

Spontaneous Emission of Radiation by Solitons in Fiber-optic Waveguides

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Abstract: We examine the dynamical behavior of light pulses displaying a soliton-like behavior, but which are affected, when entering a fiber-optic waveguide, by a slight perturbation of profile as compared to the stationary profile in the waveguide. We show that, surprisingly, certain pulses propagate while emitting a radiation whereas other pulses emit no radiation. A physical explanation of this difference of behavior is proposed, and tools of prediction of the radiating or non-radiating character of a light pulse in fiber-optic waveguides, are set up.

1 INTRODUCTION

The *soliton*, as it was discovered by Zabusky and Kruskal, (Zabusky and Kruskal, 1965), corresponds to a robust solitary wave that can propagate over large distances without profile deformation and decrease of speed. However, over time, the *soliton* terminology has acquired different meanings depending on the scientific field in which it is used. In mathematics, the soliton is an exact solution of some classes of nonlinear equations associated with completely integrable systems. Obviously, the soliton, as a mathematical object, corresponds to an idealized representation of the real world, where the solitary wave propagates through a perfect physical medium without defects or perturbations. In fact, real physical systems are always more or less perturbed (i.e., not totally integrable), and there, the *soliton* refers to an energy packet propagating without significant deformation or modification of its speed. In the present study, for sake of simplicity, we use the *soliton* terminology to designate all the light pulses displaying a soliton-like behavior, whether the pulse is affected by a perturbation or not. In this context, it is worth noting that the presence of small perturbations in a soliton system leads to many fundamental effects, such as, an alteration of the soliton profile as compared to the stationary profile in the waveguide, or the occurrence of internal dynamics within the soliton. More importantly, in certain situations which have never been really elucidated so far, the perturbed soliton generates radiation waves, i.e., wave packets of low-amplitude which follow, or sometimes precede the soliton (Gor-

don, 1992; Remoissenet, 1993; Ngabireng and Dinda, 2005). This radiation phenomenon is incontestably the most dramatic of the perturbation effects, and also one of the most detrimental to the pulse stability in many practical systems, and specifically in long-haul optical communication systems. In this work, we examine the dynamical behavior of light pulses which are affected, when entering a fiber-optic waveguide (FOWG), by slight distortions of profile as compared to the stationary profile in the waveguide. In particular, we address a fundamental question left open until now. Indeed, until very recently, the idea was widespread that a perturbed pulse necessarily emits a radiation. However, this idea has been questioned in recent studies (Ngabireng and Dinda, 2005), which demonstrated the existence of non radiating pulses. However, to our knowledge, no physical explanation related to the non-radiating behavior of certain light pulses, has been proposed in the literature so far. In the present study, we have discovered that the structure of the pulse profile at the entrance of waveguide, contains elements that explain surprisingly well, and even that can predict the presence or the absence of radiation. Furthermore, we propose theoretical tools that allow one to clearly identify the radiation waves and localize exactly their positions in the waveguide. Those tools constitute the access key to strategies of suppression of radiation in numerous systems where this phenomenon is undesirable.

2 QUALITATIVE AND QUANTITATIVE CONSIDERATIONS

As we mentioned above, during their propagation in a FOWG, light pulses are subject to a perturbed environment. Perturbations may have two main origins: They may be induced by the waveguide, i.e., be consequential to the system. Other perturbations can be external to the system, i.e., related to the action of an agent external to the system. For sake of clarity of presentation, we shall discuss separately these two situations.

