Eye-tracking Investigation During Visual Analysis of Projected Multidimensional Data with 2D Scatterplots

Ronak Etemadpour, Bettina Olk and Lars Linsen
Jacobs University Bremen, Bremen, Germany

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Abstract: A common strategy for visual encoding of multidimensional data for visual analyses is to use dimensionality reduction. Each multidimensional data point is projected to a 2D point using a certain strategy for the 2D layout. Many layout strategies have been proposed addressing different objectives and targeted at distinct domains and applications. The resulting projected information is typically displayed in form of 2D scatterplots. The user's perspective such as the role of visual attention and guidance of attention for a respective layout and task has not been addressed much. It is the goal of this work to investigate, how characteristics in the layout affect the cognitive process during task completion. Eye trackers are an effective means to capture visual attention over time. We use eye tracking in a user study, where we ask users to perform typical analysis tasks for projected multidimensional data such as relation seeking, behavior comparison, and pattern identification. Those tasks often involve detecting and correlating clusters. To understand the role of point density within clusters, cluster sizes, and cluster shapes, we first conducted a study with synthetic 2D scatterplots, where we can set the respective properties manually. We evaluate how changing various parameters affect the visual attention pattern and correlate it to the correctness of the answer. In a second step, we conducted a study where the users were asked to complete tasks on real-world data with different characteristics (image collection and document collection) that are visualized using a selection of different dimensionality reduction algorithms. We transfer the insight obtained from synthetic data to investigate the decision making with real-world data. Gestalt laws can be applied to the layout structure. We examine how certain layout techniques produce certain characteristics that change the visual attention pattern. We draw some conclusions on how different projection methods support or hinder decision making leading to respective guidelines.

1 INTRODUCTION

The goal when analyzing multidimensional data is to identify structures in the data distribution. By multidimensional data we refer to sets of points in a multidimensional space. Typical analysis tasks for gaining insight into the properties of data distribution include pattern identification such as detecting clusters, behavior comparison such as comparing characteristics of subsets, and relation seeking such as correlating subsets to each other, see Section 3. For a visual analysis of multidimensional data, it is common to use dimensionality reduction techniques that project the multidimensional points to points in a lower-dimensional visual space and typically, the projected points are displayed in form of 2D scatterplots. Since one is interested into gaining insight in data distributions in the multidimensional space, the projection method shall preserve those distributions as much as possible. In general, it cannot be avoided that some information is lost when reducing dimensionality. Therefore, different projection techniques have been established that focus on preserving certain properties of the data distribution. Consequently, they aim at supporting certain analysis tasks where those properties are crucial. To evaluate the effectiveness of preserving certain properties, various numerical and visual methods have been introduced to quantify the quality of projections with respect to preserving certain properties, thus, guiding a user to select the most appropriate projection method for their task (Sips et al., 2009). Commonly, one considers cluster preservation or separation, neighborhood preservation, or distance preservation properties. Perceptional aspects have not been in the focus of attention very much. Although projection techniques are commonly embedded into user-centric systems for interactive visual analysis, little is known about how users perceive the layouts they produce. Typical perceptional questions would be how point density within a cluster, cluster size, and cluster shapes affect the visual attention of an observer during the

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analysis process and how this guidance of attention relates to the provided answer. Automatic clustering algorithms that compute partitions of points into subsets or classes in order to maximize both the similarity among members of the same subset, and dissimilarity across classes are commonly embedded in a visual analytics setting. On the other hand, Gestalt psychologists have studied grouping as part of the general process of perception (KotBca, 1935) and formulated a set of laws to explain the perceptual processes of grouping done by humans. Our goal is to investigate how perceptual aspects influence visual analyses.

In this paper, we present a study that investigates visual attention during the analysis of multidimensional data when projected to a two-dimensional visual space and visually encoded as 2D scatterplots. Four types of seeing with different levels of attention have been introduced by Wolfe (Wolfe, 2000). Previous research has demonstrated a strong link between attention and eye movements based on the "eye-mind hypothesis" (Rayner, 1998). Thereby, to assess the allocation of visual attention, eye movement patterns were recorded. Visual attention can be influenced by many factors. Creating multidimensional data projection outputs where exactly one of these factors varies is impossible. Thus, in a first step we analyze eye movements when given typical analysis tasks for 2D scatterplots that have been generated manually. Eye movement patterns are analyzed in order to infer where visual attention was allocated. For such synthetic examples, we can tune respective parameters such as point density, cluster size, or cluster shapes. We analyze the impact of such parameters on relation-seeking tasks, see Section 4.

In a second step, we use actual multidimensional data sets from two different applications (image collections and documents collections) and project the data to two-dimensional visual spaces using different point placement techniques. Users are asked to perform typical analysis tasks on the resulting 2D scatterplots including the relation-finding tasks used for synthetic data. We investigate the users' eye movement patterns recorded with the eye-tracking system, relate those patterns to the findings with synthetic data, and correlate the patterns to the correctness of the given answers for the questions asked. We draw conclusions on how the different projection methods influence visual attention and how this supports or hinders correct task completion, see Section 5. Our two-step approach follows the reasoning of Ware (Ware, 2000) who discusses that the results of low-level processing and discovering patterns can provide design guidelines for display layouts. The results of our first step can be applied to understand how existing layouts are processed in our second step.

2 RELATED WORK

Many techniques exist to generate 2D similarity-based layouts from high-dimensional data (projections). The design goals include maintaining pairwise distances between points as implemented in multidimensional scaling (MDS) (Borg and Groenen, 2010), maintaining distances within a cluster, or maintaining distances between clusters (Tenembaum et al., 2000). Isometric feature mapping (Isomap) (Tenembaum et al., 2000) is an MDS approach that has been introduced as an alternative to classical scaling capable of handling non-linear data sets. It obtains a globally optimal solution to the distance preservation problem. Classical dimensionality reduction algorithms, such as principal component analysis (PCA) (Jolliffe, 1986), are often employed to generate similarity layouts by reducing data to lower-dimensional visual spaces. Least Square Projection (LSP) (Paulovich et al., 2008) first samples a reduced sub-set of points representative of the data distribution in the input space and projects them to the target space with a precise MDS force placement technique. It then builds a linear system from information given by the projected points and their neighborhoods. A Laplacian operator ensures that data points in a particular neighborhood remain proximate in the target space. They are based on first generating a tree that encodes similarity and then laying out the tree in a two-dimensional space. The algorithms to generate similarity layouts (Cuadros et al., 2007) are often inspired by the neighbor-joining (NJ) heuristic originally proposed to reconstruct phylogenetic trees.

