Invariant Shape Prior Knowledge for an Edge-based Active Contours
Invariant Shape Prior for Active Contours

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Abstract: In this paper, we intend to propose a new method to incorporate geometric shape prior into an edge-based active contours for robust object detection in presence of partial occlusions, low contrast and noise. A shape registration method based on phase correlation of binary images, associated with level set functions of the active contour and a reference shape, is used to define prior knowledge making the model invariant with respect to Euclidean transformations. In case of several templates, a set of complete invariant shape descriptors is used to select the most suitable one according to the evolving contour. Experimental results show the ability of the proposed approach to constrain an evolving curve towards a target shapes that may be occluded and cluttered under rigid transformations.

1 INTRODUCTION

Active contours (Kass et al., 1988; Cohen, 1991; Xu and Prince, 1997; Malladi et al., 1995; Caselles et al., 1997; Chan and Vese, 2001) have been widely used in image segmentation. One can classify them into two families : The boundary-based approach which depends on an edge stopping function to detect objects and the region-based approach which is based on minimizing an energy's functionnal to segment objects in the image. Given that these classical active contours are intensity-based models, there is still no way to caracterize the global shape of an object. Especially in presence of occlusions and clutter, all the previous models converge to the spurious contours resulted from large local gradient nearby. Many works incorporate shape prior into the active contour models. Leventon et al., (Leventon et al., 2000) associated a statistical shape model to the geodesic active contours (Caselles et al., 1997). At each step of the surface evolution, the maximum a posteriori position and shape are estimated and used to move globally the surface while local evolution is based on image gradient and curvature. Chen et al., (Chen et al., 2001) defined an energy's functional based on the quadratic distance between the evolving curve and the average shapes of the target object after alignment. This energy is then incorporated into the geodesic active contours. Bresson et al., (Bresson et al., 2003) extended (Chen et al., 2001) approach by integrating the statistical model of shape proposed by (Leventon et al., 2000) in the energy functional. Fang and Chan (Fang and Chan, 2007) introduced a statistical shape prior into the geodesic active contour to detect partially occluded object. To speed up the algorithm, an explicit alignment of the shape prior model and the current evolving curve is done to calculate pose parameters. Foulonneau et al., (Foulonneau et al., 2004) introduced a geometric shape prior into a region-based active contours (Chan and Vese, 2001) based on the Legendre moments of the characteristic function and in (Charmi et al., 2010), the authors defined a geometric shape prior for the region-based active contours after alignment of the evolving contour and the reference shape. It's well know that shape priors based on contour alignment methods force these approaches to segment only single object in the image and go without the contribution of level set, i.e. its ability to segment multiple objects at once, see (Bresson et al., 2003). Besides, contour alignment methods are not adapted to estimate the rigid transformation parameters in the case of objects with holes (which often occurs in medical imagery like MRI brain's white matter). This justifies the use of the registration methods instead of those based on contours alignment. In this work, we focus on adding a new geometric shape prior to an edge-based active contours (Malladi et al., 1995) based on phase correlation. At the begining, we assume that the shape of reference is known in advance like the work of (Zhang and Freedman,

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2003; Cremers et al., 2003; Foulonneau et al., 2004; Chan and Zhu, 2005). Then we will introduce our approach to select the best shape according to the evolving contour in situation where the shape of reference is unknown and many templates are available. The improved model can retain all the advantages of the level set approach and have the additional ability of being able to handle the case of images with multiple objects under partial occlusions and noise. The remainder of this paper is organized as follows : In section 2, we will briefly recall the used shape registration method based on phase correlation. Then, in section 3, the proposed shape prior will be presented. We assume that at this stage, the shape of reference is known in advance. In section 4, we present our method to choose the suitable shape in case of many references. Experiments will be presented and commented in order to study the robustness of the model in section 5. Finally, we conclude the work and highlight some possible perspectives in section 6.

