An Inverse Distance-based Potential Field Function for Overlapping Point Set Visualization

Jevgēnijs Vihrovs, Krišjānis Prūsis, Kārlis Freivalds, Pēteris Ručevskis and Valdis Krebs
Institute of Mathematics and Computer Science, University of Latvia, Raiņa bulvāris 29, LV-1459, Riga, Latvia

Keywords: Information Visualization, Implicit Surfaces.

Abstract: In this paper we address the problem of visualizing overlapping sets of points with a fixed positioning in a comprehensible way. A standard visualization technique is to enclose the point sets in isocontours generated by bounding a potential field function. The most commonly used functions are various approximations of the Gaussian distribution. Such an approach produces smooth and appealing shapes, however it may produce an incorrect point nesting in generated regions, e.g. some point is contained inside a foreign set region. We introduce a different potential field function that keeps the desired properties of Gaussian distribution, and in addition guarantees that every point belongs to all its sets' regions and no others, and that regions of two sets with no common points have no overlaps. The presented function works well if the sets intersect each other, a situation that often arises in social network graphs, producing regions that reveal the structure of their clustering.

1 INTRODUCTION

Point sets emerge from the study of many real-world data structures, from social networks to geographical maps. The sets identify important groups of objects inside the structure, for instance, scientific article co-authorship (Santamara and Thern, 2008) or social circles in organizations (Krebs, 2007). Overlaps often occur naturally in such graphs, with sets sharing common intermediary vertices. For example, human and biological networks rarely cluster in clean ways, and one vertex may belong to many groups.

After the identification of different point sets in the given data, there is still the problem of presenting this information in a quickly and easily comprehensible way. In order for the visualization of overlapping point sets to be effective, it should adhere to various criteria. Firstly, it should be unambiguous: the user should be able to identify the sets and their points, as well as their overlaps, without any misunderstandings. It should also represent the geometrical layout of the points themselves as closely as possible. In this paper we deal with visualizing point sets that have an arbitrary, but fixed positioning.

The common approach in visualization is to enclose the points of each set in a region that represents this set, forming an Euler diagram. There are several works that list the desirable properties of visualizations that produce such regions (Rosenthal and Linsen, 2009; Dinkla et al., 2012). Here we present a brief summary.

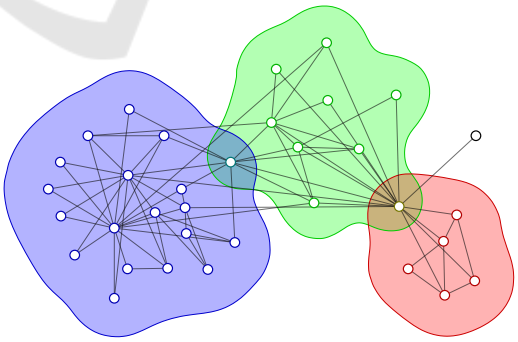


Figure 1: An example of the proposed visualization. There are three overlapping point sets, which correspond to vertex clusters in the given graph. In this example edges do not affect the visualization.

P1 Each point of a set is contained inside the region of this set.

P2 Each point not present in a set is not contained inside the region of this set.

P3 The regions of two different sets must have overlaps only where there is a point that belongs to both of the sets.

P4 The regions must have a strict boundary that sep-

Vihrovs J., Prūsis K., Freivalds K., Ručevskis P. and Krebs V..
An Inverse Distance-based Potential Field Function for Overlapping Point Set Visualization.
DOI: 10.5220/0004681100290038
In *Proceedings of the 5th International Conference on Information Visualization Theory and Applications* (IVAPP-2014), pages 29-38
ISBN: 978-989-758-005-5

arates the included and excluded space.

P5 The boundary of the region should stay close to the points of the set.

P6 The boundaries of the regions should be sufficiently smooth.

P7 The region of one set should be connected.

A widely used visualization method is to compute the regions by bounding a potential field function. The advantages are the smooth shape borders and the flexibility of the result, as the function parameters can be adjusted to obtain a better visualization. The main problems are ensuring the correct membership of the points (P1, P2) and region overlapping (P3), as well as maintaining each region connected (P7). Traditionally, most potential functions are based on the Gaussian distribution function (Blinn, 1982; Gross et al., 1997; Sprenger et al., 2000) or quadratic polinomial functions (Watanabe et al., 2007; Collins et al., 2009). These functions naturally satisfy properties P4-P6.

The main problem with such functions is maintaining the membership properties P1-P3. If each set region is calculated ignoring the influence of the foreign points, then P1 holds, but P2 and P3 are completely ignored. To mitigate this, the foreign points are assigned a negative potential, thus repelling the influence of the points of the given set. This improves P2 and P3 (though does not fully satisfy either), but violates P1, as negative potentials can overwhelm positives ones even directly at a point. To limit the effect this has on P1, smaller weights are often used for negative potentials. However, this weakens the improvements to P2 and P3. In particular, only when negative and positive potentials are equal does the following hold: if two sets share no common point, their regions do not overlap, which is important in cases with no overlapping sets.

In this work we address this problem and propose a different potential field function that always produces visualization with correct point membership, and visually still behaves much like the mentioned potential functions. It is very similar to Gaussian-based methods in isocontour appearance and satisfies the same properties P4-P6. In addition, when using our function with equal negative and positive weights as described earlier, it is guaranteed to satisfy properties P1, P2 concerning the membership of the points. Thus it also guarantees the property P3 for every two sets with no common points. We show the results of the visualization on different artificial and real-world examples and compare them to other potential functions. The function has also two parameters that allow to adjust the size and appearance of the produced regions.