Several approaches for selecting good layouts have been proposed. The approaches can be categorized into numerical approaches that compute quality measures in form of numbers or visual approaches that plot quality measures in graphical form. The silhouette coefficient (Tan et al., 2005) and the neighborhood hit (Paulovich et al., 2008) evaluate clustering capability, while the correlation coefficients (Geng et al., 2005) evaluate distances. These mathematical measures do not consider how humans perceive the layout.

User perception by conducting user studies on scatterplots is investigated in (Tatu et al., 2010). Users were asked to sort useful scatterplots among 18 instances. However, they did not look into multi-dimensional data projection as the ones mentioned above. A research by (Albuquerque et al., 2011), is attempted to find a perception-based quality mea-

sure for scatterplots. A ranking function was used to estimate the value of the projections for a specific user task in a perceptual sense, based on the data from a psychophysical study. Recently in (Sedlmair et al., 2012), the accuracy of class consistency measure (CCM) and class density measure (CDM) are discussed in scatterplots depicting multidimensional projection layouts. Their major contribution is a detailed taxonomy of factors that affect the human perception of cluster separation. In their qualitative data study, two investigators visually inspected over 800 plots to determine whether or not the measures created plausible results. In a study by (Rensink and Baldridge, 2010), the perception of correlation in scatterplots has been investigated purely from a psychological perspective developed for simple properties such as brightness. They generated a set of scatterplots with points distributed within a certain range from the diagonal and tested whether observers could discriminate pairs and concluded that perception of correlation in a scatter plot is rapid. None of these approaches used eye trackers to measure visual attention and draw conclusion on users' decisions.
Eye tracking has been a helpful tool to understand the cognitive processes of users involved in a visual analysis process. Eye tracking is used in (Burch et al., 2011), to investigate the visual behavior of participants of a user study when operating with different tree visualizations. They examined hierarchical structures represented by various tree layouts such as traditional, orthogonal, and radial node-link layouts. They examined fixation points, fixation duration, and saccades of participants' gaze trajectories. They also analyzed correctness of answers as well as completion times in addition to the eye movement data. A similar eye tracking study in (Goldberg and Helfman, 2011) is conducted to compare radial and linear graphs to support lookup tasks for one and two data dimensions. The tasks of both studies, however, are more concerned with topological distances of nodes (with respect to the tree topology) than with Euclidean distances of nodes in the layout. Hence, tasks, data, and visual encoding are different from our study.
There has also been some fundamental work on the Gestalt principles within the cognitive psychology community that relate to our work. The Gestalt principles describe psychological phenomena underlying human perception of given tasks by viewing them as organized and structured wholes. For the detection of non-spherical clusters, various researchers sought more robust ways to identify arbitrarily shaped clusters rather than the sum of their constituent parts computationally. Ahuja and Tuceryan (Ahuja and Tuceryan, 1998) studied a computational approach presented to extracting basic perceptual structures or the lowest level grouping in dot patterns with the goal to extract the perceptual segments of dots due to their relative locations. Dots were assigned perceptual roles of interior dots, border dots, curve dots, and isolated dots. Other studies investigated detection of dotted lines in a noisy background consisting of dynamic patterns of identical dots (Uttal et al., 1970). We considered in the first part of our study the work on perceptual organization.

3 MULTIDIMENSIONAL DATA ANALYSIS TASKS

We first identify multidimensional data analysis tasks for scatterplot visualizations in a projected 2D space. In a framework proposed by Andrienko and Andrienko (Andrienko and Andrienko, 2005) synoptic visual analysis tasks have been grouped into pattern identification, behavior comparison, and relation seeking. Within this framework, we identify typical analysis tasks for multidimensional data. A relation-seeking task is to investigate the similarities between subgroups (clusters or individual objects). We hence asked participants to:

Q1 Identify the closest cluster to a given object.

Q2 Identify the closest cluster to a given cluster.

In both tasks we try to determine whether the green or the blue cluster is closer to the red object(s). The colors blue and green are assigned randomly to the clusters to avoid any bias towards a specific color.

In order to assess pattern identification, participants were asked to:

Q3 Estimate the number of clusters.

Here, all points are colored in the same color (blue) as shown in the example in Figure 7.

A behavior comparison task is to compare characteristics of subsets (or clusters). In other words, we try to examine whether the objects within one cluster are more similar to each other than the objects within another cluster. Thus, we ask the subjects to compare the point densities within clusters, where density is defined as the number of points per area. The task is defined as:

Q4 Rank clusters by density.

Here, we identify three clusters in the multidimensional data set and color-code the respective projected points in the 2D scatterplot using red, green, and blue color, correspondingly. Again, the colors are assigned randomly.

4 SYNTHETIC DATA STUDY

In the first part of our study, we want to investigate the role of certain cluster properties on visual attention and task completion success. One of the modern psychological rules that was applied to visual and pattern perception is called Gestalt approaches (Wertheimer, 1923). Our goal is to examine whether it is just (Euclidean) distances that matter when visually analyzing the scatterplots or whether there are other characteristics of the clusters that influence the visual analysis from a perceptional view. The characteristics we investigated were cluster density (i.e., point density within a cluster as defined above), cluster size (i.e., the number of objects or points that belong to a cluster), and cluster shape (e.g., whether a cluster appears to be round or elongated). The synthetic scatterplots have been inspired by observations from real scatterplots.

4.1 Gestalt Laws

We formulate hypotheses based on Gestalt laws (Wertheimer, 1923) and test the hypotheses within a user study. We want to provide a short description of the four Gestalt laws that we used. Gestalt theory is based on the concept that the whole is greater than the sum of its parts. Broad observation initially identified about human perception led to a number of laws about how humans perceive groups of related information visually: (1) Law of Similarity: Objects that have similar appearance are perceived as a group. (2) Law of Proximity: Objects that share spatial proximity are perceived as a group. (3) Law of Continuity: Objects that are aligned are perceived as a group. (4) Law of Closure: Objects that are perceived to form a closed contour are treated as a group.

