A Block Size Optimization Algorithm for Parallel Image Processing

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Abstract: The aim of this work is to define a strategy for rectangular block partitioning that can be adapted to the number of available processing units in a parallel processing machine, regardless of the input data size. With this motivation, an algorithm for optimal vector block partitioning is introduced and tested in a typical parallel image application. The proposed algorithm provides a novel partition method that reduces data sharing between blocks and maintains block sizes as equal as possible for any input size.

1 INTRODUCTION

It is a well-known fact that a series of image processing operations usually involves a previous partition of the image in sections, commonly referred to as blocks, windows or neighbourhoods. This is the case of many low-level operations, such as linear filters, nonlinear processing filters like local median and rank-order filters and local histogram computing, to name a few (Bovik, 2005).

In the last decade, this Block Processing (BP) paradigm has gained special significance after multicore architectures had taken the lead as parallel Processor Units (PUs), including embedded real-time processors, FPGAs and GPUs (Bailey, 2011).

Parallel Image Processing (PIP) is probably the only valid choice if a real-time response over high resolution images is required. Given a number of available PUs, a block partition for parallel BP should be addressed in order to adapt the data to the available processors.

Nowadays, specialized technical software like MATLAB (Moler, 2007) and Mathematica (Mangano, 2010) utilizes parallel processing intensively. Both of them are used by many researchers in the field of Digital Image Processing (DIP).

But BP is also used in other disciplines –and their associated software. This is the case of statistical applications on Geographical Information Systems (GIS), for which ArcGIS may be arguably claimed to be the usual commercial software choice (de Smith *et al.*, 2013).

The ability of partition a series of data associated with some topological 2D map enables local statistical analyses via the so-called raster operations.

In GIS, a raster or grid is a spatial (geographic) data structure that divides a region into neighbourhoods (or cells) that can store one or more values for each of them, usually statistical data. A raster is often contrasted with vector data, which is used to represent points, lines and polygons.

The analogy between a raster and a digital image is straightforward. In fact, most of the data collected in a raster is usually represented with images of some kind (de Smith *et al.*, 2013).

In addition, this kind of scenario is well-suited to be addressed by a set of PUs, which should be able to work in parallel. Thus, a direct relationship between topological partition and process parallelization may be established.

The aim of this work is to devise a strategy for data block partitioning (BP) that can be optimally adapted to the number of available PUs, regardless of the input data size. An optimal partition should be able to provide minimum block size differences and minimum –if any– overlap between adjacent blocks.

The paper is organized as follows. Section 2 reviews basic background for raster operations and neighbourhood configurations for parallelized BP techniques. Section 3 describes a novel algorithm for partition an image into optimally similar area blocks in order to be used by different PUs. Experimental results are presented in Section 4. Finally, Section 5 concludes the paper.

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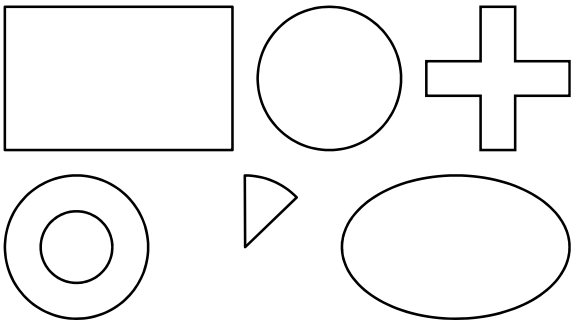


Figure 1: Some neighbourhood shapes.

2 BACKGROUND

2.1 Raster Operations

Raster operations can be classified into three groups:

- Local, where operations are performed in a cell by cell fashion;
- Neighbourhood, for which operations are computed using a moving group of cells and
- Zonal, where operations are performed using groups of similar cells (zones).

