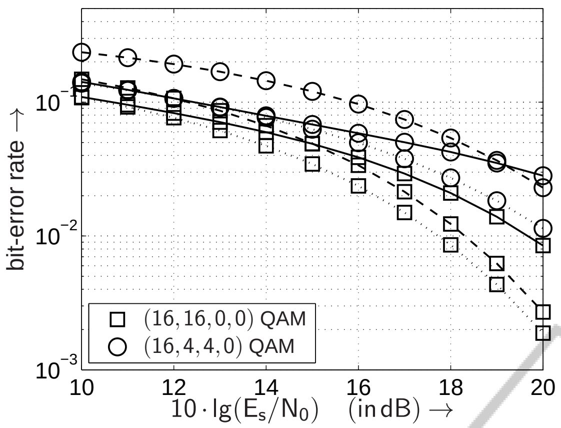

Figure 8: BER with optimal PA (dotted line), equal-SNR PA (dashed line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over channel CM-2.

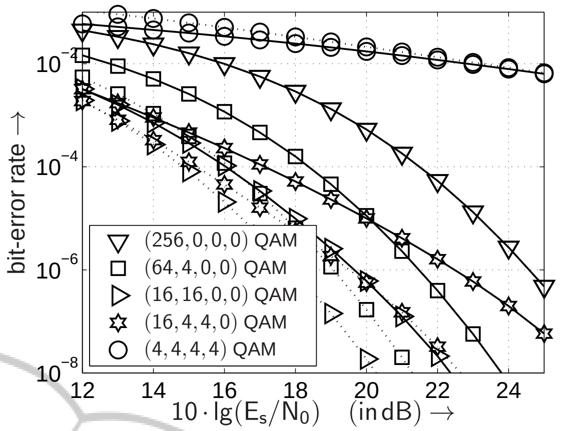

Figure 9: BER with PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over non-frequency uncorrelated selective MIMO channels.

From the simulation results it can be seen that not all the MIMO layers should be necessarily activated in order to get the best BER.

In Fig. 8 the obtained BER curves with the optimal PA are composed with the above mentioned equal quality criteria. As demonstrated by computer simulations the loss in the overall BER with the equal quality criteria is quite acceptable when using an optimized bit loading. Fig. 9 shows a comparison of the BER curves among the listed QAM transmission modes in Tab. 2 with and without PA and justifies the beforehand drawn conclusions. So far the efficiency of fixed transmission modes was studied focusing on minimizing the overall BER.

An optimized scheme would now use the specific transmission mode, guaranteeing the minimum BER for each symbol block k. As highlighting in Fig. 10 by using the best transmission mode per time slot a further minimizing of the overall BER can be obtained. The drawback is the higher signalling overhead.

Following the investigated fixed transmission modes, iterative bit and power allocation strategies seem to be the most challenging solutions when minimizing the overall BER. Here the efficiency of bit auctioning algorithms is investigated. Such solutions test the amount of transmit power needed to transmit a given numbers of bits per layer before allocating the bit to the layer and deciding for a constellation which requires the minimum power. For the iterative bit and power allocation solution investigated in this work it is first tested in which layer the allocation of an extra bit requires the minimum transmit power taking all activated layers into account ($\ell = 1, 2, \ldots, L$).

In order to find out the best bit allocation performance per time slot, it is required to find the appro-

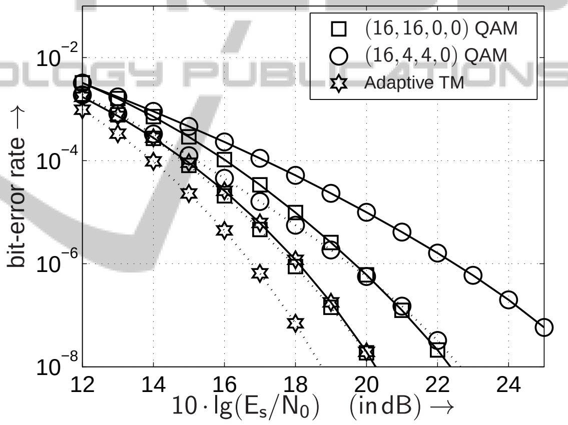

Figure 10: BER with PA (dotted line) and without PA (solid line) when using the transmission modes (TM) introduced in Tab. 2 and transmitting 8 bit/s/Hz over non-frequency selective channels ($\bigtriangledown \triangleq$ adaptive choice of the transmission mode).