2.1 Perturbation Induced by the Periodic Structure of the Waveguide

As is well known, the conventional optical soliton is the result of a delicate balance between two effects that compensate exactly, namely, the self-phase modulation and the anomalous dispersion of the fiber (Hasegawa and Tappert, 1973; L. F. Mollenauer and Gordon, 1980). In a FOWG, as the pulse propagates through the system, it undergoes losses which gradually reduce the self-phase modulation. Consequently, the balance between self-phase modulation and dispersion can no longer be maintained in the real system. In this context, several alternative strategies have been developed in order to stabilize pulse propagation in FOWGs, leading to the emergence of particularly robust pulses such as the guiding-center soliton (GCS) or the dispersion-managed soliton, which are able to propagate in a highly stable manner over several thousands of kilometers. All of those strategies have as common general feature, the fact of being based on periodically structured waveguides, i.e., which are made up of the repetition of the same basic structure called *amplification span*. Within each span, the pulse executes a relatively fast internal dynamics, before going back (at the end of the span) to a profile identical or close to the one it was having at the beginning of the span. Most of the periodically structured waveguides, admit *stationary pulses*. Here, the terminology of *stationary pulse* (SP) refers to a pulse that propagates while executing, in a periodic manner, a deformation of profile whose periodicity corresponds exactly to that of the waveguide (say, Z_A). If one disregards the internal dynamics within each amplification span (i.e., the *fast dynamic* induced by the combined actions of the exponential attenuation of the pulse peak power, the dispersion, and the amplification process), and if one considers only the pulse profile at the end of each span (*slow*

dynamic), then the SP will display a behavior identical to that of a conventional soliton (i.e., a propagation without change of profile). However, at this juncture, we wish to point out a crucial point. Indeed, there is a major qualitative difference between the ideal system, where the (conventional) soliton propagates with a perfectly smooth profile (of Sech shape), and the periodically structured waveguides, where the profile of the SP is never smooth. In fact, the internal dynamic of the pulse, which is closely related to the structure of the amplification span, significantly alters the profile of the SP, which becomes rough. To illustrate the perturbation induced by the waveguide on the profile of SPs, we will use a waveguide corresponding to a GCS (guiding-center soliton) (Hasegawa and Kodama, 1990). In this waveguide the amplification span consists of only one section of fiber with anomalous dispersion, followed by an amplifier. The choice of this waveguide is dictated only by a concern of simplicity, and does not restrict in any way the generality of the tools that we will develop afterward. Note that the technique of GCS is based on the compensation of dispersion by the self-phase modulation, but not in an instantaneous way. Indeed, as the self-phase modulation decreases gradually as the pulse propagates along the amplification span, the pulse is initially endowed with a peak power P_0 larger than that of the conventional soliton in the same waveguide (say, P_m), so that in the beginning of the span, the nonlinearity is stronger than the dispersion, and that, afterwards, the situation gets reversed within the span. So, the power P_0 is chosen so that the balance between the two effects is thus globally reached at the end of each amplification span. In this waveguide, the pulse dynamics may be described by the generalized nonlinear equation (NLSE) which follows

$$A_z = -i \frac{\beta_2(z)}{2} A_{tt} + i\gamma |A|^2 A - \frac{\alpha}{2} A + \left(\sqrt{G} - 1\right) \times \sum_{n=1}^N \delta(z - nZ_A) A, \quad (1)$$

where A refers to the electric field of the pulse, β_2 , γ and α designate the dispersion, non-linearity, and linear-attenuation coefficients, respectively. The parameter $G = \exp(\alpha Z_A)$ refers to the gain of each amplifier. Here, it is worth noting that in the literature, there exists no exact analytical expression for the profiles of SPs in FOWGs. Consequently, the profiles of SPs in real waveguides are accessible only numerically, by means of specialized techniques. By following the procedure of Ref.(J. H. B. Nijhof and Knox, 1997), we have obtained the results depicted in figures 1, for the following typical parameters: $\alpha = 0.24\text{dB/km}$, $D = 1\text{ps/nm/km}$, $\beta_2 =$

$-13 \times 10^{-4} ps^2/m$, $\Delta T = 2T_0 \ln(1 + \sqrt{2}) = 40 ps$, $\gamma = 0.002 W^{-1} m^{-1}$, $P_m = 1.2 mW$, $Z_A = 50 km$. Here ΔT corresponds to the temporal width of the pulse. Figures 1 show the profile of the SP in this waveguide (solid curve), as well as the profiles of the Gaussian and Sech pulses closest to the SP. One can observe that the temporal and spectral profiles of the SP are not smooth. In particular, the roughness of the stationary profile is clearly visible far from the central part of the SP. However, we will show below that some asperities are also present in the central part of the pulse, but they are more clearly perceptible through the profiles of the perturbation fields. Thus, figures 1 demon-