In a local raster, an output cell $Y(i,j)$ is computed as a function f of the corresponding input cell $X(i,j)$, i.e.

$$Y(i,j) = f\big(X(i,j)\big), \quad (1)$$

where f is called a mapping or point operation over input X, which is independent of the cell position (i,j) and thus can be implemented as a look-up table (LUT) (Bovik, 2005). In GIS, a local raster may be used for data reclassification (de Smith *et al.*, 2013). In Digital Image Processing (DIP), a local raster may be employed for thresholding a graylevel input image.

A neighbourhood raster utilizes for each input cell $X(i,j)$ and its associated neighbourhood $N(i,j)$, the information of the cells belonging to the region $N(i,j)$ to determine the output cell value $Y(i,j)$ via a neighbourhood operation that –again– can be modelled as a mapping function g for which, in this case,

$$Y(i,j) = g\big(N(i,j)\big). \quad (2)$$

This neighbourhood $N(i,j)$ is usually a region of X centred around the input cell $X(i,j)$, shaped as a rectangle, but that can also be defined with other shapes (see Fig. 1).

In this sense, it can be said that a local raster is just a special case of a neighbourhood raster, for which the neighbourhood N is constant for every input cell (i,j), being a rectangle of size 1×1.

A typical example of neighbourhood raster in DIP is linear spatial filtering and nonlinear local filtering. This approach is also utilized both in GIS and DIP applications for computing local statistics of the neighbours of a cell (or pixel).

Finally, a zonal raster operation involves groups of cells –called zones– that present similar values. Each one of these group can be considered to be a connected group of cells or labels. Thus, a label is defined as the group of cells for which a spatial connectivity exists, i.e. any pixel inside the label can be accessed from every other pixel in the same label by following a spatial trajectory that does not fall outside the label boundary at any step.

Typical label measurements are object perimeter and area (Bovik, 2005), but they can also include statistical information. In this context, a label can also be thought of a special kind of neighbourhood, which is topologically separated from the other labels belonging to the zone.

2.2 Parallel Processing Techniques

Virtually all image processing techniques are based on a sequence of image processing operations, i.e. they are designed as a sequential algorithm or a sequence of operations. This is a form of temporal parallelism that can be exploited in a so-called pipelined structure (Bailey, 2011).

In the pipeline depicted in Fig. 2, a separate PU is used for each operation or task. The latency of an image processing may be defined as the time between when the first input is applied to the first task of the pipeline and the corresponding output of this data is available at the end of the pipeline.

Another example of pipeline is that of a buffered video processor, in which each frame is processed by the same group of techniques. Thus, the image sequence may also be partitioned in time, by assigning successive frames to separate processors, leading to a hierarchical pipeline.

In general, the smaller the neighbourhood needed to perform the operation, the lower its latency. Thus a local raster has the lowest latency, whereas an operation which needs every pixel in the image will have the highest latency (Downton and Crooke, 1998).

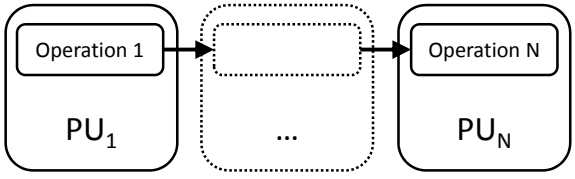


Figure 2: A pipeline for temporal parallelism exploitation.

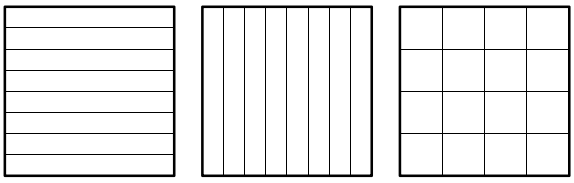


Figure 3: Spatial parallelism exploitation by block partitioning (BP): row, column and rectangular BP.

However, as the algorithm increases its complexity, the operation pipeline speedup becomes less important, mainly due to operation hierarchization and feedback. In this case, the major speedup may be obtained within a specific operation in the form of loops.