2 SHAPES REGISTRATION

Before incorporating the prior knowledge into the active contours model, the reference shape or the statistical model must be first transformed to best match the current shape. We adopt the well know method of phase correlation in Fourier space that is appropriate to estimate the translation vector and for estimating the rotation angle and the scaling factor, we use the proposed method of phase correlation in Fourier-Mellin space. We recall this method which is based on the Analytical Fourier-Mellin Transform (AFMT), see (M'Hiri et al., 2012) for a detailed description. A comparative study with other global registration methods is presented in (Sellami and Ghorbel, 2012). Let $f(r,\theta)$ be a polar representation of the image with the radius r according to the center of gravity of the image to offset translation and θ the angle according to the horizontal. It was pointed out in (Ghorbel, 1994) that the crucial numerical difficulties in computing the Fourier-Mellin transform of an image might be solved by using the Analytical Fourier-Mellin Transform (AFMT) given by

$$M_{f_\sigma}(k,v) = \frac{1}{2\pi}\int_0^{+\infty}\int_0^{2\pi} f(r,\theta) r^{\sigma - iv} e^{-ik\theta}\,\frac{dr}{r} d\theta, \tag{1}$$

where $\sigma > 0$ is a fixed and strictly positive real number. Since no discrete transform exists, three approximations of the AFMT have been designed : the direct, the cartesian and the fast algorithm, see (Derrode and Ghorbel, 2001). Let $f_{\phi_{ref}}$ and f_ϕ be two binary images associated respectively with level set functions ϕ_{ref} and ϕ. Denote by $M_{f_{\sigma,\phi_{ref}}}$ and $M_{f_{\sigma,\phi}}$ the AFMT of respectively $f_{\phi_{ref}}$ and f_ϕ with the same value of σ. $f_{\phi_{ref}}$ and f_ϕ have the same shape if and only if there is a similarity $(\alpha_0,\beta_0) \in G = (R_+^*, S^1)$ such that

$$\forall (r,\theta) \in G, f_\phi(r,\theta) = f_{\phi_{ref}}(\frac{r}{\alpha_0}, \theta - \beta_0), \tag{2}$$

The action of planar similarities in Fourier-Mellin space leads to

$$M_{f_{\sigma,\phi}}(k,v) = \alpha_0^{\sigma - iv} e^{-ik\beta_0} M_{f_{\sigma,\phi_{ref}}}(k,v), \tag{3}$$

By calculating the normalized cross-spectrum, only information on phase difference will be preserved

$$\Phi(k,v) = \frac{M^*_{f_{\sigma,\phi_{ref}}}(k,v) M_{f_{\sigma,\phi}}(k,v)}{|M^*_{f_{\sigma,\phi_{ref}}}(k,v)||M_{f_{\sigma,\phi}}(k,v)|} = \alpha_0^{-iv} e^{-ik\beta_0}, \tag{4}$$

Phase correlation of two images represented respectively by $f_{\phi_{ref}}$ and f_ϕ is defined as

$$C_{Tfm}(\alpha,\beta) = \int_0^{+\infty} \sum_Z \Phi(k,v) \alpha^{iv} e^{ik\beta} dv, \tag{5}$$

We can deduce the images transformation's parameters (α,β) by estimating (α_0,β_0) that maximize the correlation function C_{Tfm}. Having the parameters of rigid transformation between the two binary images, we perform the registration of the image $f_{\phi_{ref}}$ according to the following formula (Chan and Zhu, 2005)

$$f^{reg}_{\phi_{ref}}(x,y) = \alpha f_{\phi_{ref}}\left(\frac{(x-a)\cos\theta + (y-b)\sin\theta}{\alpha}, \frac{-(x-a)\sin\theta + (y-b)\cos\theta}{\alpha}\right), \tag{6}$$

where (a,b) represents the translation vector, θ the rotation angle and α is the scaling factor. On the resulting image (Fig.1), the pixels in black (resp. white) correspond to positive areas (resp. negative) of the signed distance map which is associated to level set function. The image on the right of Fig.1 shows the product function given by

$$f_{prod}(x,y) = f^{reg}_{\phi_{ref}}(x,y) \cdot f_\phi(x,y), \tag{7}$$

By construction, the function f_{prod} is negative in the areas of variability between the two binary images (occlusion, clutter, missing parts etc.) whereas in positive regions, the objects are similar. Thus, in what