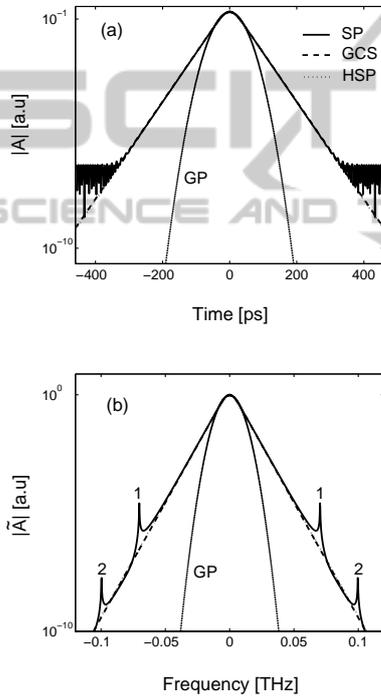


Figure 1: Profile of the stationary pulse (SP), and profiles of the Gaussian pulse (GP) and Hyperbolic secant pulse (HSP) closest to the SP.

strate that the GCS proposed in Ref. (Hasegawa and Kodama, 1990), as well as the Gaussian and Sech pulses, in spite of their exceptional robustness, do not correspond rigorously to SPs in the waveguide, because of their smooth profiles. Here we have used the GCS whose profile between two consecutive amplifiers Z_A^n and Z_A^{n+1} , is given by (Hasegawa and Kodama, 1990):

$$A_{GCS}(z, t) = V_0 \sqrt{P_m} \exp \left[-\frac{\alpha}{2} (z - Z_A^n) \right] \text{sech} \left(\frac{t}{T_0} \right) \times \exp \left(i \frac{z}{2Z_C} \right), \quad (2)$$

where V_0 is the enhancement factor of the input peak power, P_m is the average power of pulse over the amplification span, and $Z_C = T_0^2 / |\beta_2| = 1 / \gamma P_m$. One of the most outstanding results in figure 1(b), is the presence of several pairs of sidebands (indicated by numbered labels), which constitute the most clear distinguishing mark of the waveguide effects on the profile of the SP. These sidebands, which are sometime called *Kelly bands* (Kelly, 1992), result from a process in which two photons of the soliton, of frequency ω_0 , are destroyed simultaneously to create two new photons at frequencies $\omega_0 - \Omega$ and $\omega_0 + \Omega$. This process satisfies the following phase-matching condition: $2k_0 = k_s + k_a + 2k_l$ where $k_l = \frac{2\pi p}{Z_A}$ represents the wave vector corresponding to the harmonic of order p of the oscillation of the pulse peak power, while k_s, k_a and k_0 are respectively the Stokes, anti-Stokes, and soliton wave vectors. One can easily obtain the sideband frequencies that fulfill the phase-matching condition:

$$\Omega_p = \pm \sqrt{\frac{1}{|\beta_2|} \left(\frac{4\pi p}{Z_A} - \frac{1}{Z_C} \right)} \quad (3)$$

The frequencies (3) coincide perfectly with those of the sidebands in figure 1 (b). Thus, the presence of Kelly bands constitutes one of the most dramatic perturbation effects induced by the waveguide on the profile of the SPs. The growth of Kelly bands is systematically accompanied by a specific radiation.

2.2 External Perturbation to the System

The complexity of the profile of SPs in FOWG, is detrimental to the development of those waveguides, because complexes profiles of light pulses are not feasible with currently available optical devices. In practice, for a better stability of the pulse propagation in the waveguide, one endeavors to make so that the input pulse, say $A(0, t)$, is as close as possible to the SP $A_S(0, t)$. At this juncture, it is crucial to realize that the injection of the field $A(0, t)$, which is different but very close to $A_S(0, t)$, is felt by the waveguide as a perturbation of the SP, by a perturbation field $q(0, t)$ such as:

$$q = A - A_S, \quad (4)$$

where $|q(0, t)| \ll |A_S(0, t)|$. In other words, everything happens as if, when entering the waveguide the SP $A_S(0, t)$ collides with the perturbation $q(0, t)$. We show below that, in fact, the input profile of the perturbation field $q(0, t)$ contains a set of special signs allowing the prediction of the general dynamical behavior of the pulse, and specifically, the prediction of the radiating and non-radiating character of the pulse.