4.2 Hypotheses

Considering Task Q1 that compares distances between a point and a cluster (set of points), the Law of Proximity would postulate that the point is perceptually grouped with the closer cluster. Here, the clusters have equal size to the point and we consider the case that one of the cluster is denser than the other. The Law of Similarity leads to the assumption that the point would be grouped to the less dense (or sparser) cluster, as the distance of the point to the clusters is closer to the distances within the sparser cluster than those within the denser cluster. We expect that visual attention is drawn towards the sparse cluster. Hence, we formulate the hypothesis:

H1. Sparser clusters are looked at for a longer overall time and are considered closer to the given reference object.

Similar results we expect for Task Q2. However, here, the single reference object is replaced by a third reference cluster, which itself has a certain point density. Thus, we may assume from the Law of Similarity that the cluster whose density is more similar to the density of the reference cluster is more likely to be chosen as being closer. We phrase the hypothesis:

H2. Visual attention and decision is affected by the density of the reference cluster.

The influence of cluster shape on the given tasks can be described with respect to the Law of Continuity. It can be assumed that a reference point (or cluster) appears closer to a cluster, if it is located in the continuation of that cluster's principal direction, i.e., being aligned with it. On the other hand, if the reference point (or cluster) is located in a direction orthogonal to the principal direction, it is expected to appear farther. Consequently, we formulate the hypothesis:

H3. Reference points (or clusters) appear closer to clusters they are aligned with.

To test for the influence on cluster size, we use two clusters of same shape and density and varied size (i.e., the number of points). Based on the Law of Continuity, we assume that the reference point (in Task Q1) or cluster (in Task Q2) is more likely to be perceptually merged with the larger cluster. Hence, we formulate the hypothesis:

H4. Reference points (or clusters) appear closer to larger clusters.

4.3 Design of User Study

In a study on perception of random dot interference patterns (Glass et al., 1973) is shown that varying both the local and global parameters describing the interference patterns, the functional organisation of the visual system can be probed and new perceptual affects discovered. In a study by (Healey et al., 1996), is stated that if a visualization tool was being used to display multiple independent data values, interference among features should ideally be eliminated. If a visualization tool was being used to investigate a specific relationship, like finding similarity here, the "strongest" feature should be used to encode that relationship. Secondary features used to encode additional data values must not interfere with the task-relevant feature. Thus, we needed examples where only one of the parameters varies while the others remain constant. As it is impossible to obtain projections of multidimensional data into a 2D visual space, where exactly one of the parameters cluster density, cluster size, and cluster shape varies, we manually generated 2D scatterplots.

We created 20 synthetic images that show scatterplots with manually defined properties. The images are targeted at the evaluation with respect to Tasks Q1 and Q2. We define two clusters that are equally far (with respect to the 2D Euclidean distance) from a reference point (Q1) or a third reference cluster (Q2). Hence, if only distances matter, we expect that subjects in about 50% of the cases choose the first cluster and in about 50% of the cases choose the second cluster as being closer to the reference point (or cluster). However, we modify the characteristics of the two clusters, i.e., they differ in density, in size, or in shape. We also added a control scatterplot image, where all parameters (density, size, and shape) are identical for both clusters. The scatterplots are generated by defining shape and number of points per clusters and, then, randomly placing the points inside the shape.

We conducted a controlled user study involving 20 subjects (12 female and 8 male) with different educational background and normal vision. We did not provide any statistical analysis across the gender. The subjects were not familiar with visual multidimensional data analysis, but received a short introduction. Based on their assigned study ID, each subject was presented ten (Task Q1) or twelve (Task Q2) images with 2D scatterplots. Each task was presented in written form on a slide and subjects had the chance to ask in case of any necessary clarification. The experimenter was present and manually recorded the answers, which were given verbally by the subjects. There was no time limit to fulfill the tasks.

During the experiment, a Tobii T60 eye tracking system was used to record eye movements and sequences of gaze fixations of the subjects on the visuals. The system consists of a 17-inch computer monitor with a video camera built in which tracks the user's eye movements at 60 Hz. It did not constrain users' motion allowing subjects to move freely and naturally while they looked at the screen and answered questions. Each subject received a brief description of the eye-tracking system. The data recording session began with an eye-tracking calibration, which consisted of the user looking at the screen and following a moving dot with their eyes. Questions and scatterplot images were embedded in a slide show on the Tobii system monitor.

4.4 Analysis Methods

To test for statistical significance of deviations from a theoretically expected distribution of observations into two categories, two-tailed binomial tests have been used. ANOVA test was used for computing statistical significance when comparing more than two groups.

To analyze the visual attention patterns we used the EyeC software system (Ristovski et al., 2013). It computes heat maps from fixation durations, which maps to each pixel a color ranging from blue (no fixation) to red (highest fixation duration), see Figure 1 (mid). Moreover, the user can select AOIs and retrieve statistical information about them, see Figure 1 (right). It shows the difference between the accumulated fixation times for the selected areas of interest (AOIs) and selected participants. The AOI labels in the heat maps are inserted manually for the images shown. The shapes are shown by their contours. Finally, one can also analyze visual attention sequence encoded over defined AOIs (not shown in the figure).

4.5 Results

Influence of Cluster Density. In our experiments for *Task Q1*, we created six different scatterplots. Two images show scatterplots where both clusters had the same roundish shape and the same size, while the density varied, see Figure 1 (left). Since density is related to size, we also looked into varying size in addition to density and created two further images where the denser cluster had more points and another two images where the denser cluster had less points. To analyze the fixation patterns, we defined five AOIs: The area around the reference object (AOI 1), the space between the reference object and the sparser cluster (AOI 2), the sparser cluster (AOI 3), the denser cluster (AOI 4), and the space between the reference object and the denser cluster (AOI 5), cf. Figure 1 (mid). We selected those five AOIs, as we assumed that the cognitive process includes examining the clusters to find the point closest to the reference point and examining the respective distances to the reference point.