Maintaining property P7 is a topological problem, and cannot be solved by a specific potential function design. Still, it can be ensured using standalone techniques such as including the edges of each set's spanning tree when computing the potential function (Collins et al., 2009), but usually at the cost of ignoring correct overlap properties P2 and P3. We show how this method works with the proposed function. However, as the proposed function is designed to improve properties P1-P3, this leads to unusual visual behavior. Without this property the region of each set can consist of multiple disconnected shapes. In our examples, we use color-coding to differentiate between set regions.

2 RELATED WORK

There have been a substantial number of works that deal with overlapping point set visualization. The problem is highly relevant in visualizing graph vertex clusters in graph drawing, and many approaches are described in the context of graph clusters (Gansner et al., 2010; Balzer and Deussen, 2007; Van Ham and Van Wijk, 2004; Heer and Boyd, 2003).

The main tasks pursued in most visualizations are maintaining connectivity of the regions of a set (P7) and point region membership (P1-P3). The connectivity is very important in the sense that it allows the user to quickly identify a particular set. The correctness of the sets' overlaps may be crucial in understanding the membership of the points and the relationships of the sets. Both these properties may be conflicting: if the positioning of the points is fixed, it is frequently not possible to ensure connectivity and eliminate all unnecessary region overlaps. It is also important whether the layout of the points can be changed or not; various algorithms of point positioning may greatly improve the readability of the visualization.

The most straightforward approach for obtaining the region of a set is to obtain its convex hull, used, for example, in (Heer and Boyd, 2003; Santamara and Thern, 2008). Here, the boundaries of the convex hulls are smoothed by using Bézier curves. However, the convex hull approach ignores the P2 and P3 properties, and many points may happen to reside inside foreign set regions. This is addressed in (Byelas and Telea, 2009), which describes visualization of UML diagrams. The areas where foreign elements overlap the convex hull are excluded from the region heuristically, and afterwards the resulting polygon is shrunk inwards to better represent the layout of the elements in the set. The borders of the regions are also softened which achieves a more natural look of the shapes.

A more sophisticated method that also computes discrete hulls is proposed in (Simonetto et al., 2009). The intersection graph of the sets (which is planar) is built and positioned using a force-directed algorithm (Di Battista et al., 1998). Then a polygonal skeleton is built around the graph vertices, and the points are inserted inside the corresponding polygons; these polygons are then expanded by applying the force-directed layout algorithm several more times. The resulting visualization has correct point membership, and each set region is connected, thus all the constraints P1-P3, and P7 are fully satisfied. Still, this visualization can be applied only when the positioning of the points is not important.

Another method is to use the Voronoi diagram of the given points to create the space partitioning for the visualization. A simple application of such approach for visualizing self-organizing maps can be found in (Matsumoto et al., 2008). The result fully ensures all the membership constraints P1-P3, and is very simple to implement. However, the regions have many sharp edges and all borders are shared (P6 does not hold). Another problem is the long and sharp region parts that do not contain any set vertices, which are common in Voronoi diagrams, as these parts distort the regions and lessen readability, violating P5. GMap (Gansner et al., 2010) mitigates this by inserting many artificial points in the diagram around the areas of interest. A particular technique for smoothing the boundaries of the regions is described in (Rosenthal and Linsen, 2009), which also uses Voronoi diagrams. They create the region boundaries by using the Voronoi diagram to obtain a hull of the set and then drawing a distance field around this hull. This does not, however, work on boundaries between adjacent or intersecting regions. A problem with all Voronoi-based methods is that they do not ensure region connectivity.

The approach used in this work is to compute the regions for point sets by using a potential field function. The value of influence of a point in the given space is defined by the potential function, and the total influence of a point set in some location is the sum of influences for all the points in the set. Then the region for a set is obtained by thresholding the total influence of this set.

The properties of such visualization depend on the choice of the potential field function. Most standard potential functions are based on the Gaussian distribution. Its main advantage is the contour smoothness of the regions, they look similar to hand-drawn. One of the first analyses of such a potential function appears in (Blinn, 1982). It also describes how the "blobbiness" of the visualization can be adjusted using the parameters of the function.

Another type of potential functions are polynomial approximations of Gaussians that are faster to compute and produce nearly identical results, e.g. (Balzer and Deussen, 2007). Also used are bounded quadratic functions (Watanabe et al., 2007; Collins et al., 2009), they generate similar results.

This approach has been widely applied in point set visualization. Some works visualize and cluster point sets in 3D (Sprenger et al., 2000; Balzer and Deussen, 2007). The latter also focuses on hierarchical depiction of a clustered graph. To avoid incorrect overlaps of different set regions, one can assign negative potential to point of foreign sets, which is noted in (Blinn, 1982), and used in (Gross et al., 1997; Watanabe et al., 2007; Collins et al., 2009).

Bubble Sets (Collins et al., 2009) directly focus on 2D point set visualization; while it also uses a potential field function and negative influences for foreign set points, it maintains P7 by using the edges of a spanning tree of each set in addition to the vertices when computing the potential functions. However, these edges from different sets can overlap, violating P3.

Other novel methods include Euler diagrams with connected regions (Riche and Dwyer, 2010), as well as Kelp diagrams (Dinkla et al., 2012), which focus primarily on region connectivity in visualizing points with fixed geometrical layout.

3 THE POTENTIAL FUNCTION

We are given a set of points and the subsets we have to visualize, along with a geometrical positioning of the points in an Euclidean space. For clarity, we will call the given points the *vertices* (from the use cases of representing social graphs), to avoid confusion with spatial points. For each set, we independently compute a function which assigns a real number to each point denoting the set's influence on it. Vertices belonging to the set positively influence the value of the function, whereas other vertices influence it negatively. The amount a vertex influences a point is determined by their distance.