Once the stationary profile of the pulse is known, and if we have a pulse that fits at best to this stationary profile, then one can easily obtain the input profile of the perturbation field. Thus, if we choose to propagate Gaussian or Sech pulses [$A_g(0,t)$ or $A_{sech}(0,t)$] that fit at best to $A_S(0,t)$, we can then deduce the corresponding perturbation fields, $q_g(0,t) = A_g(0,t) - A_S(0,t)$ and $q_{sech}(0,t) = A_{sech}(0,t) - A_S(0,t)$, associated with $A_g(0,t)$ and $A_{sech}(0,t)$, respectively. The perturbation field for the GCS is given by $q_{GCS}(0,t) = A_{GCS}(0,t) - A_S(0,t)$.

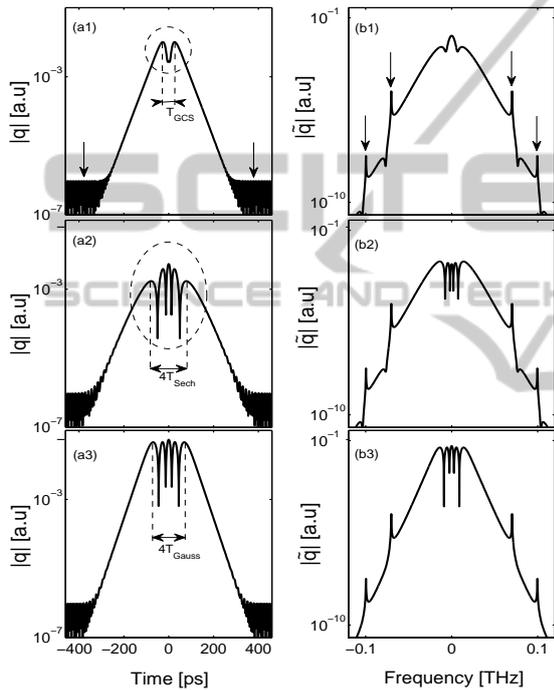


Figure 2: Plot of the perturbation fields associated to the GCS [(a1) and (b1)], the Sech pulse [(a2) and (b2)], and Gaussian pulse [(a3) and (b3)] closest to the SP.

Figures 2 show the temporal and spectral profiles of the perturbation fields, for the three types of pulses under consideration. In particular, the temporal profiles of those perturbation fields exhibit an oscillatory structure [surrounded by the dashed lines in figures 2(a1) and 2(a2)], which is an indication of the roughness of the central part of the SP profile. More importantly, a careful inspection of this oscillatory structure, has enabled us to set up a procedure for the identification of the radiating or non-radiating character of the pulse. Indeed, we have found that the pulse is capable of generating sidebands of radiation only if this oscillatory structure contains a minimum of full periods of oscillations, of the order of four periods. Thus, as figure 2 (a1) shows, the oscillatory structure for

the perturbation field associated with the GCS, contains only a single period of oscillation. We predict that this pulse will be unable to generate sidebands of radiation. Quite in contrast, as figures 2(a2) and 2(a3) show, the perturbation fields associated with the Sech and Gaussian pulses, are endowed with four full periods of oscillation; which is largely sufficient to activate a spectral reorganization leading to the generation of sidebands of radiation. We then predict that the Sech and Gaussian pulses belong to the category of radiating solitons. In general, we have discovered that the central part of the perturbation field always contains an oscillatory structure which constitutes the germ of an eventual radiation process in the FOWG. The more the size of this germ is big, the more the capacity of radiation of the pulse is high. This observation is the most important result of our study. By the way, it is worth noting that the Kelly bands are also clearly visible in the spectra of the perturbation fields $q(0,t)$ [see Figs 2(b1), 2(b2), and 2(b3)].