The findings showed that in all six scatterplots the sparse cluster (AOI 3) were *significantly* looked at more than all the other AOIs, cf. Figure 1 (right). In accordance with this finding, in 75.83% the sparser cluster has been reported as the closer one by the 20 subjects, which is *significantly* more than the to be expected 50%. Hence, Hypothesis H1 is confirmed. At this point, we want to mention that for the control scatter plot, where both clusters have the same density, size, and shape, there was no significant difference in the visual attention and the answers from the expected 50%.

We can also conclude that density seems to be perceptually more relevant than cluster size, as changing the cluster size did not change the fact that the sparser cluster got more attention. Another observation is that the single point (AOI 1) did not need much attention, where the fixation durations sometimes were negligible. Also, AOI 5 was looked at more than AOI 2, which means that the space between the dense cluster and the single object was more recognizable. The sequence analysis shows that eyes frequently moved from AOI 3 to AOI 5, before proceeding to AOI 4.

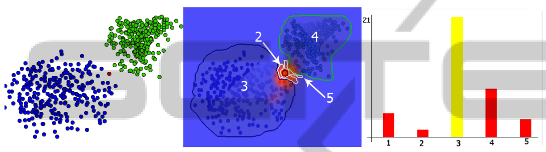


Figure 1: Task Q1: Finding closest cluster to reference point for synthetic data with varying cluster density.

The findings for *Task Q2* revealed that in five out of the six scatterplots subjects still looked at the sparser cluster most, cf. Figure 2(a). Also, in 86.4% of all cases subjects started their analysis by looking at the sparser cluster, which is statistically significant. In some cases the reference cluster was actually looked at most. A general observation was that the reference cluster got substantially more attention when moved to the center between the two other clusters. Despite these findings, the answers that the sparser cluster is closer dropped to 56,6%, which is actually not statistically significant anymore. While for the example shown in Figures 2(a) the sparser cluster was generally reported as closer, all subjects reported the denser cluster to be closer in the example shown in Figure 2(b). When looking at the visual attention, the sparser cluster has still the highest mean fixation duration in example (b). However, as opposed to example (a), in example (b) the space between the reference cluster and the sparser cluster (AOI 6) is actually looked at more than the space between the reference cluster and the denser cluster (AOI 5). (c) AOI 1 got substantially more attention than AOI 2, i.e., the part closer to the reference cluster is investigated more. Hence, the subjects recognized AOI 6 more, which let them decide for the denser cluster to be closer. Now, when investigating the reference cluster's density in the examples, it can be seen that in example (a) it seems closer to that of the sparser cluster, while in example (b) it seems closer to the denser cluster. Investigating all six different scatterplots, Hypothesis H2 has been confirmed.

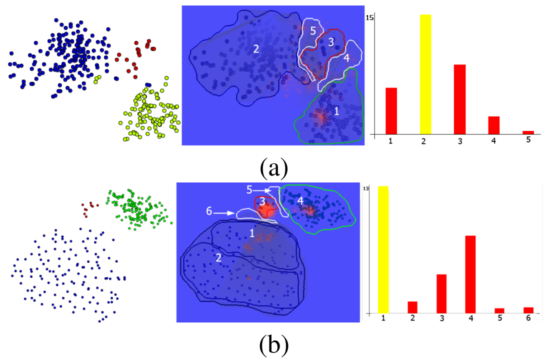


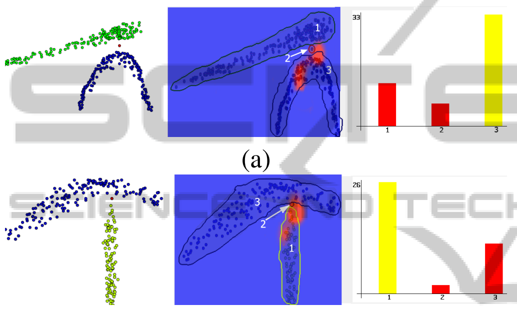
Figure 2: Task Q2: Examples of finding closest cluster to reference cluster for synthetic data with varying cluster density.

Influence of Cluster Shape. We first considered *Task Q1* and generated eight scatterplots that targeted two investigations. First, we used two clusters, where one was more roundish the other more longish, while density and size were the same. With this set-up, we placed the reference point in continuation of the more longish cluster or orthogonal to that direction. Second, we looked into two longish clusters, where one was bent and the other straight. The bending may be in the direction away or towards the reference point.

For the first case, in 80.7% of the cases the subjects chose the roundish cluster as the closer one, which is what we expected. However, 75% of the subjects looked at the longish cluster more. In the case of a roundish and a longish cluster, where the reference point is in the continuation of the longish clusters (is not shown here), in 100% of the cases the subjects chose the longish cluster as the closer one, which again is what we expected. Here, 50% of the total subjects looked at the longish cluster most (among all AOIs defined as before). One may speculate that the elongated structure requires more attention than the more compact, roundish structure to comprehend the shape. To follow up on this, we investigated two longish structures with one being bent as shown in Figure 3(a). Here, the more complicated structure, i.e., the curved one, is again the one that is looked at more. However, the straight cluster was the one that was seen closer by all subjects. This may relate to the Law of Closure, as the curved cluster seems to define a closed area, to which the reference point does not belong.

Finally, we investigate the role of orientation of the bending. We have a similar set-up as in Figure 3(a), but now the bending is towards the reference point, cf. Figure 3(b). In this case the reference point would lie inside the closure of the curved cluster. On the other hand, the reference point lies in the continuation of the straight cluster. Hence, the Laws of Continuity and Closure are competing. The findings were that the straight cluster (AOI2) has the highest fixation time, but that in 71% of all cases the subjects chose the curved cluster as the closer one.

Hence, we conclude that Hypothesis H3 was confirmed. When considering two longish clusters with one of them being curved, the Law of Closure seems to be dominant, i.e., the orientation of the bending is most relevant for the decision. The visual attention patterns do not deliver such a consistent view as for the varying densities, but it seems that more compact clusters need less attention.



(a)

(b)

Figure 3: Task Q1: Finding closest cluster to reference point for synthetic data with varying cluster shape.