The outermost loop within each operation usually iterates over the pixels within the image, because many operations (e.g. spatial filtering) perform the same function independently on many pixels.

This is spatial parallelism, which may be exploited by partitioning the image and using a separate processor to perform the operation on each partition (Bailey, 2011).

2.3 Block Partition Schemes

An important consideration when partitioning an image is to minimize the expected communication between PUs, i.e. between the different considered partitions.

Typical partitioning schemes split the image into blocks of rows, blocks of columns or rectangular blocks, as illustrated in Fig. 3.

For low-level DIP operations such as spatial filtering, the performance improvement approaches the number of processors, as the communication may be reduced to zero if a non-overlapping partition scheme is utilized.

However, in higher level processes this performance will be degraded as a result of communication overheads or contention when accessing shared resources. In high-level operations, these shared resources may just not be pixel values.

Partitioning is therefore most beneficial when the operations only require data from within a local region, which is defined by the partition boundaries. For that reason, each processor must have some local memory to reduce any delays associated with contention for global memory (Bailey, 2011).

If the operations performed within each region are identical, this leads to a SIMD (single instruction, multiple data) parallel processing architecture.

On the other hand, a MIMD (multiple instructions, multiple data) architecture is better suited for higher level image operations, where latency may vary for each block of data.

In these cases, better performances may be achieved by having more partitions than processors, utilizing a Pipeline Processor Farm (PPF) approach (Fleury and Downton, 2001). In a PPF, each partition is dynamically allocated to the next available PU, thus reducing idle process latencies related to block data dependencies.

In the next section, a procedure for block partitioning will be devised to be optimally suited to a PPF approach.

3 AN ADAPTIVE BLOCK PARTITIONING PROCEDURE

3.1 Overlapping Neighbourhoods

Rectangular block partitioning is by far the main shape choice in low-level image operations (Davies, 2012). Two types of rectangular neighbourhoods are commonly considered: overlapping and non-overlapping.

In spatial linear filtering, a series of sums and products are needed for each pixel within the input image, with $P \times Q$ pixels, as a result of applying a small kernel of $p \times q$ weight elements over the pixel surrounding neighbours. In this case, each pixel and its neighbourhood can be processed by a single PU and no inter-process communication is needed.

Except for the border pixels of the image, where additional data –usually zero values– is needed to complete the neighbourhood, each PU can read the corresponding pixel information directly from the image and return its own data as output.

This same concept applies to focal statistics (de Smith *et al.*, 2013), where a neighbourhood raster is applied. The algorithm visits each cell in the raster and calculates a specific statistic over the identified neighbourhood. As neighbourhoods can overlap, cells in one neighbourhood will also be included in any neighbouring cell's neighbourhood.

This situation may enable data reutilization in the raster (Huang *et al.*, 1979), thus reducing shared memory accesses in a PPF environment.

On the other hand, non-overlapping partitions may be used for parallelization speedup in hierarchical parallel schemes. This paradigm is best suited for distributed memory MIMD architectures, in which each PU handles its own local memory

(Fleury and Downton, 2001).

In DIP, non-overlapping block processing is widely used in applications such as image scaling (e.g. image pyramids), compression (e.g. DCT computing) and recognition (e.g. ridge orientation assessment in fingerprint representation) (Ratha *et al.*, 1996).

In GIS applications, this approach is also known as block statistics. In block statistics, the algorithm performs a neighbourhood raster that calculates a statistic for input cells within a fixed set of non-overlapping neighbourhoods.

The statistic (e.g. dynamic range, average or sum) is calculated for all input cells contained within each neighbourhood. The resulting value for an individual neighbourhood or block is assigned to all of its cell locations.

Since the neighbourhoods do not overlap, any particular cell will be included in the calculations for only one block. In other words, distributed parallel computation of each block is possible with no further cost.