After calculating the function for a set, the shape for this set is extracted using a fixed threshold. We describe the potential function in Sect. 3.1. In Sect. 3.3, we discuss the specifics of the implementation.

3.1 Function Description

Suppose we are given N non-empty sets $S_1 \dots S_N$ where $\forall i : S_i \subseteq V$, and V is the set of all the vertices.

Some vertices can also belong to no set. These vertices in any case negatively influence the value of the function for all sets.

For the visualization, we need the vertices to be geometrically positioned. The positioning can be expressed by a function $P : V \to \mathbb{R}^n$ that assigns a position to each vertex in a n-dimensional Euclidean space. Clearly, in practice, the visualization can be meaningful to users when $n \leq 3$. For use case without a predefined layout, constructing the function P is itself a separate, well-studied problem(Di Battista et al., 1998). In this work, we do not propose any new positioning methods, and the visualization itself does not require any special positioning. For clustered graphs, we will often use a positioning based on a force-directed layout algorithm.

The field function approach works as follows: first, define the *influence* of a vertex v with respect to a point p, which is a function $I(v,p)$. Then the *influence* of a vertex set S on a point p is defined as the sum of the influence of its vertices:

$$I(S,p) = \sum_{v \in S} I(v,p). \tag{1}$$

The potential field function for set S_i at a point p is defined as the difference between the influence of this set and the total influence of all other vertices:

$$F_i(p) = I(S_i,p) - w \cdot I(V \setminus S_i, p) \tag{2}$$

All the vertices that belong to the i-th set influence the value of the function positively, whilst all other vertices influence it negatively. The nonnegative parameter w is the weight of negative influence, which is usually less than 1 (i.e. the weight of the positive influence).

To obtain a shape that represents the i-th set, use a nonnegative threshold t. The region of the vertex set is defined to be the set of points such that for any point p in the set the inequality $F_i(p) > t$ holds. By adjusting the value of t the shapes can be made smaller or larger. In practice, it can be difficult to estimate the shape size from t, therefore we adjust the desired radius of the shape of a single vertex R_t and set t to be the influence of a single vertex at distance R_t.

Since the influence of the vertices belonging to S_i is taken with a positive sign and the influence of other vertices with a negative sign, vertices not belonging to the set repel its shape.

As the function $I(v,p)$ we use the following:

$$I(v,p) = \max(\|P(v) - p\|^{-b} - m, 0) \tag{3}$$

where b and m are nonnegative constants (parameters). We also use the weight $w = 1$ (we show later this contributes to an important property of the function).

When $\|P(v) - p\| = 0$, influence $I(v,p)$ is defined as $\lim_{x \to 0} \frac{1}{x} = +\infty$.

There can also be a situation when two or more vertices have equal positions, where by (3) it is not clear what value should F_i take. For example, there can be one vertex v_1 from set i and two other v_2, v_3 from set j in the same point p. For all three vertices the influence in this point is equal to infinity, so F_i formally is undefined. In this case F_i should be equal to $-\infty$, since for all other points q the value of F_i is $-I(v_3,q) = I(v_1,q) - I(v_2,q) - I(v_3,q)$. So in such situations, if the number of set i vertices in point p is less than the number of foreign set vertices in this point, then $F_i(p)$ should be equal to $-\infty$, and $+\infty$ or 0 if it is greater or equal respectively. This may not be very important as it affects only individual points but should be noted during implementation.

Parameter b regulates the slope of the function. When b grows larger, the function decreases much faster when distance is greater than 1. Visually this adjusts how strongly the blobs of the vertices interact with each other. Fig. 2 shows the graphs of the function with different b values ($m = 0$). The threshold b was chosen so that for each function the radius of a single blob would be constant. When m is small, the blobs of nearby vertices meld together. When b is large, they appear as overlapping circles.

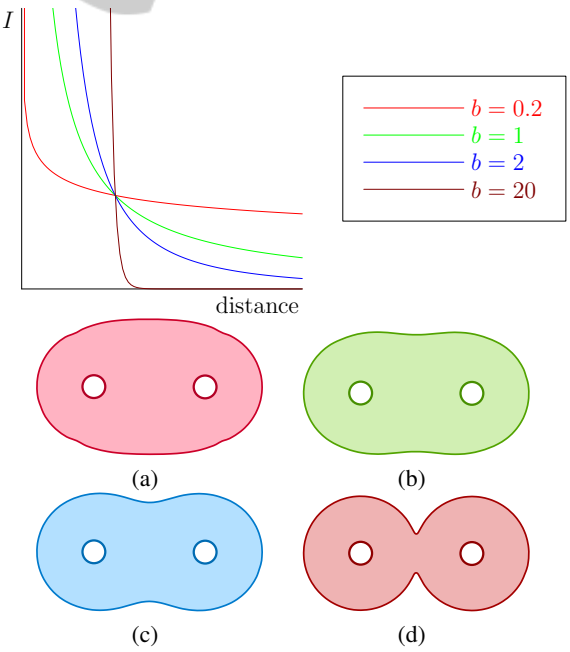


Figure 2: Graphs of the influence function with different b values. This example shows how b affects blob interaction: **(a)** $b = 0.2$; **(b)** $b = 1$; **(c)** $b = 2$; **(d)** $b = 20$. The radius of a single blob R_t is the same in all examples.

The value m is subtracted to set function values

to 0 starting from a certain distance. The purpose of this is to limit the maximum effect radius of a single vertex, preventing large concentrations of vertices from accumulating a large radius of the corresponding region with respect to other regions that contain less vertices. This is shown in Fig. 3: m normalizes the radiuses of the small and large set regions. Again, as with threshold, we regulate m with the use of the value R_m, which is defined as the distance at which the value of influence of a single vertex is equal to 0. It is clear that R_m should be greater than R_t; we mostly set the value of R_m to be a multiple of R_t.