For *Task Q2*, we investigated four scatterplots with the two set-ups from above. I.e., we consider one more roundish and one more longish cluster, where the reference cluster is either located in continuation of the longish cluster or in an orthogonal direction. For the case, where the reference cluster is in continuation of the longish cluster, the longish cluster was chosen as the closer one in 82.35% of all cases (which is statistically significant). Consistently, the space between reference cluster and longish cluster had less attention than the space between reference cluster and roundish cluster. For the case, where the reference cluster is in direction orthogonal to the longish cluster, the roundish cluster was chosen as the closer one in 87.5% of the cases (which is statistically significant). Consistently, the space between reference cluster and roundish cluster had less attention than the space between reference cluster and longish cluster. We can conclude that these results also approve Hypothesis H3.

Influence of Cluster Size. For *Task Q1*, in 75% of the cases the larger cluster is indeed chosen as being closer to the reference point and the respective AOI has the highest mean fixation duration (which is statistically significant). The reference cluster in some cases actually got most attention. For *Task Q2*, even in 89% of the cases the larger cluster is chosen as being closer to the reference cluster (which is statistically significant). However, we can report that the space to the smaller cluster is looked at more than the space to the larger cluster, which is consistent with our earlier findings. We can conclude that Hypothesis H4 was confirmed.

5 REAL DATA STUDY

In the second part of the study, we investigate actual multidimensional data. We identified two application fields, where the multidimensional data sets exhibit different characteristics. The first application is the visual analysis of document collections. Each document represents an object. The corresponding multidimensional point is a feature vector that represents the frequency of occurrences of certain keywords in the document. The second application is the visual analysis of image collections. Each image represents an object and the corresponding multidimensional point is a vector of features that are derived from the image using image processing steps. Document data are typically of very high dimensionality when compared to the number of objects, which imposes a certain data sparseness. Image data are typically of significantly lower dimensionality, which leads to a generally denser distribution.

5.1 Hypotheses

Considering distance-based tasks on real data, we can formulate the following hypothesis based on the findings of the preceding section:
H5. Cluster density, shape, orientation, and size influence distance estimation.
Next, we look into how visual attention matches the analysis tasks. For cluster identification, we formulate the hypothesis:
H6. There is a strong correlation between the visual attention pattern (locations of AOIs) and the provided answer when trying to identify clusters.
Concerning density-based tasks, we assume that sparser clusters get more visual attention, based on the findings of the preceding section. As the densities of the clusters are examined in 2D scatterplots, the densities in projected space are the ones that influence the perception. How well the answers match the cluster densities computed in high-dimensional space also depends highly on how well the projection methods manage to maintain the cluster density properties during projection. Our hypothesis is the following:

H7. The sparser the clusters in the scatterplot, the higher the visual attention.

5.2 Design of User Study

We picked four techniques as representatives of modern and classic strategies for embedding data in two dimensions. *Principal component analysis (PCA)* has been included in the study because it is a classical dimension reduction strategy often employed to generate visual embeddings of data. *Isomap* is effective on data that present non-linear relationships, that both PCA and classical scaling typically fail to detect. *LSP* is a modern dimension reduction technique that presents precisely the results achieved with sampling by clustering. Finally, we picked the *neighbor-joining (NJ) tree layout* (Paiva et al., 2011) as a tree layout for point placement to investigate whether their good grouping and distance properties would be perceived by users in the same way as the projections if the edges are removed from the layouts (i.e., if visually encoded as a scatterplot).

We use two document and two image data sets. The first document data set - referred to as CBR - contains 680 objects with 1,423 dimensions. The document information includes title, authors, abstract, and references from scientific papers in four different subjects[1]. The second document data set - referred to as KDViz - contains 1,624 objects with 520 dimensions and four highly unbalanced labels generated from an Internet repository[2]. The first image data set - referred to as Corel[3] - contains 1,000 objects with 150 dimensions. The images are photographs on ten different themes (Li and Wang, 2003). The second image data set - referred to as Medical - contains 540 objects with 28 dimensions (features) including Fourier descriptors and energies derived from histograms as well as mean intensity and standard deviation computed from the images themselves.

We conducted a controlled user study involving the same subjects as above. Each subject was presented 56 images with 2D scatterplots of projected multidimensional data using the four presented projection methods and asking one of the four identified tasks. We had to exclude a few cases from our study such as some tasks when PCA is applied to KDViz because of severe visual clutter that made it impossible to identify clusters and AOIs. The set-up of the experiments including eye tracking were as above. Actually, both parts (synthetic and real data) were executed in one session. The entire experiment did not take longer than 42 minutes for any of the subjects.

[1] http://vicg.icmc.usp.br/infovis2/DataSets

[2] http://vicg.icmc.usp.br/infovis2/DataSets

[3] UCI KDD Archive, http://kdd.ics.uci.edu

5.3 Analysis Methods

For the analysis of the correctness of the answers using real data, we computed the ground truth (distances, densities, and clusters) in the multidimensional space. Pairwise distances (Tasks Q1 and Q2) are computed using cosine distances for document data and Euclidean distances for image data to identify smallest distances. Clusters (Tasks Q1 and Q2) were computed using an X-means approach (Pelleg and Moore, 2000) and picking clusters with good properties that adhere to the given labeling. Densities (Task Q4) are computed as the inverse of the average edge length in the minimum spanning tree of each cluster, which is a simple distance-based measure that is sufficient for comparative analysis. Moreover, it scales well to high dimensions, is not biased towards any shape, and insensitive to density changes. Statistical methods and eye tracking analysis methods were the same as above.