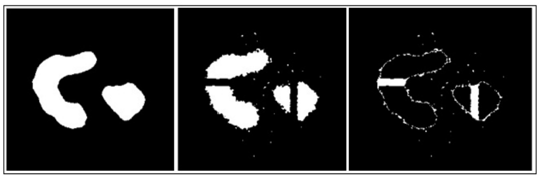

Figure 1: Left : $f^{reg}_{\phi_{ref}}$, Middle : f_ϕ , Right : f_{prod}.

follows, we propose to update the level set function ϕ only in regions of variability between shapes to make the evolving contours overpass the spurious edges and recover the desired shapes of objects. This property recalls the Narrow Band technique used to accelerate the evolution of the level set functions (Malladi et al., 1995). In many works like those of (Foulonneau et al., 2004; Foulonneau et al., 2006) and (Leventon et al., 2000; Fang and Chan, 2006; Fang and Chan, 2007), all the pixels in the image, called n, are invoked in the process of incorporating the shape prior which may increase the calculus complexity $(O(n^2))$ of the model. In our work, only pixels of the region of variablity, called k, are invoked. Generaly $k << n$, hence the calculus complexity (in our case $O(k^2)$) is reduced. This remark will be more developed in section 5.

3 THE PROPOSED SHAPE PRIOR

Geometric active contours are iterative segmentation methods which use the level set approach (Osher and Sethian, 1988) to determine the evolving front at each iteration. Working with this approach makes it possible to manage topology changing of the contour like splitting and merging, and consequently the segmentation of an arbitrary number of objects in the image. In (Malladi et al., 1995), the level set method is used to model the shape of objects with an evolving front. The evolution's equation of the level set function ϕ, which is the embedding function associated to the active contour, is

$$\phi_t + F|\nabla\phi| = 0, \tag{8}$$

F is a speed function of the form $F = F_0 + F_1(K)$ where F_0 is a constant advection term equals to (± 1) depending of the object inside or outside the initial contour. The second term is of the form $-\varepsilon K$ where K is the curvature at any point and $\varepsilon > 0$, is a constant real. To detect objects in the image, the authors proposed the following function which stops the level set function's evolution at the object boundaries

$$g(x,y) = \frac{1}{1+|\nabla G_\sigma * f(x,y)|^p},\ p >= 1 \tag{9}$$

where f is the image and G_σ is a Gaussian filtre with a deviation equals to σ. This stopping function has values that are closer to zero in regions of high image gradient and values that are closer to unity in regions with relatively constant intensity. Hence, the discrete evolution equation is

$$\frac{\phi^{n+1}(i,j)-\phi^n(i,j)}{\Delta t} = -g(i,j)\ F(i,j)\ |\nabla\phi^n(i,j)|, \tag{10}$$

It's obvious that the evolution is based on the stopping function g which depends on the image gradient. That's why this model leads to unsatisfactory results in presence of occlusions, low contrast and even noise. To make the level set function evolves in the regions of variability between the shape of reference and the target shape, we propose the new stopping function as follows

$$g_{shape}(x,y) = \begin{cases} 0,\ if\ \psi(x,y) >= 0, \\ sign(\phi_{ref}(x,y)), else, \end{cases} \tag{11}$$

where $\psi(x,y) = \phi(x,y)\cdot\phi_{ref}(x,y)$, ϕ is the level set function associated to the evolving contour, while ϕ_{ref} is the level set function associated to the shape of reference after registration. As it can be seen, the new proposed stopping function only allows for updating the level set function in the regions of variability between shapes. In these regions g_{shape} is either 1 or -1 because in the case of partial occlusions, the function is equals to 1 in order to push the evolving curve inward (deflate) and in case of missing parts, this function is equals to -1 to push the contour towards the outside (inflate). This property recalls the Balloon snake's model proposed by Cohen in (Cohen, 1991) in which the direction of evolution (inflate or deflate) should be precised from the beginning. In our work, the direction of evolution is hundled automatically based on the sign of ϕ_{ref}. The total discrete evolution's equation that we propose is

$$\frac{\phi^{n+1}(i,j)-\phi^n(i,j)}{\Delta t} = -(w\,g(i,j) + (1-w)\,g_{shape}(i,j))\ \ F(i,j)|\nabla\phi^n(i,j)|, \tag{12}$$

where w is a weighting factor between the image-based force and the knowledge-driven force. To illustrate the ability of the proposed shape prior to constrain geometrically an active contour, we show the evolution of the contour under the influence of the proposed shape prior term only (i.e. $w = 0$). We present in Fig.2 an example of successive evolutions between several shapes of different topologies. We have chosen for initial curve a green square. The first shape of reference is a tree leaf. An intermediate step in this evolution is shown by the first row and the final curve is presented by the last column. This last configuration of the contour is used as an initial curve for the next experiment by taking the image of the left and right ventricles of the heart as a reference and then finally in the same way by taking the shape of a pen and a ring as shapes of reference.