Somewhere between these two methods, a small overlap between adjacent neighbourhoods is also used in several DIP techniques, such as (Kuwahara *et al.*, 1976). This approach is a trade-off between the fully overlapping and non-overlapping neighbourhood paradigms, which enables the user to control the speedup with the aid of an overlap factor.

With a controlled neighbourhood overlap, a raster may estimate a particular statistic by performing a smaller amount of calculations, with the cost of a later interpolation stage (Davies, 2012).

In addition, this approach shares the benefits and drawbacks of both overlapping and non-overlapping PPF design.

3.2 Definitions

3.2.1 Rectangular Neighbourhood

For a particular input cell or pixel located at (i, j) within a $P \times Q$ discrete digital input X, a rectangular neighbourhood N_{rs} of size $p \times q$ may be defined as the set

$$N_{rs}(i,j) = \{X(k,l)\}, \tag{3}$$

for which

$$\max(|k-i|) \leq r \text{ and } \max(|l-j|) \leq s, \tag{4}$$

where r and s are positive integers that set the vertical and horizontal radius of the neighbourhood around the central position (i, j), respectively. Thus, a pixel neighbourhood N_{12} yields a 3×5 rectangular block of input cells.

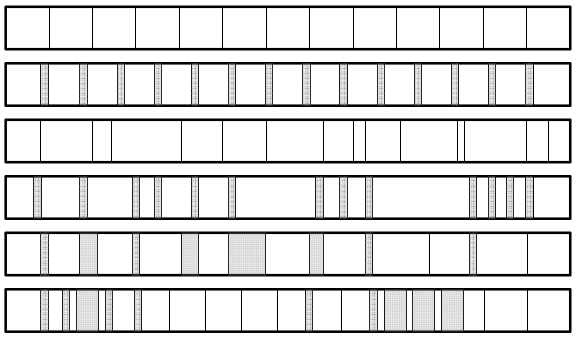


Figure 4: Complete vector partitions (top to bottom): regular non-overlapping, regular fixed overlapping, irregular non-overlapping, irregular fixed overlapping, irregular loose overlapping and regular loose overlapping.

3.2.2 Vector Partitions

Let $\bar{x}$ be a single-dimensional discrete vector of length L. A partition P of $\bar{x}$ into p non-empty parts, $P(\bar{x}, p)$, may be defined as a group of p subsets $\bar{x}_i$ of $\bar{x}$, i.e. $P(\bar{x}, p) = \{\bar{x}_i\}$ for $0 < i \leq p$, where each part $\bar{x}_i$ is a subset of $\bar{x}$ of length L_i, with the following property:

Given any two elements of $\bar{x}_i$ with vector positions i_u and i_v, where $0 < i_u < i_v \leq L_i$ and their corresponding positions in $\bar{x}$, j_u and j_v, with $0 < j_u < j_v \leq L$, the following condition is hold:

$$j_u - j_v = i_u - i_v. \tag{5}$$

In other words, each part $\bar{x}_i$ keeps the original relative positions of their elements in $\bar{x}$ and no element of $\bar{x}$ is missed between the first and the last element in any part $\bar{x}_i$.

A partition $P(\bar{x}, p)$ is said to be complete if and only if (iff) the set union of all the parts of $\bar{x}$ yields $\bar{x}$, i.e. iff $\bigcup_i \bar{x}_i = \bar{x}$. Otherwise, it is said to be incomplete. From this point on, our discussion will only deal with complete partitions.

Table 1: Vector Partition Types.

Type	Condition
Regular	$L_i = L_j, \forall i, j$
Pseudo-regular	$\max\lvert L_i - L_j \rvert = 1, \forall i, j$
Fixed	$0 < v_i = v_j, \forall i, j$
Tight	$\max\lvert v_i - v_j \rvert = 1, \forall i, j$

In a complete partition, every element of $\bar{x}$ belongs to at least one of its parts. In Fig. 4 a series of six different complete vector partitions is illustrated.