priate QAM constellation with the minimum transmit power. For that, firstly from equation (4) it is required to solve the SNR parameter resulting in

$$\rho_{\text{IBPA}}^{(\ell,k)} = 2 \cdot \text{inverfc}^2 \left(\frac{P_{\text{b}}^{(\ell,k)} \cdot \log_2(M_\ell)}{2 \cdot \left(1 - \frac{1}{\sqrt{M_\ell}}\right)} \right) \quad . \qquad (26)$$

The half-level transmit amplitude on each MIMO layer results together with (3) and (5) in

$$(U_{\text{s IBPA}}^{(\ell)})^2 = \frac{\rho_{\text{IBPA}}^{(\ell,k)}}{\xi_{\ell,k}} \cdot U_{\text{R}}^2 \quad . \qquad (27)$$

Consequently, using (26) and (27), the transmit power per MIMO layer (6) can be obtained as

$$P_{\text{s IBPA}}^{(\ell,k)} = \frac{4}{3}\,(M_\ell - 1)\,\frac{U_{\text{R}}^2}{\xi_{\ell,k}}\,\text{inverfc}^2\left(\frac{P_{\text{b}}^{(\ell,k)}\cdot\log_2(M_\ell)}{2\cdot\left(1-\frac{1}{\sqrt{M_\ell}}\right)}\right) \tag{28}$$

With a given noise power per quadrature component U_{R}^2, the layer-specific weighting $\xi_{\ell,k}$ and a reference bit-error rate $P_{\text{b}}^{(\ell,k)}$, the transmit power per layer can be calculated for a given M_ℓ.

In a first approach the proposed IBPA solution focuses on minimizing the overall BER at a fixed data rate. Fig. 12 illustrates the working principle at a fixed data rate of 8 bit/s/Hz using channel CM-2. According to Fig. 11 firstly the bit-error rate level is fixed. Next the bit allocation process is started with two bits. The QAM constellation is transmitted over the layer with the minimum power. Here layer $\ell = 1$ and the $(4,0,0,0)$ transmission mode is selected. After that, as the overall transmit power $P_{\text{s IBPA}}^{(k)} = \sum_{\ell=1}^{L} P_{\text{s}}^{(\ell,k)}$ is lower than one, another two bits are allocated. Resulting in the $(16,0,0,0)$ transmission mode depicted in the second row from Fig. 12. In the third row, as another two more bits are allocated, the $P_{\text{s IBPA}}^{(k)}$ exceeds the power limitation so the BER level is increased to 10^{-6}, being $(16,4,0,0)$ the updated QAM transmission mode. In the fourth row, the last two bits are allocated due to the 8 bit/s/Hz limitation. In this last iteration the BER level is increased to 10^{-3}. This results in the $(64,4,0,0)$ transmission mode. As the available 8 bits have been allocated, the bit allocation process is finished. After the bit allocation process, power allocation can be used to share the remaining power if $P_{\text{s IBPA}}^{(\ell,k)} < 1$ and to balance the bit error probabilities in the number of activated MIMO layers. Since the transmit power is not uniformly distributed, the beforehand developed power allocation schemes can not be used any longer, with $P_{\text{s IBPA}}^{(\ell,k)} \neq P_{\text{s}}^{(k)}/L$. In this case the transmit power allocation parameter has to fullfill the following boundary condition

$$P_{\text{s}}^{(k)} \quad = \quad \sum_{\ell=1}^{L} p_{\text{IBPA}}^{(\ell,k)}\cdot P_{\text{s IBPA}}^{(\ell,k)} \quad . \tag{29}$$