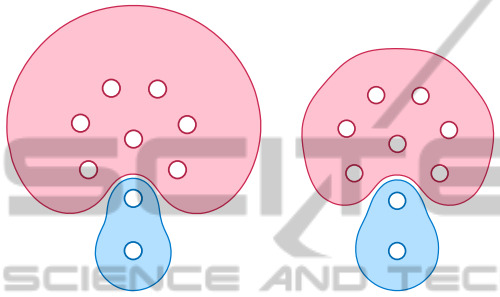


Figure 3: The effect of parameter m on the consistency of region radiuses: **(a)** R_m is unbounded ($m = 0$), the larger set region has a much larger radius then the smaller set region; **(b)** $R_m = 2.5R_t$, the radiuses of the regions are almost the same.

3.2 Properties and Comparison

In this section we discuss how the presented function deals with properties P1-P3 and how it compares to already known functions. As influence function $I(v,p)$ we also consider the Gaussian distribution function $e^{-\frac{x^2}{2\sigma^2}}$ and quadratic polynomial function $(\max(r-x,0))^2$, where $x = \|P(v)-p\|$. For Gaussian distribution function, standard deviation parameter σ also adjusts blobbiness. Overall the proposed function keeps the desired properties of these functions, and in addition guarantees correct point region membership because the potential function diverges to plus infinity at $x = 0$.

A simple example in Fig. 4 shows that the proposed function maintains the visual appearance of Gaussian distribution and quadratic polynomial potential functions. Of course, each function has its parameters that adjust the blobbiness and size of the regions, so each can be used to produce different visualizations with the same example case. However, the figure shows that all three functions can be adjusted to produce similar-looking results.

Further we examine how these functions satisfy

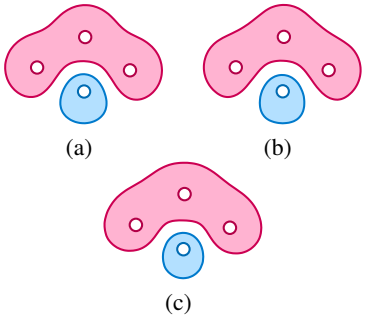


Figure 4: A simple example of how the proposed function maintains the appearance of common potential functions: **(a)** Gaussian distribution; **(b)** quadratic polynomial; **(c)** proposed function.

the membership properties P1-P3. In contrast to other functions, the proposed function guarantees properties P1, P2. It also ensures property P3 for sets with no common points.

The property P1 is easy to establish for any potential function by taking $w = 0$ in (2), i.e. ignoring foreign vertex influence for the current set. This introduces many unwanted region intersections, thus properties P2 and P3 are not maintained. This is shown in Fig. 5 (a) with Gaussian distribution (of course it happens with any potential function): the blue and red regions almost completely overlap (they should not since the two sets do not have any vertex in common), besides the blue region swallows all vertices of the red region.

This means that if we want to remove unnecessary region overlaps or ensure correct vertex membership, we have to adjust the value of w. Fig. 5 (b) shows the same example with Gaussian distribution and $w = 0.5$. The vertex membership is correct, but the regions still overlap. Fig. 5 (c) uses Gaussian with $w = 1$, i.e. the negative potential has the same weight as the positive. In this case the two regions do not overlap, but the red region swallows the single blue vertex.

That is, in fact, an important property when $w = 1$, which holds for any potential function. If two sets have no vertices in common, then their regions do not overlap (this is actually a special case of the P3 property). The proof is trivial: suppose the total positive influences of the two sets at some spatial point are I_1 and I_2. Only one of the values $I_1 - I_2$ and $I_2 - I_1$ can be positive, thus only one can be greater than threshold t. Other set vertices do not change this property, since for each spatial point the influence from them is subtracted from both of the given set total influences.

This property is very important if we need to visualize sets with no intersections: then no two regions will have any overlaps. Therefore we would want to keep it while also ensuring correct vertex member-

ship. We examine this property for a small example with a fixed R_t.

Using a Gaussian function with a fixed parameter σ, as it is shown in Fig. 5 (a)-(c), all of the mentioned properties at once are not achievable. In the given example, we can change σ (from 2.5 to 3.5), and obtain a correct visualization, see Fig. 5 (d); however, this is an unstable improvement and slightly adjusted vertex positions again produce an incorrect visualization even with the new σ, see Fig. 5 (e). In addition, σ also regulates blobbiness and it is not desirable to adjust it in order to maintain correctness.

With the proposed function all of the above is guaranteed in the visualization, see Fig. 5 (f). Firstly, we keep weight $w = 1$, so non-intersecting sets will have no overlaps between their regions. Secondly, the potential function is based on inverse distance and reaches positive infinity at distance 0, regardless of the function parameters. Thus properties P1 and P2 are ensured: each vertex will always be inside the regions of the sets it belongs to and will not be inside the regions of the sets it doesn't belong to. This allows using the parameters b and m to adjust the visual quality of the result, while the membership of the vertices will always be correct.

The only case when P1 and P2 do not hold is in the same case if several different set vertices are in the same position p. However, it is obviously impossible to satisfy these properties with any visualization in this case.

Using a quadratic polynomial function produces results similar to those of using a Gaussian function, see Fig. 6. Parameter r can be adjusted to regulate the blobbiness. The red region swallows the blue vertex in Fig. 6 (a), when $r = 1.4R_t$. Again, it can be solved by adjusting r to $1.02R_t$ (see Fig. 6 (b)), and again this solution is unstable and doesn't work when the vertex positions are changed slightly (see Fig. 6 (c)).