A complete partition $P(\bar{x}, p)$ is said to be regular iff each part $\bar{x}_i$ has the same length, i.e. iff $L_i =$

$L_j, \forall i,j$. Thus, in this case $L = p \cdot L_i$. Otherwise, the partition is called irregular. In Fig. 4, the first, second and sixth partitions are regular.

A special case of irregular partition is also of interest. A pseudo-regular partition $P(\bar{x},p)$ is an irregular complete partition whose different part lengths obey the following expression:

$$\max\left|L_i - L_j\right| = 1, \forall i,j. \quad (6)$$

A complete partition $P(\bar{x},p)$ is said to be non-overlapping iff the set intersection of any two parts of $\bar{x}$ is the empty set, i.e. iff $\bar{x}_i \cap \bar{x}_j = \emptyset, \forall i \neq j$. Otherwise, it is called overlapping. In Fig. 4, overlapping partitions are depicted with their overlapping parts shaded.

An overlapping partition $P(\bar{x},p)$ may, of course, be regular iff $L_i = L_j, \forall i,j$. However, if the overlap sections shared between two adjacent parts $\bar{x}_i$ and $\bar{x}_{i+1}$, for $0 < i < p$, have constant size $v_i > 0$, the overlapping partition is said to be fixed, i.e. if $0 < v_i = v_j, \forall i,j$. Otherwise, it is said to be loose. Both second and fourth partitions illustrated in Fig. 4 are fixed.

An overlapping fixed partition may be represented with the special notation $P(\bar{x},p,v)$, where its main parameters, L, p and v are positive integers bound to the equation

$$L = \overline{L} - v(p-1), \quad (7)$$

where $\overline{L} = \sum_{i=1}^{p} L_i$ is called the limit length of the partition, i.e. the accumulated length of its p parts. In addition, if $\overline{L} > L$ and $p > 1$, then the positive condition for v is always met,

$$v = \frac{\overline{L}-L}{p-1} > 0. \quad (8)$$

Finally, a special kind of overlapping partition is also considered. A tight overlapping partition $P(\bar{x},p)$ is a loose overlapping partition whose different overlap section sizes v_i follow the relation

$$\max\left|v_i - v_j\right| = 1, \forall i,j. \quad (9)$$

For convenience, a summary of the previous definitions is collected in Table 1.

3.3 A Vector Partition Algorithm for Parallel Block Processing

Given an input vector $\bar{x}$ and a fixed part size l, an optimal partition $P_o(\bar{x},p)$ is sought. The optimality of the partition is based on the discussion of Sections 2.2 and 2.3, where a PPF is supposed to be used to perform some operation over the input vector. Thus, our main interests are: 1) to keep partition parts as equal in size as possible and 2) to reduce overlapping to a minimum.

Moreover, the optimal partition selection algorithm will be performed as follows:

- First, if a regular non-overlapping partition exists, it will be chosen as optimal;
- Otherwise, if a non-overlapping pseudo-regular partition is possible, it will be chosen in the second place,
- Third, if none of the above possibilities are available, an overlapping fixed partition will be selected, with the minimum amount of overlap.

From the previous section, we know that if $p = L/l$ is a positive integer $p > 0$, then $P(\bar{x},p)$ will be a regular non-overlapping partition.

However, for this preferred case to occur, l must be an integer divisor of L and L must not be prime. Otherwise, a pseudo-regular partition should be chosen.

The following algorithm is proposed for obtaining a non-overlapping pseudo-regular partition $P(\bar{x},p)$ of L elements with fixed part length, l:

Let $m = \mathrm{mod}(L,l)$ with $m > 0$.
If $m < (l-1)/2$,
  Let $p = \lfloor L/l \rfloor$, $n_b = m$ and $l' = l + 1$.
Else
  Let $p = \lceil L/l \rceil$, $n_b = l - m$ and $l' = l - 1$.
End

In the previous algorithm, the number of parts p of the partition is defined such that the number of pseudo-parts n_b of length l' is minimal and condition (6) is maintained. Thus, a pseudo-regular partition $P(\bar{x},p)$ is obtained.