Again, a suboptimal power allocation can be found which concentrates on the argument of the complementary error function resulting in

$$\rho_{\text{IBPA}}^{(\ell,k)} = \xi_{\ell,k}\cdot p_{\text{IBPA}}^{(\ell,k)}\cdot\frac{(U_{\text{s }\ell})^2}{U_{\text{R}}^2} \quad , \tag{30}$$

which is assumed to be equal for all activated MIMO layers per data block k, i.e., $\rho_{\text{IBPA}}^{(\ell,k)} =$ constant $(\ell = 1,2,\ldots,L)$. The new power parameter $p_{\text{IBPA}}^{(\ell,k)}$ is calculated for each activated MIMO layer ℓ and transmitted data block k, taking into consideration the transmit power $P_{\text{s IBPA}}^{(\ell,k)}$ per MIMO layer (28), the boundary condition (29) and the SNR per quadrature component (30). The new power parameter $p_{\text{IBPA}}^{(\ell,k)}$ can be shown to be calculated as follows

$$p_{\text{IBPA}}^{(\ell,k)} = \frac{1}{P_{\text{s IBPA}}^{(\ell,k)}}\cdot\frac{M_\ell - 1}{\xi_{\ell,k}}\cdot\frac{1}{\sum\limits_{\nu=1}^{L}\frac{(M_\nu - 1)}{\xi_{\nu,k}}} \quad . \tag{31}$$

Using the power parameter $p_{\text{IBPA}}^{(\ell,k)}$ from equation (31) to be allocated in each transmitted MIMO symbol, the half vertical eye opening changes to

$$U_{\text{A IBPA}}^{(\ell,k)} = \sqrt{p_{\text{IBPA}}^{(\ell,k)}}\cdot\sqrt{\xi_{\ell,k}}\cdot U_{\text{s IBPA}}^{(\ell)} \quad , \tag{32}$$

and in consequence the MIMO SNR per layer becomes

$$\rho_{\text{IBPA}}^{(\ell,k)} = p_{\text{IBPA}}^{(\ell,k)}\cdot\xi_{\ell,k}\cdot\frac{(U_{\text{s IBPA}}^{(\ell)})^2}{U_{\text{R}}^2} \quad . \tag{33}$$

With the obtained transmit power per MIMO layer ℓ and the symbol block k, the half level transmit amplitude results in

$$U_{\text{s IBPA}}^{(\ell)} = \sqrt{\frac{3}{2}\cdot\frac{P_{\text{s IBPA}}^{(\ell,k)}}{(M_\ell - 1)}} \tag{34}$$

Consequently, using (33) and (34), the bit-error rate per MIMO layer $P_{\text{b}}^{(\ell,k)}$ is modified to

$$P_{\text{b IBPA}}^{(\ell,k)} = \frac{2\cdot\left(1-\frac{1}{\sqrt{M_\ell}}\right)}{\log_2(M_\ell)}\,\text{erfc}\left(\sqrt{\frac{3\cdot p_{\text{IBPA}}^{(\ell,k)}\cdot\xi_{\ell,k}\cdot P_{\text{s IBPA}}^{(\ell,k)}}{4\cdot U_{\text{R}}^2\cdot(M_\ell - 1)}}\right) \tag{35}$$

Afterwards, the PA algorithm is applied according to Fig. 11 and Fig. 12. Firstly the remaining non allocated power is equally distributed over the MIMO active layers. Secondly, an equal-SNR PA technique for Non-Equal Power Distribution is introduced.

The drawback of the so far investigated IBPA solution, the fixed data rate per time slot, can be avoided for improving the overall performance. First four bits are equally allocated like the first four bits in the IBPA with fixed data rate method. Within this research the 8-QAM is excluded since the transmit power per MIMO layer expression do not consider the transmit power of an 8-QAM accurate enough. The updated QAM constellation size is $(16,0,0,0)$. The QAM constellation with the minimum transmit power is selected among the different transmission modes achieved due to the bit allocation in the different layers. After that, bits are individually allocated as long as the overall transmit power $P_{\text{s IBPA}}^{(k)}$ is lower than one.