The example uses the same R_t in all cases, but a vertex positioning producing similar results can be obtained for any R_t. In practice, changing the radius of the blobs is also unwanted if we need to obtain shapes of particular size. With the proposed function there is no need for fine-tuning to obtain a correct vertex membership visualization.

It should be noted that the situation with P3 is different if two sets do have some vertices in common. As it was shown that any vertex belonging to a set will reside inside the region of this set, a vertex belonging to two sets will also always reside in some intersection of their regions. However, among all overlaps of the regions, there can be some that contain no vertices from any of the given sets. These overlaps may be unwanted since they do not hold any semantic meaning

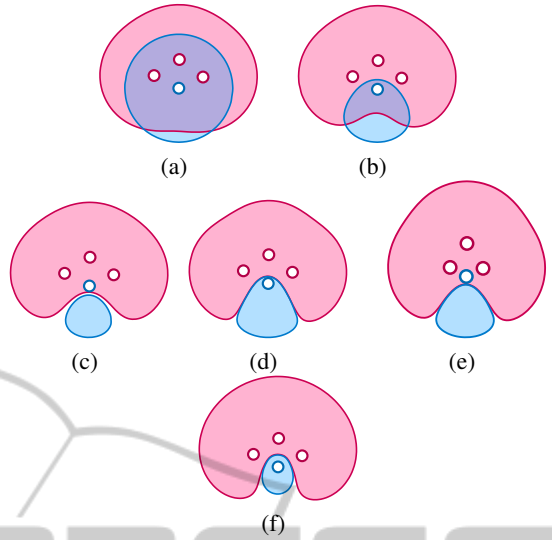


Figure 5: Comparison with the Gaussian distribution function. The example shows two non-intersecting sets: 1 vertex (blue); 3 vertices (red). In this example the threshold radius R_t is the same for all six cases. **(a)** Gaussian, $\sigma = R_t/2.5$, $w = 0$. **(b)** Gaussian, $\sigma = R_t/2.5$, $w = 0.5$. **(c)** Gaussian, $\sigma = R_t/2.5$, $w = 1$. **(d)** Gaussian, $\sigma = R_t/3.5$, $w = 1$. **(e)** Different vertex positioning, Gaussian, $\sigma = R_t/3.5$, $w = 1$. **(f)** The proposed function, $b = 2$, $R_m = 2R_t$.

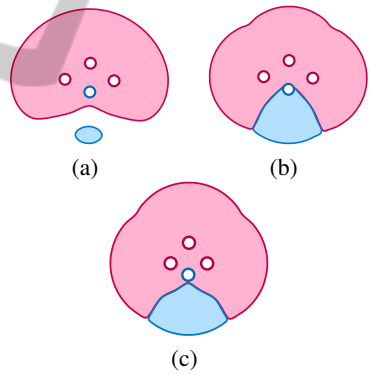
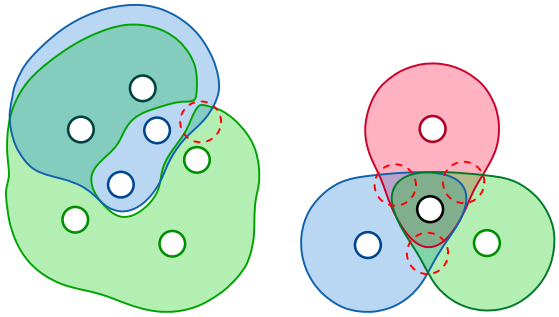


Figure 6: Quadratic polynomial function $(\max(r-x,0))^2$ results with the same example and the same R_t: **(a)** $r = 1.4R_t$; **(b)** $r = 1.02R_t$; **(c)** Different vertex positioning, $r = 1.02R_t$.

of set relationship, see Fig. 7 (a). Still, such overlaps can also contribute to the smooth region borders, see Fig. 7 (b); without them, the image would not look natural. Generally, this problem has a topological nature and most probably cannot be solved solely through potential function design.

3.3 Implementation

In this section we discuss the implementation specifics of the proposed potential function. We use a radial sweep algorithm to find the region boundaries



(a) (b)

Figure 7: Unnecessary region overlaps (with the proposed potential function): **(a)** the highlighted overlap is unwanted; **(b)** the highlighted overlaps are not necessary, but mandatory for smooth shape borders. Vertices that have black borders in each example belong to all sets.

for each set (marching squares could be used as well), which are simplified using the Douglas-Peucker algorithm (Douglas and Peucker, 1973). For high-quality images (as in this paper) we also interpolate these polygons with splines (Hobby, 1986).

One aspect of the implementation of our potential function is the possibility of infinity values. In fact, due to datatype restrictions (for example, floating point) such values could be achievable not only exactly at a vertex, but also within some ε-distance from the vertex. This problem can be solved by adjusting the used datatype for our function specifics:

- if the operation would cause the value of a variable to overflow, it should take the special ∞-value with an appropriate sign (the floating point standard guarantees this);
- if an ∞-value is added a value which is not an ∞-value with the opposite sign, the result should remain the same ∞-value (the floating point standard guarantees this);
- if an ∞-value is added to an ∞-value with the opposite sign, the result should be 0 (this is different from the floating point standard).

Still, in our implementation we used the standard floating point datatype, as it is fully sufficient in practice and produces no visible errors, because such ε-regions are negligible in size.