5.4 Results

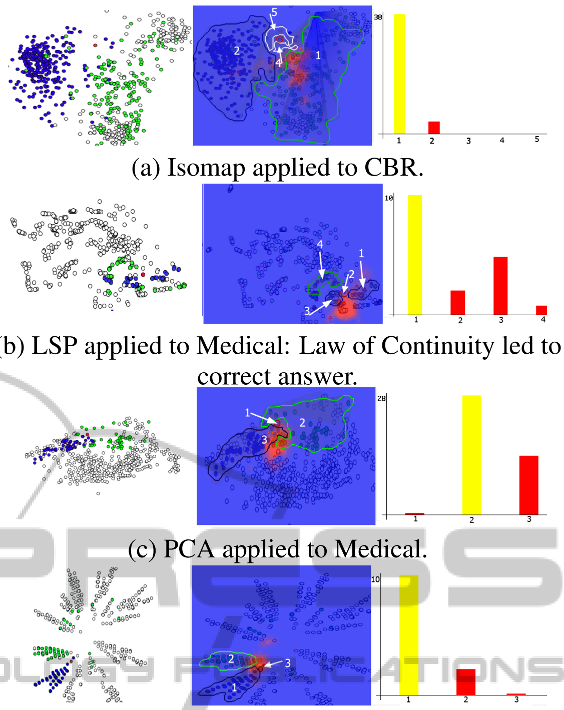
Closest Cluster to Reference Point. Task Q1 is again concerned with the identification of the closest cluster to a reference point. However, now the reference point is not equally distant from the cluster. The correct answer is computed in the high-dimensional space before projection. Also, the clusters have been computed before projecting. It is clearly visible from the examples in Figure 4 that different projection methods did differently well in preserving and separating the clusters. Also, there are severe difference between the results of different data sets for the same projection method. According to the mean correctness of the given answers for Task Q1 considering all dataset, LSP got the highest correctness (58.33%), closely followed by Isomap (53.125%), while PCA (19.79%) and Tree (9.375%) had lower correctness and Anova test showed significant difference among them (P=0.034). We investigated visual attention and cognitive processes for the individual examples. For the scatterplots generated using *Isomap* we observed a consistent visual attention pattern to our synthetic data, as the sparser cluster was the most looked at AOI, while the reference point got almost no attention. Figure 4(a) shows the example of Isomap applied to CBR, where we have two conflicting properties. The green cluster is sparser, but the blue cluster is larger. According to our findings from synthetic examples, density was dominant over size. Here, however, the clusters are not completely separated and only 37.5% of the subjects reported the green cluster as closer, although this would have been the correct answer.

For *LSP* the visual attention patterns when considering document data are similar to Isomap. However, we observed interesting cases for the image data sets. When LSP is applied to Corel, the reference point lies in the continuation of the blue cluster and subjects followed the Law of Continuity and incorrectly chose the wrong cluster as the closer one. Correctness dropped to 33.33%. In Figure 4(b), on the other hand, LSP is applied to Medical, and the reference point is aligned with the blue cluster. The Law of Continuity made 100% of the subjects choose correctly the blue cluster as the closer one.

For *PCA* the clusters were generally not well preserved or separated, which led to lower correctness. In Figure 4(c), for the Medical data set, 62.5% of the subjects correctly reported the green cluster being closer and it becomes evident that the green cluster is the sparser one. Subjects followed our earlier identified pattern, as the sparser cluster (AOI 2) is the AOI with most visual attention. We want to note that for some examples, we could only identify three meaningful AOIs (reference point, cluster 1, cluster 2), as it is not obvious what the space between the clusters and the reference point would be.

The *Tree* layout created the least correct results for Task Q1 on average. Figure 4(d) shows the worst case when Tree is applied to Corel leading to 0% correctness. The Tree layouts are, in general, most affected by the Gestalt laws, as the generated branches - according to the Law of Continuity - create the perception of a whole even when not drawing the edges of the tree. In Figure 4(d), the reference point happens to be included in a branch that otherwise contains only points of the blue cluster. The reference point (AOI 3) is not looked at explicitly, as it is perceived as being part of the whole (the branch with the blue cluster). Consequently, all subjects incorrectly answered that the reference point is closer to the blue cluster. In summary, we can conclude that also for real (i.e., projected multidimensional) data cluster properties influence the answers of the subjects. Hence, Hypothesis H5 is approved.

Closest Cluster to Reference Cluster. Task Q2 is concerned with the identification of the closest cluster to a reference cluster. Again, ground truth is computed in high-dimensional space. An additional aspect that comes in here is that the multi-dimensional reference cluster itself may not be well preserved during projection. For this task, we also want to test Hypothesis H5.



(a) Isomap applied to CBR.

(b) LSP applied to Medical: Law of Continuity led to correct answer.

(c) PCA applied to Medical.

(d) Tree applied to Corel.

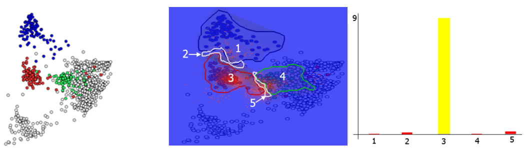
Figure 4: Task Q1: Finding closest cluster to reference point for real data.

Isomap was the projection method that created best results in terms of correctness of the answers with 82.29% for the whole datasets. Again, the sparser clusters are the AOIs that are observed most in almost all the cases. The only exception is shown in Figure 5(a) when applied to Corel. Here the reference cluster is the one with higher mean fixation duration. As it is shown, the red reference cluster actually spreads over a large area of the scatterplot. It can further be observed that the reference cluster mixes with the green cluster. Based on the Proximity law, the red and the green cluster are perceived as a whole and accordingly 87.5% of the subjects selected the green cluster as the closer one. Accumulated fixation times were higher for AOIs 3, 4, and 5 than for AOIs 1 and 2.

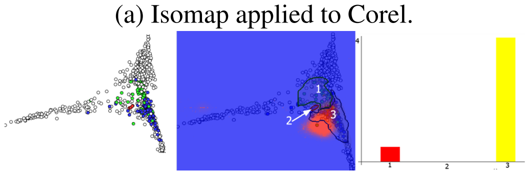
LSP also produced good correctness values with an average of 73.96%. The sparser clusters were looked at most for all four data sets but the correctness was lowest for the Medical data set. Investigating the eye tracking data showed that the denser cluster also got a large amount of attention for this example. The reason why this example was often answered incorrectly is most likely the fact that the density of the reference cluster matched the density of denser cluster and based on the Law of Similarity, subjects reported the denser cluster as the closer one incorrectly.

PCA had the weakest performance on Task Q2 with a correctness of only 20.83%. Figure 5(b) gives the example of PCA applied to Corel, which had a correctness of 33.33%. The green cluster does not exhibit a clear structure but is widely spread. AOI 3, which represents the much more coherent and denser blue cluster is examined longer. Although the reference cluster (AOI 2) is in proximity to both the green and the blue cluster, its density is similar to the blue cluster. According to the Law of Similarity this led to the incorrect conclusion that the reference cluster belongs to the blue cluster.