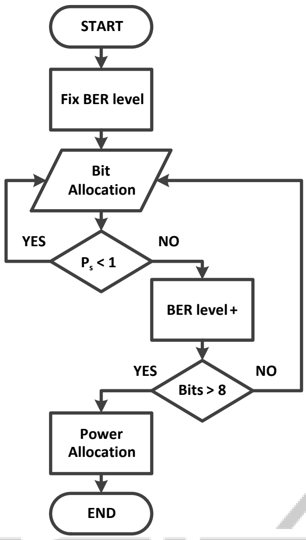

Figure 11: Iterative Bit-and Power Allocation scheme at a fixed data rate.

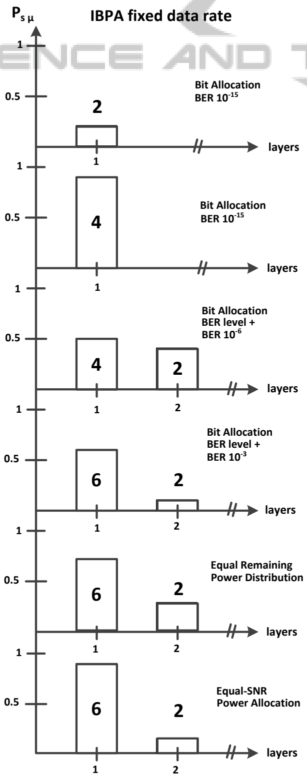

Figure 12: Iterative Bit-and Power Allocation at a fixed data rate.

Because of that the QAM transmission mode becomes $(16,2,0,0)$ in the third row of the Fig. 14, $(32,2,0,0)$ in the fourth row, $(32,4,0,0)$ in the fifth, $(64,4,0,0)$ in the sixth and $(128,4,0,0)$ in the seventh one. In this moment, because of allocating an extra bit means exceed the transmit power limit, the definitive QAM constellation results $(128,4,0,0)$. Besides, according to the Fig. 13 and the Fig. 14, PA techniques are applied. The remaining not allocated power is equally distributed over the MIMO active layers. And additionally, an equal-SNR Power Allocation (SNR-PA) technique for Non-Equal Power Distribution is introduced.

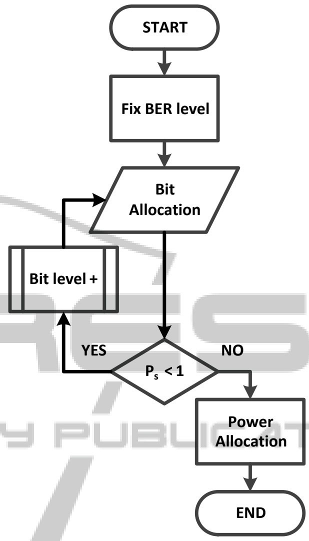

Figure 13: Iterative Bit-and Power Allocation scheme with variable data rate.

4 RESULTS

The investigation described in this paper has analysed different resource allocation techniques aimed at increasing the degree of design freedom, which largely affects the bit-error probability performance on correlated MIMO communication systems. In general, thanks to the selection of the most favourable QAM transmission mode and the optimal transmit power allocation per active layer the best BER performance is achieved. Firstly, fixed QAM constellations with and without equal SNR-PA are analysed and compared to frequency non-selective (4×4) MIMO channels with weak and strong correlation. Secondly, iterative bit and power allocation set-ups are investigated and compared with fixed QAM constellation sizes in order to achieve the optimal resources allocation performance with the minimum bit-error probability.