Another aspect of the implementation is choosing appropriate values for b, m and t. Recall the following auxiliary parameters we used in the previous section: R_t — the desired radius of the shape of a single vertex; R_m — the distance where the influence of a single vertex reaches 0. These parameters can be used in the implementation to adjust the look of the visualization. We advise to set R_m to be some multiple of R_t, for example, $R_m = 2R_t$. We use the following rules to compute the actual parameters for the function:

- $b = 2$, but adjust in real-time if needed.
- $m = R_m^{-b}$.
- $t = R_t^{-b} - m$, provided $R_m > R_t$.

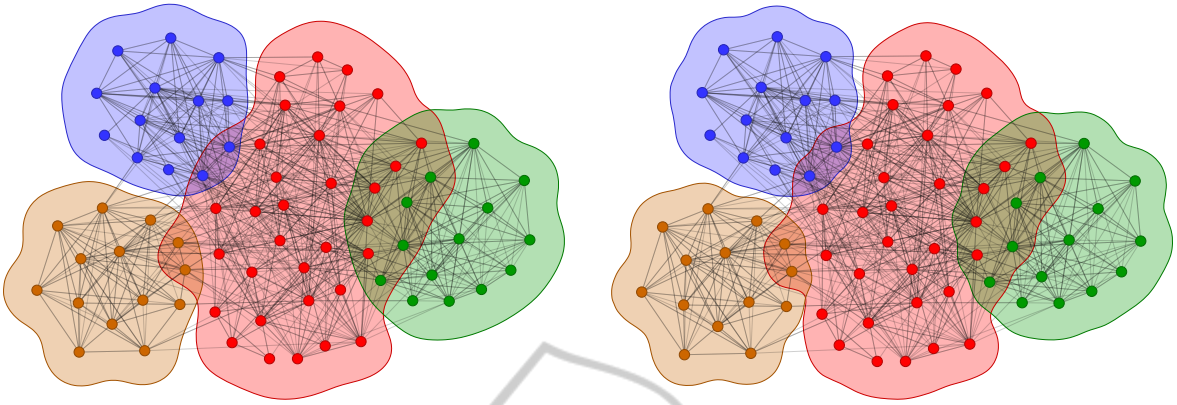
The running time of the implementation is mainly dependent on the particulars of the radial sweep or marching squares algorithm used. The only time-consuming part connected with the function is the calculation of the influence on the given scalar field, which is essentially $O((\text{set count}) \cdot (\text{vertex count}) \cdot (\text{field size}))$. There are a few worthy optimizations to this:

- since $F_i(p) = I(S_i, p) - I(V \setminus S_i, p)$, first calculate only $I(S_i, p)$: if it is less than the used threshold t, there is no need to calculate $I(V \setminus S_i, p)$;
- if the distance $\|P(v) - p\|$ is greater than R_m, then the influence is 0 and there is no need to take v into account for point p;
- to avoid expensive exponentiation in $\|P(v) - p\|^{-b}$, the value of $b = 2$ can be used, leaving only the inverse of the easily computable squared distance between v and p.

4 CASE STUDIES

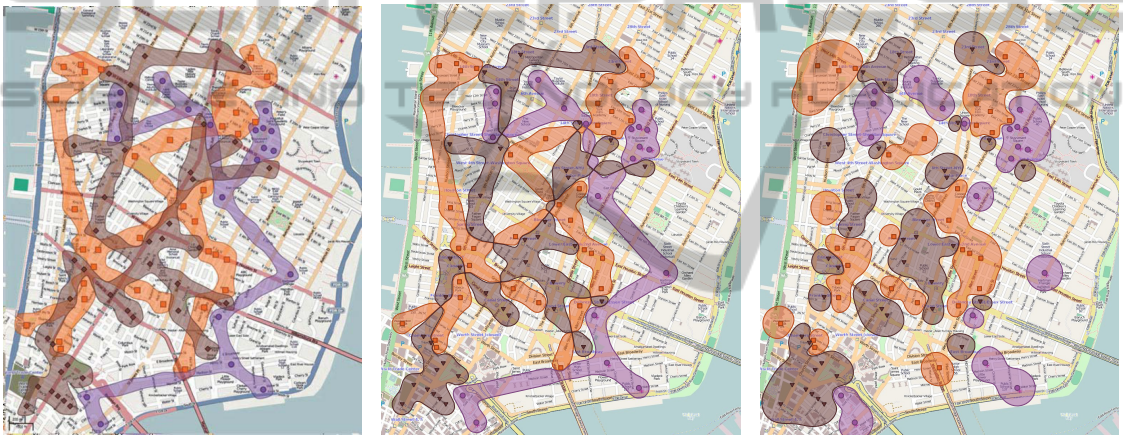
In this section we demonstrate our visualization on several real-world examples and compare it to the Gaussian distribution-based visualization. We show that in cases where the Gaussian distribution works well, so does the proposed function. In addition, we show that our function can be used to achieve a good result where that is not possible using the Gaussian distribution. We focus on the Gaussian distribution function, as quadratic polynomial potential functions produce essentially the same results.

The first example shows that the proposed function can be used to obtain similar results to those of the Gaussian distribution function, see Fig. 8. The latter works well in this example, and we have produced essentially the same result using our function. The example itself demonstrates a small company with four work locations. The central red cluster corresponds to the company headquarters. Each vertex in the graph represents an employee, colored according to the location they work at. Graph edges denote frequent, work-related communications between employees. Cluster overlaps reveal which employees frequently interact with other locations. Besides the comparison of the functions, this example shows how this visualization can be used to depict graph overlapping clustering (Krebs, 2007).



(a) (b)

Figure 8: This figure illustrates a visualization of a simple real-world overlapping set example: **(a)** Gaussian distribution; **(b)** the proposed function. There are no difficulties in ensuring correct point membership for both functions in this case. The example shows that our function is able to produce results that are very similar to the visualization with Gaussian distribution function. Both cases illustrate that this visualization method works well with graph clustering.