However, the specific distribution of the n_b pseudo-parts still remains undefined. Between all the possibilities, a symmetric distribution with maximal distance between pseudo-parts is proposed. This kind of distribution should enable the best balance of any possible side effect as a result of the pseudo-regular kind of the partition.

Let B be a n_b-elements vector of pseudo-part positions $b_k = B(k) \in [0,p], \forall k$ in $P(\bar{x},p)$. A symmetric distribution of the pseudo-parts in a pseudo-regular non-overlapping partition can be obtained with the following algorithm:

If n_b is odd,
  Let $B(\lfloor n_b/2 \rfloor) = \lfloor p/2 \rfloor$.
End
If $n_b > 1$,
  Let $s = p/(n_b - 1)$ with $s > 1$.
  For $k = 0$ to $\lfloor n_b/2 \rfloor$

Let $B(k) = \lfloor k \cdot s \rfloor$,
$B(n_b - k) = p + 1 - B(k)$.

End
End

Thus, if the number of pseudo-parts is odd, one of them should be the central part of the partition. In addition, each pseudo-part would be separated by s parts, including the first and last parts of $P(\bar{x}, p)$.

The previous algorithm would only yield pseudo-regular non-overlapping partitions iff $s > 1$. Otherwise, the obtained partition cannot ensure optimal criteria (e.g. block pseudo-regularity).

In these cases, a third kind of partition should be chosen. This new partition scheme should have, at least, pseudo-regular overlapping parts with fixed overlap v.

With the aid of (7), it is possible to apply the previous algorithm to a new non-overlapping partition with length equivalent to the limit length $\overline{L}$ of an overlapping fixed partition $P(\bar{x}, p, v)$, for increasing v. Once the equivalent pseudo-regular partition is obtained, a fixed overlap is performed in each of its parts.

3.4 *N*-D Block Partitions

The optimal vector partition algorithm discussed in Section 3.3 may be separately applied to each dimension of a *n*-D discrete input data.

In this case, the obtained *n*-D partition is called a *n*-D block partition (BP).

Table 2: 16-element Vector complete Partitions.

l	p	n_b	v	$\overline{L}$
1	16	0	0	16
2	8	0	0	16
3	6	2	0	16
4	4	0	0	16
5	3	1	0	16
6	3	2	0	16
7	2	2	0	16
8	2	0	0	16
9	2	2	0	16
10	2	0	4	20
11	2	0	6	22
12	2	0	8	24
13	2	0	10	26
14	2	0	12	28
15	2	0	14	30
16	1	0	0	16

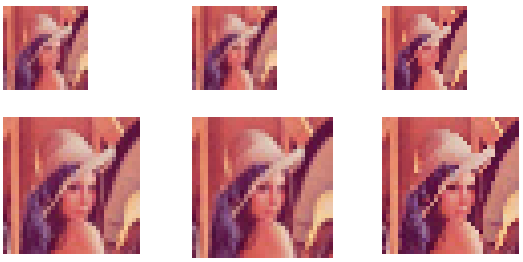
4 PARTITIONING EXAMPLES

In the first example, the optimal partition algorithm of Section 3.3 is applied to a vector with $L = 16$ elements. The algorithm is executed for part lengths with $l = 1, 2, \ldots, L$. The resulting partitions are presented in Table 2. Not until a relatively large part size, $l = 10$, the algorithm delivers an overlapping partition as optimal. From this part length on, two overlapping parts are set as the optimal partition for the test vector with minimum overlap v.

In the second test, two optimal partitions for a block decimation colour scale pyramid are computed for *Lena* image, of 512×512 px. The chosen decimation factors for this example are 1/15 and 1/25, thus block partitions for l = 15 and l = 25 are computed using the algorithm of Section 3.3.