4.1 Fixed QAM Constellation Sizes Combined With Equal-SNR PA

Pre-establishing the QAM constellation, depicted on Tab 2, a fixed data rate is assured per data block (8

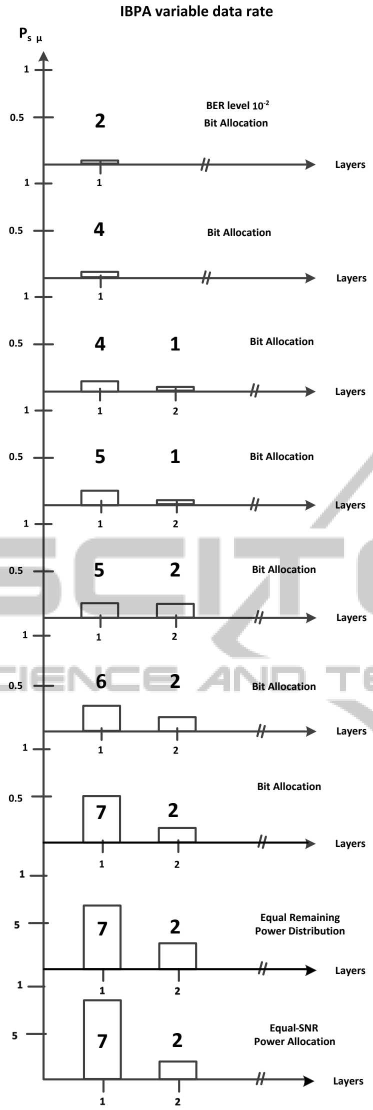

Figure 14: Iterative Bit-and Power Allocation with variable data rate.

bit/s/Hz). Fig. 15 shows the PDF of the transmission modes in bold introduced in Tab 2 and transmitting 8 bit/s/Hz over uncorrelated, weak and strong (channels profiles listed on Tab. 1) frequency non-selective (4×4) MIMO channels. Here is highlighted how the correlation effect modifies the QAM transmission mode chosen. The resultant BER curves with fixed QAM constellation are depicted in Fig. 16 and Fig. 18 for the two different investigated channel profiles from Tab. 1, when transmitting at a bandwidth efficiency of 8 bit/s/Hz. Here it is highlighted that not all MIMO layers have to be activated in order to achieve the best BERs. The target is find out the best combination of the QAM mode and the number of MIMO layers, which gives the best possible BER performance at 8 bit/s/Hz bandwidth efficiency. Subsequently, PA techniques were applied for minimizing the bit-error probability.

On the other hand, by comparing Fig. 16 and Fig. 18 the BER increase due to antennas correlation is noticed.

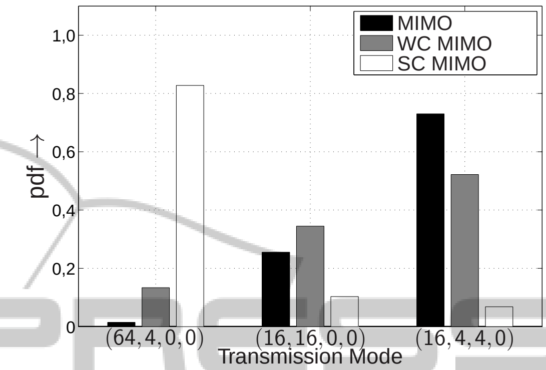

Figure 15: PDF (probability density function) of choosing different transmission modes when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over uncorrelated, weak correlated (WC) and strong correlated (SC) frequency non-selective (4×4) MIMO channels.

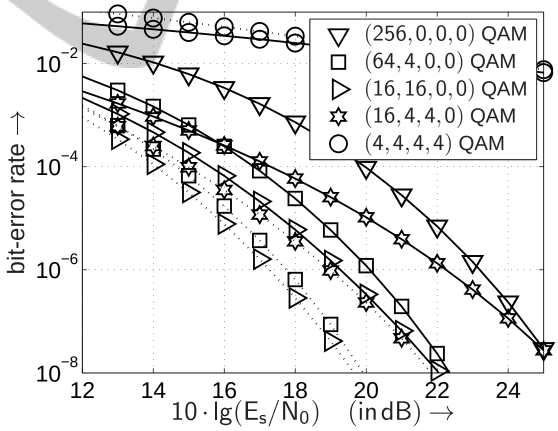

Figure 16: BER with equal-SNR PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over frequency non-selective (4×4) MIMO channels under weak antenna correlation.