(a) (b) (c)

Figure 9: Hotels (orange), subway stations (brown), and medical clinics (purple) in Manhattan: **(a)** Bubble Sets (Collins et al., 2009); **(b)** our function with spanning tree edges added to the visualization; **(c)** our function without any modifications. This example shows how region connectivity can be ensured while using our function.

In the second example we compare our visualization with Bubble Sets (Collins et al., 2009), see Fig. 9 and illustrate how their method of ensuring region connectivity works with the proposed method. The example depicts the locations of hotels, subway stations and clinics in Manhattan. Case (a) shows the Bubble Sets result. It uses a quadratic polynomial function as the potential field function, and ensures region connectivity by assigning potential to the edges of a (not necessary the minimum) spanning tree of the set vertices in addition to the vertices themselves. When computing the influence of an edge on a point the standard point-segment distance is used. As a result the regions have unwanted overlaps, many of which, however, contribute to region connectivity. In case (b) we have applied the same spanning tree method with our function. In addition to maintaining connectivity, in our visualization there are also no region overlaps. The function properties lead to interesting behavior at edge intersection points, where each of the regions is connected by a single point. We leave it to the reader to decide whether such a result is aesthetically pleasing. In case (c) we apply our visualization without any modifications.

In the last example we visualize a part of the human disease network described in (Goh et al., 2007), see Fig. 10. In this graph, diseases are linked by common genetic associations, with the sets denoting various types of disease. This graph contains no set overlaps, so using equal weights for positive and negative influence ensures that there are no region overlaps. We show that our visualization works better than

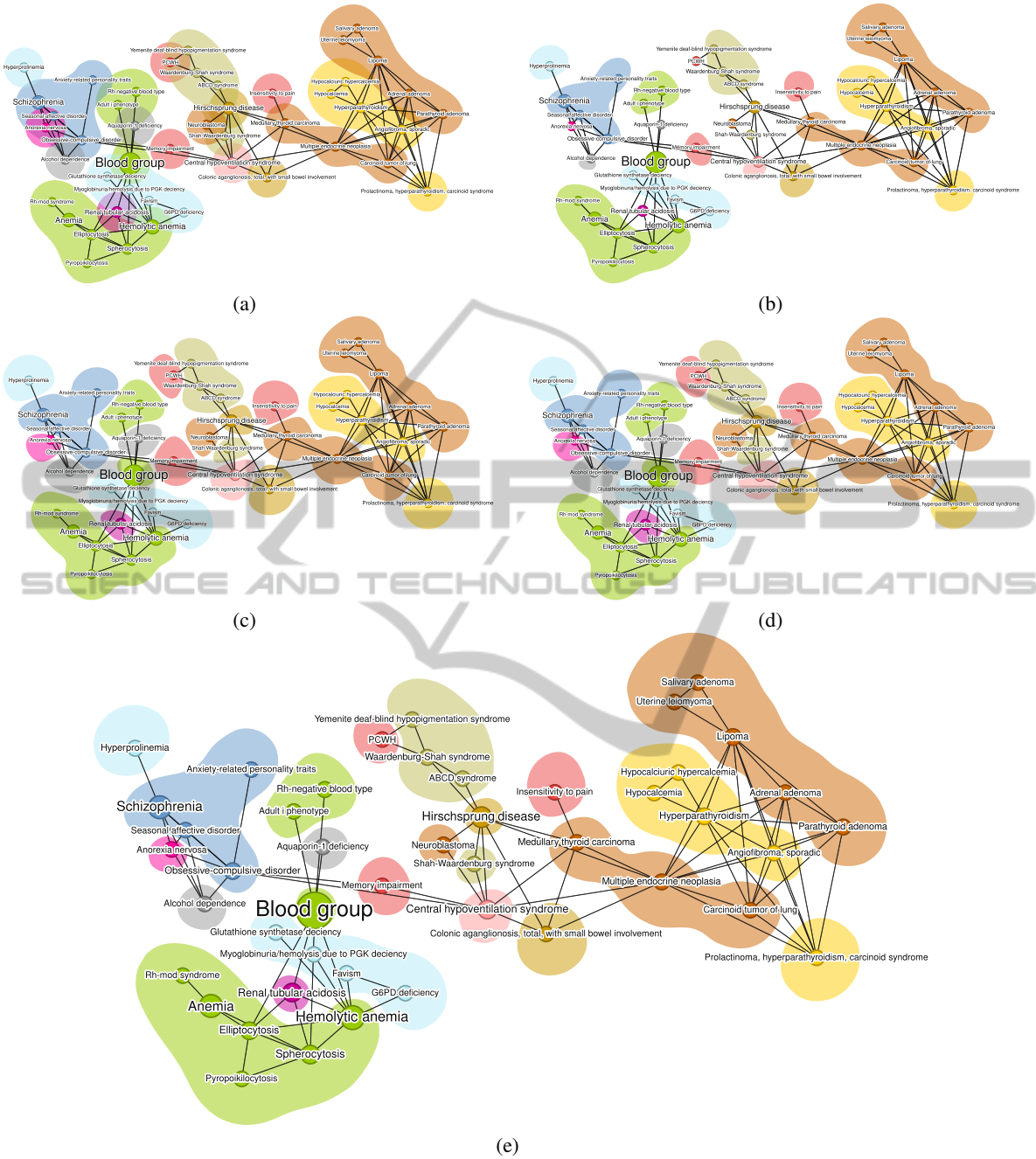


Figure 10: A part of the human disease network (Goh et al., 2007). Case **(a)** is recreated from the diseasome poster available at http://diseasome.eu/poster.html and uses a Gaussian distribution function with no negative weights. In this case there are many unnecessary region overlaps. Cases **(b)** and **(c)** show the Gaussian distribution function with negative weights. In **(b)**, we tried to preserve the blobbiness of the shapes in (a), however this results in some regions disappearing completely. In **(c)** this is remedied at the cost of the blobbiness of the regions. Cases **(d)** and **(e)** show the results of the proposed method. **(d)** closely resembles the results of the Gaussian function in (c). However, **(e)** combines the smooth regions of (a) with correct overlaps, a result not achievable using a Gaussian function.

the Gaussian in this example. We were also interested in improving the visualization performed by Mathieu Bastian and Sébastien Heymann from Gephi (available online at http://diseasome.eu/poster.html).