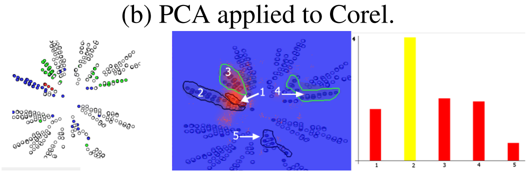
Tree layouts were correctly analyzed in 53.125% of all cases. Least correct (0%) was the example when Tree is applied to Corel, see Figure 5(d). Subjects tend to investigate branches of the tree individually. For example, AOIs 3 and 4 belong to the same cluster but were not looked at sequentially. The same holds true for AOIs 2 and 5. AOI 2 got the most visual attention, which is based on the Law of Proximity, as it is the one closest to the reference cluster. From the sequence of fixations, one may even conclude that AOI1 and AOI2 were looked at together. Consequently, all subjects answered incorrectly that the reference cluster is closer to the blue cluster. In conclusion, Hypothesis H5 was also confirmed for Task Q2.



(a) Isomap applied to Corel.



(b) PCA applied to Corel.



(c) Tree applied to Corel.

Figure 5: Task Q2: Finding closest cluster to reference cluster for real data.

Cluster Identification. Task Q3 is concerned with identifying the clusters and reporting back the number of identified clusters. According to the given application, CBR had 4 labels (or classes), KDViz had 4 classes, Corel had 10 classes, and Medical had 12 classes, as indicated by the color coding in Figure 6. However, when presenting the scatterplots to the subjects, color coding of classes was removed. In the following, we investigate how eye movement patterns relate to the given answers. In particular, we look into how many AOIs can be seen in the heat maps and compare that to the answers given.

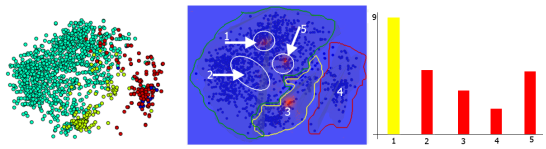
For *Isomap* we can state that the subjects' answers were close to their eye movement patterns. For CBR, the subjects reported 3.5 clusters on average and we could identify four hot spots (AOIs) in the heat map, where the heat map result contains the eye movements of all subjects. For KDViz, the heat map shows five hot spots, while the average answer was four. We further examine this case in Figure 6(a). In the sequence analysis, we observed a large amount of back-and-forth movement between AOIs 1 and 2. Because of the Laws of Proximity (AOIs 1 and 2 are close to each other) and Similarity (AOIs 1 and 2 have similar density), we can conclude that they have been perceived as one cluster, which explains the answer four instead of five. For Corel, seven hot spots can be seen in the heat map and subjects reported 7.83 clusters on average. For Medical, nine hot spots can be seen in the heat map and subjects reported 9.89 clusters on average. We can draw the conclusion that the hot spots in the visual attention match very well the answers that were given. However, the reported numbers are not necessarily the exact number of classes, as the projection may fail to keep clusters sufficiently separated. We observe that the reported numbers for Isomap are lower or equal than the actual number of classes. Moreover, the visual attention pattern reveals that even when the correct answer is given, it may be that the perceived clusters do not match the projected classes. For example, in Figure 6(a), the blue and red classes are highly overlapping and have been perceived as one cluster, while the dark green class has been split into two clusters. Consequently, the answer is correct despite the two perceptual mismatches.

For *LSP* the answers also matched well the number of hot spots. For CBR four hot spots were observed and subjects reported 4.85 clusters on average, for KDViz five hot spots were observed and subjects reported 4.375 clusters on average, for Corel nine hot spots were observed and subjects reported 11.14 clusters on average, and for Medical eight hot spots were observed and subjects reported 8.375 clusters on average. Obviously, for the examples with larger number of clusters, it gets more difficult to distinguish the hot spots and identify AOIs. In Figure 6(b), we try to investigate some AOIs for the example of the Medical data set. AOI 1 got most attention, as it is a sparser structure that needs longer investigation to make a decision. On the contrary, the dense cyan cluster in AOI 7 was obvious and did not need to be looked at intensively. We also deduce that AOI 5 (despite being quite dense) needs quite some attention because of its complex, non-concave structure.

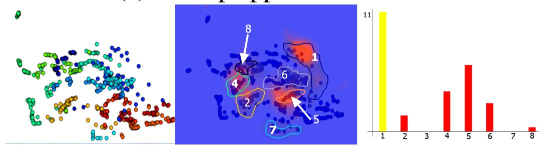
For *PCA* we obtained the following results: For CBR three hot spots were observed and subjects reported 2.67 clusters on average, for KDViz two hot spots were observed and subjects reported 2.375 clusters on average, for Corel three hot spots were observed and subjects reported 3.0 clusters on average, and for Medical six hot spots were observed and subjects reported 4.12 clusters on average. Again, there is a pretty good match between the visual attention pattern and the number of reported clusters. However, it becomes very obvious that the numbers are generally lower for PCA. For example, looking at the fixation sequence when applying PCA to CBR, we can deduce that overlapping areas were considered as one cluster, which explains why we only had three hot spots and on average 2.67 reported clusters. Perceptually merging these areas is reasonable, as they have similar density (Law of Similarity) and are well aligned (Law of Continuity). The general problem of PCA is that it does not manage very well to keep clusters separated. The cluttered clusters are, then, perceived as a single big cluster.

Finally, for *Tree*, subjects reported 11.64 clusters for CBR, 13.875 clusters for KDViz, 10.875 clusters for Corel, and 10.625 clusters for Medical. Obviously, numbers are generally higher for Tree. The fixation times reflect that the hot spots match the branches of the tree layout. Figure 6(d) shows the example of applying Tree to CBR. Groups that belong to one class are perceptually separated when split to two branches, e.g., AOIs 3 and 7. Hence, the general problem of Tree is that it does not manage very well to preserve clusters. Clusters are split over multiple perceptionally separated groups.