4 CASE OF MANY REFERENCES

In presence of many templates, we have to choose the

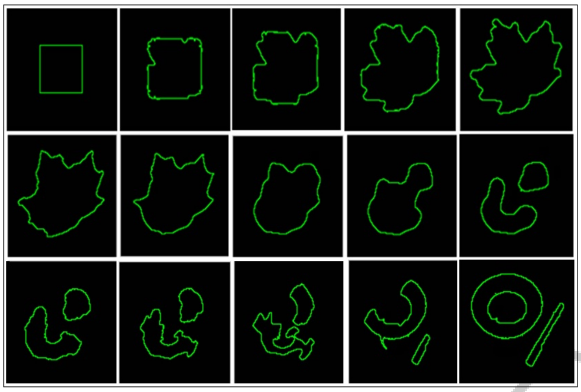

Figure 2: Curve evolution under the proposed shape prior.

most suitable one according to the evolving curve. In (Fang and Chan, 2006), a new statistical shape prior is proposed for robust object detection. This prior model is capable to handle multiple shape states of the object. A Gaussian Mixture Model (GMM) is used to estimate the data distribution in the feature subspace and a Bayesian classifier is used to assign the currently detected object to the most similar shape cluster. A shape prior is then constructed by using the statistical properties of that cluster to constrain the subsequent curve evolution. This model requires a preliminary step which consist in aligning the training data. Besides PCA must be applied. In (Charmi et al., 2009), the authors proposed a geometric approach to add prior information to the snake model in case of many references. A set of complete and locally stable invariants to Euclidean transformations, based on Fourier transform of the contour, is used to define new force which makes the snake overcome some well-known problems. Motivated by (Charmi et al., 2009), we propose in this section to use as criterion the distance between a complete family of similarity invariant features based on the AFMT suggested in (Ghorbel, 1994; Derrode and Ghorbel, 2001). This family can be easily rewritten and applied to any strictly positive value σ in the following way, $\forall(k,v) \in (Z,R)$:

$$I_{f_\sigma}(k,v) = M_{f_\sigma}(0,0)^{\frac{-\sigma+iv}{\sigma}} e^{-ikArg(M_{f_\sigma}(1,0))} M_{f_\sigma}(k,v), \tag{13}$$

Completeness is recognized as an important criterion for full shape discrimination and reconstruction from features. Since this invariant set is convergent for square summable functions, it can be shown in (Ghorbel, 1994) that the following function defines a true mathematical distance between shapes :

$$d_2(I_{f_\sigma}, I_{g_\sigma}) = \left(\int_{-\infty}^{+\infty} \sum_{k\in Z} |I_{f_\sigma}(k,v) - I_{g_\sigma}(k,v)|^2 dv\right)^{\frac{1}{2}}, \tag{14}$$

f and g represent two gray-scale objects. Due to numerical sampling and approximation, we never have exactly zero and the value of the distance is used for the quantification for the similarity between objects, regardless of their pose, orientation and size in the image.

5 EXPERIMENTAL RESULTS

In this section, the proposed model with shape prior will be applied to the segmentation problem. Consequently the model will evolves under both data and prior terms. In order to reduce the computational complexity and to have a good estimation of the parameters of the rigid transformation as in (Foulonneau et al., 2004; Fang and Chan, 2007), we first evolve the active contour without shape prior until convergence (i.e. $w = 1$). This first result provides an initialization for the model with prior knowledge. To promote the convergence to the target shape, we generally give more weight to prior knowledge (i.e $w <= 0.5$). In the next experiment, we compare our model to that proposed by Fang and Chan in (Fang and Chan, 2007). The shape of reference is provided by image (a). Images (b) and (c) represent repectivelly the results obtained by our model and the model of Fang and Chan. It is visually clear that in region of variability (occlusion), the proposed approach gives a better result. By