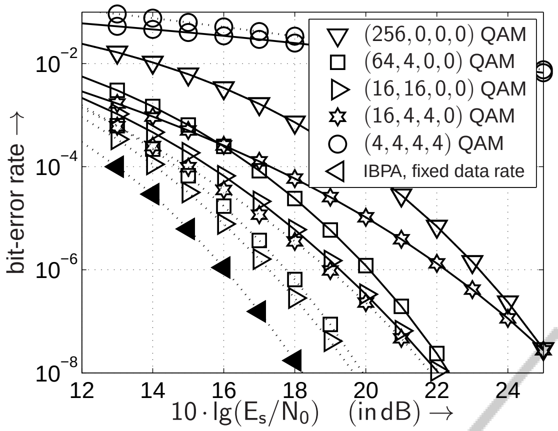

Figure 17: BER with equal-SNR PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over frequency non-selective (4×4) MIMO channels under weak antenna correlation.

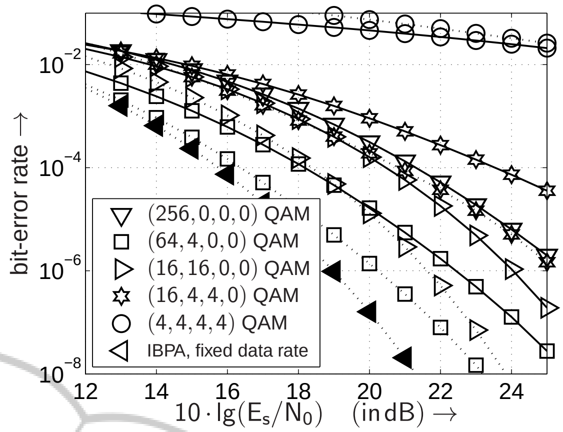

Figure 19: BER with equal-SNR PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over frequency non-selective (4×4) MIMO channels under strong antenna correlation.

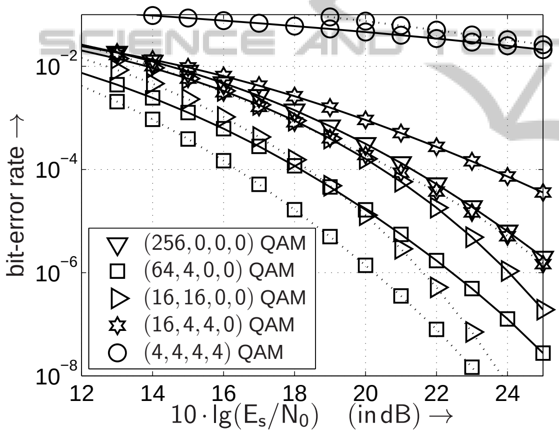

Figure 18: BER with equal-SNR PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over frequency non-selective (4×4) MIMO channels under strong antenna correlation.

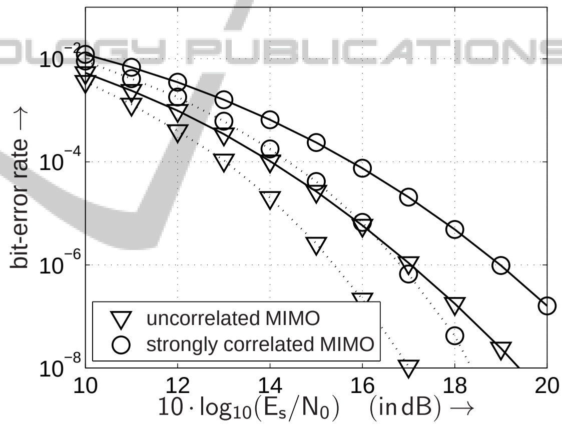

Figure 20: BER based IBPA at a fixed data rate (solid line) and at variable data rate (dotted line) when transmitting an average data throughput of 8 bit/s/Hz over frequency non-selective (4×4) MIMO channels with and without antenna correlation.