5 CONCLUSIONS

We presented a new potential field function for overlapping point set visualization in the form of Euler diagrams. In contrast to the most widely used potential functions, the proposed function ensures correct point membership in the set regions. Moreover, it retains all desired Gaussian-based potential field function properties, e.g., the set region shapes are smooth and visually pleasing. Set regions are easily identifiable and closely match the layout of the points. The smoothness and size of the regions can be also adjusted using the parameters of our function.

We have applied our function on different real-world examples and compared the result to the earlier methods. The proposed function is very effective in cases with no intersecting sets, since then the regions are guaranteed not to overlap. It also works well with overlapping sets, with regions creating an easily comprehensible Euler diagram, retaining correct point membership. We have demonstrated that our function works well in cases where it is not possible to obtain a good result using the Gaussian potential function. We have also illustrated how the overall approach can be successfully used to visualize overlapping graph clustering.

REFERENCES

Balzer, M. and Deussen, O. (2007). Level-of-detail visualization of clustered graph layouts. *2007 6th International AsiaPacific Symposium on Visualization*, pages 133–140.

Blinn, J. F. (1982). A generalization of algebraic surface drawing. *ACM Trans. Graph.*, 1(3):235–256.

Byelas, H. and Telea, A. (2009). Towards realism in drawing areas of interest on architecture diagrams. *Journal of Visual Languages Computing*, 20(2):110–128.

Collins, C., Penn, G., and Carpendale, S. (2009). Bubble sets: Revealing set relations with isocontours over existing visualizations. *IEEE Transactions on Visualization and Computer Graphics*, 15(6):1009–1016.

Di Battista, G., Eades, P., Tamassia, R., and Tollis, I. G. (1998). *Graph Drawing: Algorithms for the Visualization of Graphs*. Prentice Hall PTR.

Dinkla, K., van Kreveld, M. J., Speckmann, B., and Westenberg, M. A. (2012). Kelp diagrams: Point set membership visualization. *Computer Graphics Forum*, 31(3pt1):875–884.

Douglas, D. H. and Peucker, T. K. (1973). Algorithms for the reduction of the number of points required to represent a digitized line or its caricature. *Cartographica The International Journal for Geographic Information and Geovisualization*, 10(2):112–122.

Gansner, E. R., Hu, Y., and Kobourov, S. (2010). GMap: Visualizing graphs and clusters as maps. *2010 IEEE Pacific Visualization Symposium PacificVis*, pages 201–208.

Goh, K.-I., Cusick, M. E., Valle, D., Childs, B., Vidal, M., and Barabsi, A.-L. (2007). The human disease network. *Proceedings of the National Academy of Sciences of the United States of America*, 104(21):8685–8690.

Gross, M. H., Sprenger, T. C., and Finger, J. (1997). Visualizing information on a sphere. In *In Proceedings of the 1997 Conference on Information Visualization*, pages 11–16.

Heer, J. and Boyd, D. (2003). Vizster: visualizing online social networks. *IEEE Symposium on Information Visualization 2005 INFOVIS 2005*, 5(page5):32–39.

Hobby, J. D. (1986). Smooth, easy to compute interpolating splines. *Discrete and Computational Geometry*, 1(2):123–140.

Krebs, V. (2007). Managing the 21st century organization. *IHRIM Journal*, XI(4):2–8.

Matsumoto, Y., Umano, M., and Inuiguchi, M. (2008). Visualization with Voronoi tessellation and moving output units in self-organizing map of the real-number system. *Neural Networks*, 1:3428–3434.

Riche, N. H. and Dwyer, T. (2010). Untangling Euler diagrams. *IEEE Transactions on Visualization and Computer Graphics*, 16(6):1090–1099.

Rosenthal, P. and Linsen, L. (2009). Enclosing surfaces for point clusters using 3D discrete Voronoi diagrams. *Computer Graphics Forum*, 28(3):999–1006.

Santamara, R. and Thern, R. (2008). Overlapping clustered graphs: Co-authorship networks visualization. *Lecture Notes in Computer Science*, 5166:190—199.

Simonetto, P., Auber, D., and Archambault, D. (2009). Fully automatic visualisation of overlapping sets. *Computer Graphics Forum*, 28(3):967–974.

Sprenger, T. C., Brunella, R., and Gross, M. H. (2000). H-BLOB: a hierarchical visual clustering method using implicit surfaces. In *Visualization 2000. Proceedings*, pages 61 –68.

Van Ham, F. and Van Wijk, J. J. (2004). Interactive visualization of small world graphs. *IEEE Symposium on Information Visualization*, pages 199–206.

Watanabe, N., Washida, M., and Igarashi, T. (2007). Bubble clusters: an interface for manipulating spatial aggregation of graphical objects. In *Proceedings of the 20th annual ACM symposium on User interface software and technology*, UIST '07, pages 173–182, New York, NY, USA. ACM.