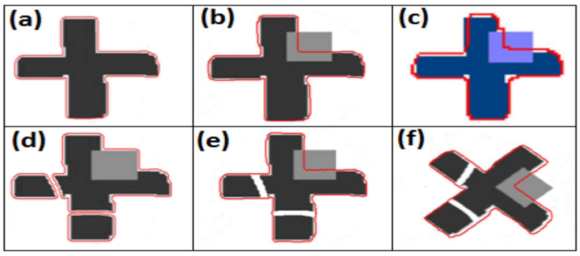

Figure 3: (a) : The reference, (b) : Result obtained by our model, (c) : Result obtained by Fang and Chan model, (d) : Segmentation without shape prior, (e) and (f) : Segmentation with the proposed model.

the second row, we handle a situation where the object to be detected is no longer connected (due to missing parts or hole). In such situations, methods based on contours alignment does not allow to estimate the parameters of the rigid transformation. Thanks to registration by phase correlation, our model can handle this case and consequently the detection of the target object. In order to illustrate how the calculus complexity can be reduced by updating the evolving level set function only in the regions of variability between the reference and the target objects, we compute the execution time for the image (b) until convergence with

Table 1: Execution time depending on the image size.

	128x128	256x256	512x512
Our model	9.743	30.671	151.998
Fang's model	10.157	41.153	299.3

different sizes (128x128, 256x256 and 512x512). We recall that for the model proposed by (Fang and Chan, 2007), the proposed shape prior is as follows :

$$\phi^{n+1}(i,j) = \phi^{n}(i,j) + \Delta t(\phi^{*}(i,j) - \phi^{n}(i,j)), \quad (15)$$

where ϕ^* is the shape model. We set the same values for the weighting factor and time step and we consider that the shape of reference (the shape model) is given. The table below presents the needed execution time (in seconds) to have satisfactory results. It's clear that for an important image size, our approach needs less iterations to reach good results. In Fig. 4, the case of real image with several objects under partial occlusions and different types of noise (Gaussian, Salt and Pepper and Speckle) is considered. Segmentation without shape prior fails to detect the familiar objects (second image of first row). However, using the shape prior, the proposed model succeeds in segmenting the desired objects (third image of first row). For the second row, a rotation of $-90°$ with differents kind of noise are applied. Results seems to be satisfactory.

In what follows, our model is applied to medical images obtained from Brain Web Simulated Data Base[1]. We focus on the segmentation of white matter of slice 56 that contains holes. The first image represents the reference segmentation. We have chosen to initialize the model with a green curve. The obtained contours without the constraint of shape are presented by the third image. Starting with this result and after the registration step, final segmentation based on prior knowledge and image-based information is presented by the last image. Table 2 shows the value of RMSE (Root Mean Square Error), execution time until convergence and the time for parameters estimation (in seconds) of the rigid transformation for different values of w. The image size is 256 x 256. We set the total number of iterations equals to 500. The prior knowledge is introduced at the iteration 400 for different values of w. For the second part of the experiments, we assume that the reference shape is unknown and we rely on the set of invariants presented in section 4 to choose the most appropriate template. Thus the proposed algorithm is as follows :

1. Segmentation of the target object using the active contours model (Malladi et al., 1995) without shape prior.

[1]http://mouldy.bic.mni.mcgill.ca/brainweb/

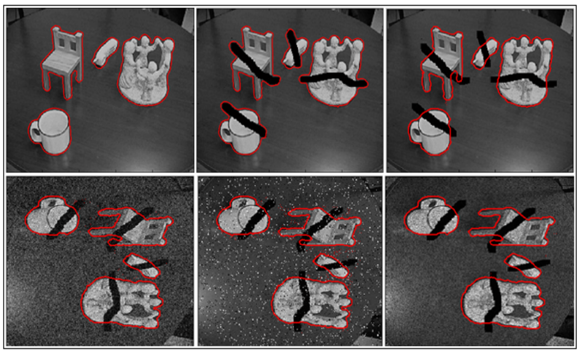

Figure 4: Several object detection under partial occlusion and noise.