The above predictive analysis is remarkably confirmed by the numerical simulations of propagation of the considered pulses. Figures 3 (a1) - (b1), which show the evolution of the perturbation field associated with the propagation of the GCS over a distance of 6000km, confirms our prediction on the absence of radiation. We can clearly observe in figure 3(a1), which results from the propagation of the GCS, that the pulse executes a restructuring of profile, which results in a progressive modification of the perturbation field. But the spectral restructuring of the pulse is not sufficient to activate the process of radiation [as shown in figure 3(b1)]. One can finally observe in figure 3(a1) that the pulse absorbs the perturbation, but without being able to contain it over all the propagation distance. Consequently, the perturbation field widens continually (while flattening) during the propagation. Figures 3 (a2)-(b2), which show the evolution of the perturbation field associated with the propagation of the pulse having initially a Sech profile, confirm our prediction on the existence of a radiation process. The temporal profile of the perturbation field in figure 3 (a2), shows clearly two waves of radiation moving away from the center of the pulse rest frame. At the center of this frame, one can clearly distinguish a trapped field (corresponding to the non radiating part of the perturbation field). Figures 3(a3)-(b3) show that injection of the Gaussian pulse leads to a behavior qualitatively similar to that of the Sech pulse in figures 3 (a2)-(b2), namely, the radiation of a part of the perturbation and the trapping of the other part. We have noticed only a quantitative difference in the amplitude of the trapped field, which is higher in the case of the Gaussian pulse.

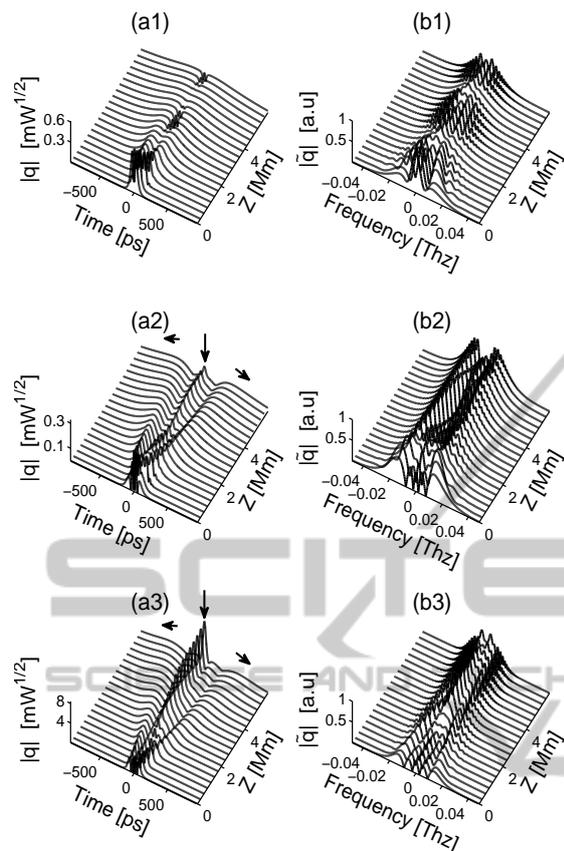


Figure 3: Propagation of the perturbation fields generated by the guiding-center soliton [(a1) and (b1)], the hyperbolic secant closest to the SP [(a2) and (b2)], and the Gaussian profile closest to the SP [(a3) and (b3)]. In figures (a2) and (a3) the horizontal arrows indicate the radiated waves. The vertical arrow indicates the part of the perturbation which is trapped within the pulse.

3 CONCLUSIONS

We have examined the dynamical behavior of a light pulse near its stationary state, and in particular, the physical processes that generate radiation waves. It emerges from our analysis that a light pulse (endowed with a non-stationary profile in the waveguide), always executes a restructuration of profile in order to get closer to the stationary profile. It is during this process of restructuration of profile that certain pulses emit a radiation while other pulses emit no radiation. We have shown that the ability to radiation is determined by the initial structure of the perturbation field, defined as the disagreement of profile between the input pulse and the SP in the waveguide. We have established the existence of an oscillatory structure in the central part of the perturbation field, whose size determines the ability to radiate. Non-radiating pulses

are characterized by an oscillatory structure containing only a few periods of oscillation (typically, less than four periods of oscillation). Radiating pulses are characterized by an oscillatory structure containing a large number of periods of oscillation (typically, at least four periods of oscillation). Finally, the fact that we can clearly identify the radiation and localize exactly its position in the waveguide, leads us to consider its suppression as feasible in certain practical systems where this phenomema is undesirable, such as in long-haul optical communication systems, or mode-locked fiber lasers.

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