Clearly, neither 15 nor 25 are integer divisors of 512. Thus, the obtained optimal partitions are both pseudo-regular non-overlapping, as depicted in Fig. 5. These partitions are optimal as they yield minimum overlap and minimum amount of blocks with different lengths. In this case, n_b = 2 for l = 15 and n_b = 13 for l = 25.

In Fig. 6, the resulting output decimated block images are displayed. Both local mean and median are used for computing each pixel in the decimated image. For reference purposes, an additional bicubic-decimated image is also shown in Fig. 6. This latter filter is not based on the proposed optimal blocks.



a. b. c.

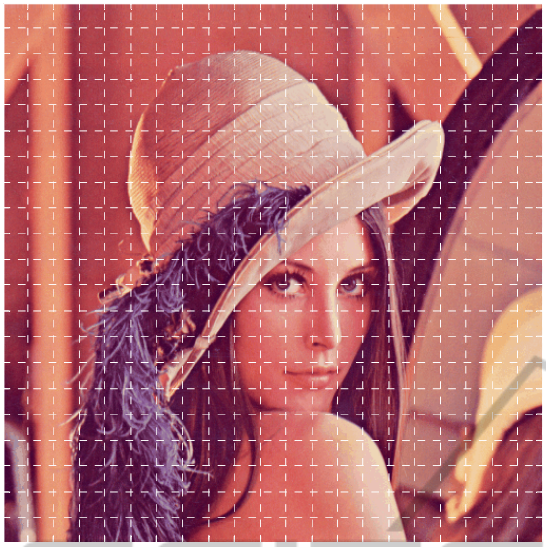
Figure 6: 1/25 and 1/15 block decimation for *Lena* (2x scale): a) block mean, b) bicubic interpolation and c) block median.

5 CONCLUSIONS

In this paper, a vector partition algorithm with applications in parallel processing environments, such as PPF, has been introduced.

From a theoretical discussion on the description and classification of vector partitions, which can be

a.



b.

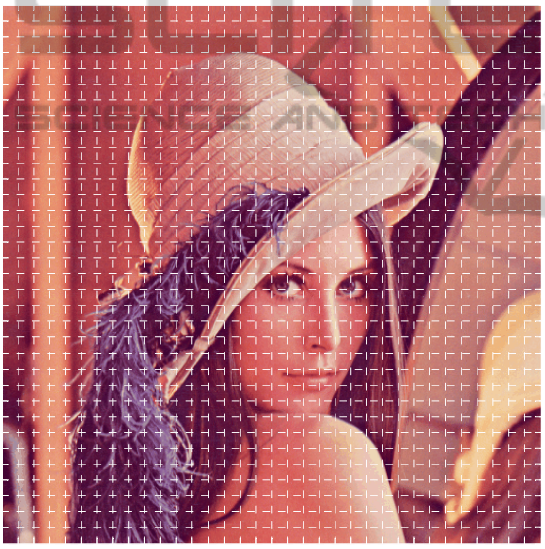


Figure 5: Optimal block partitions for *Lena*: a) $l = 25$ and b) $l = 15$.

extended to the n-dimensional case, the application of the proposed algorithm has been tested on a typical image decimation stage.

The algorithm produces a block partition which is optimized to be processed in PPF frameworks. The proposed optimization procedure focuses in both minimizing possible size differences in process loads, and maintaining inter-block data sharing at a minimum, by selecting the minimum amount of overlap between adjacent blocks.

In combination with time parallelization, possible applications of the proposed algorithm in real-time processing platforms may accelerate some common high load pre-processing tasks in computer vision, such as statistical analysis from local histograms.

In a parallel processing machine, the proposed algorithm should enable a BP scheme which may be real-time adapted to the instantaneous availability of PUs in the environment.

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