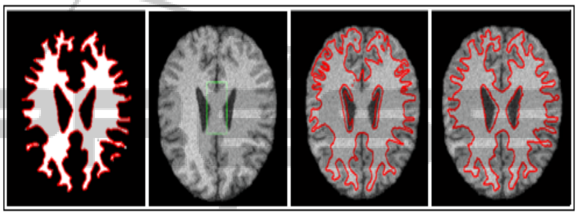

Figure 5: Segmentation of brain's white matter.

Table 2: Variation of the RMSE and the execution time depending on w.

w	RMSE	Execution time	Motion estimation
1	0.549	24.563	-
0.8	0.237	26.264	0.430
0.4	0.230	27.009	0.470
0.1	0.222	24.798	0.375
0	0.216	27.161	0.526

2. When the evolving curve becomes stable, we compute the set of invariant descriptors associated with the target object.
3. We compare this set of descriptors to those associated with the available templates computed at an off-line step, then we select the reference shape presenting the smallest Euclidean distance.
4. Then, we perform the registration step and we compute the proposed shape prior.
5. Finally, we evolve the model under both image and prior forces.

In what follows (Fig.6, second row), we have occluded some objects from the COIL database (Columbia University) and for every object, we seek for the best template from the available ones (first row). For each shape, we compute its associated invariant descriptors to choose the appropriate template. Table 3 summarizes the obtained results and Fig.7 shows the final segmentation without and with our proposed method.

Table 3: Shape distance between every occluded object and the set of the available references.

	(a)	(b)	(c)	(d)
(a) + occlusion	***0,471***	1,376	1,698	0,924
(b) + occlusion	1,460	***1,113***	2,088	1,445
(c) + occlusion	1,616	2,038	***1,161***	1,494
(d) + occlusion	1,519	1,854	1,933	***1,419***

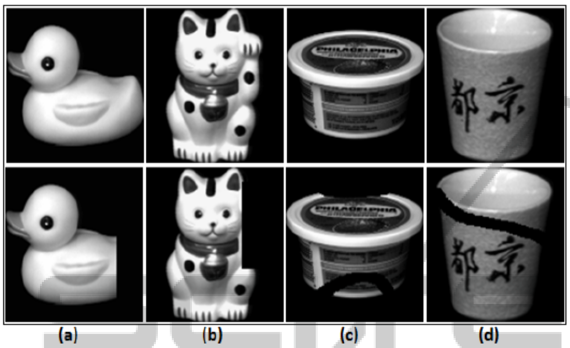

Figure 6: Selected objects from the COIL database (Columbia University), First row : the original objects, second row : objects after partial occlusions and missing parts.

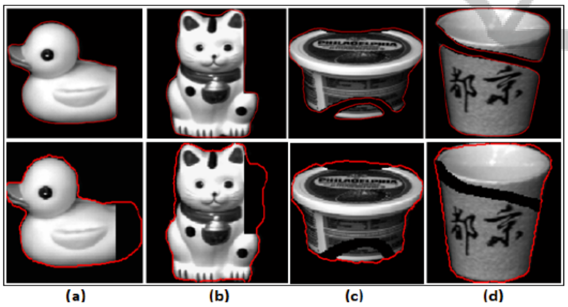

Figure 7: Detection of the familiar objects without (first row) and with the proposed method (second row).

For this example, the selected objects were extracted from the background. Hence we directly compute the value of shape descriptors on the gray-scale images. Then we select the template of every target image. In real situation, we have to extract the target object from the background using the active contours model in order to compute the set of invariant shape descriptors. The following experiment illustrates the proposed algorithm. Fig.8 presents several images of reference. The objects of interest were isolated from the cluttered background of the image using the edge-based active contours (Malladi et al., 1995). We propose to detect the true edges of the target shapes (1) and (2) which are partially occluded (see Fig.9, images (a) from first and second rows). Table 4 shows the Euclidean distance between the target object's invariants and those associated with the available shapes of reference.

Table 4: Shape distance between the two target images and the set of available reference objects.