4.2 IBPA Combined With Equal-SNR PA

Due to the QAM transmission modes are previously defined in the fixed QAM constellation approach, some kind of rigidity in terms of resource allocation is introduced. In order to increase the degree of freedom in the MIMO performance design, iterative bit allocation with adaptive QAM constellation sizes per time slot is developed in this contribution. Furthermore, a optimal transmit power allocation with equal-SNR not uniformly distributed per active layers is used for minimizing the overall BER.

From Tab. 4 it is possible to compare the effect of the investigated correlated channel profiles listed in Tab. 3 over BER performance in (4×4) MIMO system with IBPA fixed data rate. Due to the increasing correlation effect from CM-1 to CM-2, the bit-error probability increases.

The resultant BER curves are depicted in Fig. 17 and Fig. 19 when transmitting at 8 bit/s/Hz using the iterative bit and power allocation algorithm previously described. Comparing the IBPA fixed data rate BER curves, in both investigated channels profiles weak and strong correlated, it is possible to show the significant BER performance improvement compared with the fixed QAM constellation sizes after applying optimal resources allocation techniques.

The constraint in the investigated IBPA algorithm

Table 4: Investigated channel profiles assuming a (4×4) MIMO system with IBPA fixed data rate at $10\log_{10}(E_s/N_0) = 20$ dB.

Profile	bit/s/Hz	QAM TM	BER
CM-1	8	$(64,4,0,0)$	$1.6 \cdot 10^{-4}$
CM-2	8	$(64,4,0,0)$	$1.8 \cdot 10^{-4}$

concerning to the requirement of fixed data rate at a given bit/s/Hz per time slot can be modified for a variable data rate approach where the given bit/s/Hz in average is assured with the aim to improve the overall BER performance. From Tab. 5 it is possible to compare the effect of the investigated correlated channel profiles, listed on Tab. 3, over BER performance in (4×4) MIMO system with IBPA variable data rate. Depending on the transmitting number of bits the QAM transmission mode is modified. The higher the transmit data rate the higher the BER performance.

Fig. 20 shows the comparison between MIMO system performances when using fixed and variable data rate when transmitting an average data throughput of 8 bit/s/Hz over frequency non-selective (4×4) MIMO channels with and without antenna correlation. As highlighted by the BER curves, from the variable data rate schema a noticeable better BER performance is obtained.

Table 5: Investigated channel profiles assuming a (4×4) MIMO system with IBPA variable data rate at $10\log_{10}(E_s/N_0) = 20$ dB.

Profile	bit/s/Hz	QAM TM	BER
CM-1	7	$(32,4,0,0)$	$1.4 \cdot 10^{-6}$
CM-1	8	$(32,4,2,0)$	$3.2 \cdot 10^{-5}$
CM-1	12	$(64,16,4,0)$	$7.2 \cdot 10^{-3}$
CM-2	7	$(32,4,0,0)$	$5.8 \cdot 10^{-6}$
CM-2	8	$(64,4,0,0)$	$1.8 \cdot 10^{-4}$
CM-2	9	$(128,4,0,0)$	$2.5 \cdot 10^{-3}$

5 CONCLUSIONS

The chance to design and determine the number of active MIMO layers, the number of the overall data rate, the QAM modulation schema per layer and per time slot and all combined with the appropriate allocation technique of the transmit power can remarkably improve the performance of a correlated MIMO communication system. In this work a noteworthy BER performance improvement has been accomplished by using optimal bit and power allocation techniques based on iterative allocation methods over frequency non-selective (4×4) MIMO channels with and without antenna correlation. Additionally, in order to develop these iterative power allocation algorithms (due to the transmit power is not uniformly distributed over active layers any longer) a new approach of suboptimal power allocation parameter $p_{\mathrm{IBPA}}^{(\ell,k)}$ has been perfected from the previous uniformly distributed power parameter $p_{(\ell,k)}$.

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