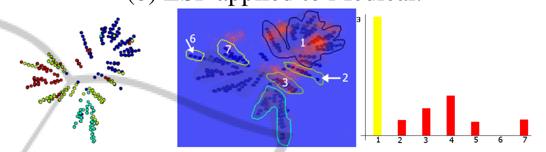
We conclude that the visual attention pattern matches well the given answers, which confirms Hypothesis H6. Moreover, we have seen that PCA and Isomap produced better results in form of preserving and segregating clusters during projection. PCA often produces scatterplots, where clusters were not separated well, while Tree produces scatterplots, where clusters were not well preserved, i.e., split over multiple clearly separated groups. Cluster separation and segregation is a highly studied topic when using projections for multidimensional data visualization. A commonly used quality measure which measures the cohesion and separation between groups of objects on the layout is the silhouette coefficient (Tan et al., 2005). Given an object p_i, its cohesion a_i is the average distance between p_i and all other objects belonging to the same group as p_i. Its separation b_i is



(a) Isomap applied to KDViz.



(b) LSP applied to Medical.



(d) Tree applied to CBR.

Figure 6: Task Q3: Estimate number of clusters for real data.

the minimum distance between p_i and all the other instances belonging to the other groups. The silhouette coefficient of a projection is obtained by averaging the silhouette coefficients of its n objects. Resulting values vary in the range -1 and 1, with 1 meaning that groups are perfectly separated. When computing the silhouette coefficients for the four projection methods when applied to the four data sets, Isomap and LSP indeed had on average the best values (0.215). PCA has the worst average silhouette coefficient (0.145) and even a negative one for KDViz. Tree's average silhouette coefficient lies in between (0.19). Hence, the silhouette coefficient results confirm our findings.

Cluster Ranking. For Task Q4, we asked the subjects to compare the density of clusters in the scatterplot. We picked three clusters in the multidimensional space and encoded them visually using red, green, and blue color. The subjects had to rank them by density. We also assume that the visual attention pattern matches the rankings reported by the subjects. In general, we observe a visual attention pattern where sparser clusters in the scatterplot get looked at more. In 12 out of the 16 scatterplots that were examined, the subjects started their investigation by looking at the sparsest cluster and, on average, also had the longest fixation duration for the sparsest cluster. Also, the densest clusters were, on average, looked at least when comparing the fixation durations of the three highlighted clusters. When trying to match the answers to the visual attention pattern, we can report that this worked best for Isomap, where in 33.33% of all cases the reported ranking matches precisely the ranking of average fixation duration. For the other methods the respective numbers are 29.16%

for LSP, 24.10% for Tree, and 18.75% for PCA. When only seeking the densest cluster, the match occurs in 41.67% for Isomap, 39.58% for both LSP and PCA, and 37.05% for Tree. Considering the correctness of the answers with respect to densities computed in the multi-dimensional space, the results are as follows: Isomap achieved highest correctness (65.62%), followed by PCA (47.92%), LSP (46.88%), and finally Tree (42.85%).

For the PCA projection, the sparser cluster in the 2D scatterplot is the one looked at most and is reported by the subject as sparsest. However, in the multidimensional space the sparsest cluster is actually the densest. The comparative density properties among clusters are not preserved by PCA. For the Tree layout, the cluster that spreads over the entire scatterplot got the highest amount of visual attention. However, since there are densely populated branches the overall density of the cluster was rated as high. Consequently, the majority of subjects answered it as the densest. The outliers that are part of branches (e.g. AOI 6 in Figure 6(c)) with dominantly yellow points here are not perceived as outliers, as the respective branches are seen as a whole.

In summary, we observed that the projection method may change comparative density properties of clusters. In the scatterplot, there was a tendency to have more visual attention for sparser clusters, as it was postulated in Hypothesis H7. However, this tendency was not as strong as expected, as other factors like cluster separation, size, and shape also influence perception here.

6 CONCLUSIONS

We have presented a study on the role of visual attention when interpreting scatterplots that were obtained by projecting multidimensional data into 2D visual spaces. In a first part of our study, we considered synthetic scatterplots, which allowed us to vary only one perceptual factor at a time. Our hypotheses made use of the Gestalt Laws of Proximity, Similarity, Continuity, and Closure to postulate that cluster properties such as density, shape (and also orientation), and size influence perception when interpreting distances in scatterplot. Density turned out to be more influential than size. For distance tasks, there was a clear tendency that the space between the reference and the perceptually farther cluster was looked at more than the space between the reference and the perceptually closer cluster. Our hypotheses were confirmed. There was a clear correlation between this visual attention pattern and the given answer.

In a second part of our study, we formulated respective hypotheses for visual analyses of projected multidimensional data. Investigating the role of cluster characteristics in real-world data, we were able to also confirm those hypotheses and we can conclude that there are multiple factors that influence perception (or visual attention) and that perception plays an important role in interpreting the scatterplots. We also performed a comparative analysis of four projection methods on two types of data, which led to some guidelines for their usage. In particular, continuity can influence the answers significantly, where the Tree layout was most affected by this due to the branching structure. Isomap and LSP, on the other hand, had a tendency to create more roundish clusters (of course, with exceptions), which led to less misinterpretations. PCA had problems with cluster segregation, while Tree had issues with cluster preservation. Hence, projection methods should also be investigated with respect to how well they maintain these properties. For example, when two clusters are being projected, where one is denser than the other, the projected denser cluster should also be denser than the projected sparser cluster and not vice versa. We also want to mention that we initially had included a fifth projection method in our study, namely Glimmer (Ingram et al., 2009). In Glimmer iterative point placement procedure is highly optimized by clever usage of GPU hardware combined with a multilevel strategy that operates on a hierarchical model of the underlying particle-spring system. However, as shown in the example in Figure 7, the projection and the visual attention pattern was scattered and we have not been able to identify any meaningful AOIs for Glimmer and, therefore, excluded it from our study. The silhouette coefficients for Glimmer when applied to our four data sets was negative.

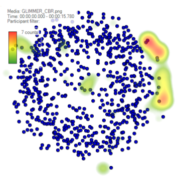


Figure 7: Glimmer applied to CBR, overlaid with eye fixation pattern for Task Q3 (using a green-to-red color map).

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