	(a)	(b)	(c)	(d)	(e)
Target 1	***0.426***	0.988	1.208	1.020	0.805
Target 2	1.308	1.509	***1.160***	1.364	1.483

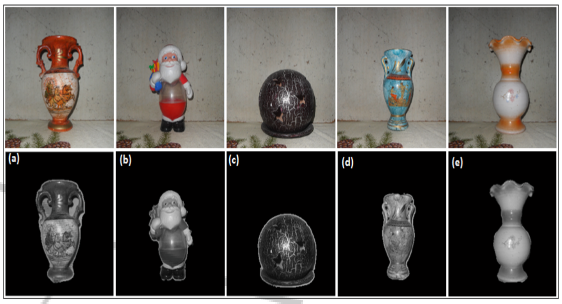

Figure 8: The reference images and the associated objects of interest.

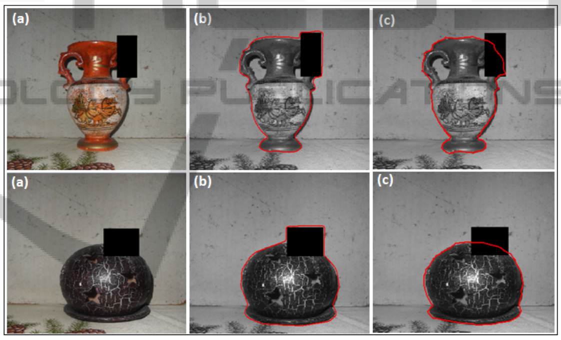

Figure 9: Object detection (b) without shape prior, (c) with shape prior.

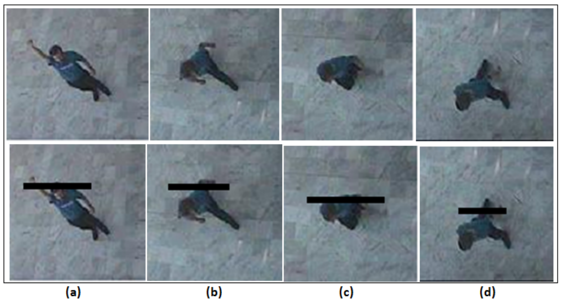

Figure 10: Different human states under partial occlusions.

Once the reference shape with small distance is selected, its associated level set function is used to constrain the contour evolution towards the true contours of the object of interest. For the last experiment, we consider the segmentation of human shape under different states and partial occlusions. Fig.10 shows the considered human shape states. The used images were obtained from[2]. We note that unlike the work

[2]http://homepages.inf.ed.ac.uk/rbf/CAVIAR/

of (Fang and Chan, 2006), the use of invariant features allows to avoid the registration step at the learning stage and the computation of PCA on the training data in order the estimated the appropriate number of clusters in a low dimensional feature subspace. The final detection results are given by Fig.11.

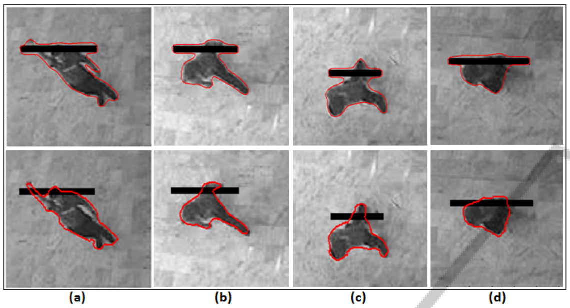

Figure 11: Detection of partially occluded human shapes in video images. First row shows the detection results obtained by the traditional edge-based active contours and the Second row shows the result obtained by our proposed method.

6 CONCLUSIONS

New method of geometric active contours with shape prior is presented in this research. This approach uses the registration by phase correlation and a set of invariant descriptors to define prior knowledge. Experiments have shown the ability of the new added term to improve the robustness of the detection process in presence of missing parts and partial occlusions of the target objects. The addition of shape prior has not increased significantly the execution time given that the proposed approach does the registration only once and it is done by the Fast Fourier Transform unlike (Foulonneau et al., 2006; Charmi et al., 2008) where at each iteration shape descriptors are calculated for a given order which has to be set empirically. In fact, a small order gives unsatisfactory results and a big one increase significantly the execution time. As future perspectives, we are working on applying our model in the context of medical application where the shape of reference is given by medical atlas in order to aid in the diagnosis. Also, we plan to extend this approach to more general transformations such as affine transformations.

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