

Monitoring the Mental Status of Football Players

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Keywords: Heart Rate, Football, Modelling, Real-time.

Abstract: This work has tested heart rate to measure anxiety during a penalty shootout. Until now, anxiety is measured through questionnaires, where online monitoring is not possible. Therefore there is a need for physiological parameters to represent anxiety online. Since it is proven that the level of anxiety is a good predictor of penalty outcome, it was hypothesised that this outcome can be estimated with heart rate and activity. To test this hypothesis an experiment has been conducted with 54 participants (age= 23±4,54 years). They each performed three sessions of a penalty shootout, where heart rate and activity were measured. An adapted version of the State-Trait Anxiety Inventory was used as reference for anxiety level. The data have been analysed using a static and dynamic approach. These resulted in parameters that were used to predict the anxiety level and penalty performance of the participant with a multinomial logistic regression model. The results show that 47,11% of the participants were correctly classified into three classes of anxiety. Based on a classification into penalty performance 55,11 % of the participants were correctly classified. It can be concluded that heart rate in combination with activity shows promising results as predictor for anxiety.

1 INTRODUCTION

In the past, a lot of research is done with the focus on the physical aspect of sports. Today, the focus is shifting towards the mental aspect. The reason for this shift is that it got to researchers' attention that mental health can play an important role in performance.

The mental aspects in sports are defined here as mental influences that have an impact on the performance of the athlete. Generally, these influences can be divided in two main groups. These are on the one hand perception of effort and on the other hand feelings of anxiety.

Perception of effort means that an exercise at the same intensity feels harder after some time (Knicker et al., 2011). De Morree et al., (2012) state that it is the conscious awareness of the central motor command sent to the active muscles. This interpretation of perception of effort is called the corollary discharge model (Marcora, 2009). When humans undertake action, it is preceded by brain activity. Specifically for voluntary actions, which are present in sports, this brain activity takes place in the motor areas. It is this action in the central motor

system that is sensed and is reflected by perception of effort. The second mental influence is anxiety. One of the most important causes for presence of anxiety is stress. Stress is defined here as an athlete's ability, or lack of ability, to deal with competitive pressure (Mateo et al., 2012).

Different methods have been developed to measure perception of effort and the level of anxiety. These methods are mainly based on surveys. For the measurement of perception of effort the Borg rating scale is the oldest and the most widely used instrument (Chen et al., 2002). This scale is a general measure of exercise intensity (Zamunér et al., 2011). It is an equidistant interval chain that starts at 6 (no exertion at all) and ends with 20 (maximal exertion) (Borg, 1998). To measure the level of anxiety the state-trait anxiety inventory (STAI) is most commonly used. As the name suggests, the STAI measures both state and trait anxiety (Horikawa and Yagi, 2012). The test consists of two forms with each 20 items. The items are rated on a four-point Likert scale (Horikawa and Yagi, 2012). Based on the sum of the quotations on each item, a measure of both state and trait anxiety is provided.

Since these inventories are quite devious and

always require some time to fill out, it is the goal of many researchers to find physical parameters that perform as good as the questionnaires, but are easier to measure. For perception of effort, extensive work has already been done to find a link of the gold standard with physical parameters (Chen et al., 2002). Chen et al., (2002) concluded from their work that the physiological variable that correlates best with the Borg scale is breath rate. For anxiety however still some work is necessary. There are some physical and biochemical parameters identified such as heart rate variability (Mateo et al., 2012), blood pressure (Frazier et al., 2002), epinephrine, norepinephrine and cortisol (Hoehn et al., 1997). The problem with these parameters however is that none of them can be measured online.

It is in this context that this research tries to make a contribution. A very specific scenario in football where there is a need for an online measuring method of anxiety is used. Since it is proven that the outcome of a penalty is highly dependent on the level of anxiety of the player (Jordet, 2009), it could be interesting for coaches to know the level of anxiety of each player before deciding who will take a decisive penalty. The use of inventories would in this case be too time consuming and clumsy. In this research, heart rate in combination with activity is examined as a possible physiological parameter. The activity is represented by the acceleration signal. Since it is proven that the level of anxiety is a good predictor of penalty outcome (Jordet, 2009), it is hypothesised that this outcome can be estimated based on the measurement of heart rate and activity. If this hypothesis is confirmed, coaches will have an interesting tool in deciding who will take the penalties in competition.

2 MATERIAL AND METHODS

To examine the hypothesis that the outcome of a penalty can be estimated based on the measurement of heart rate and activity, an experiment was conducted. In the first section an overview of the set-up of this experiment and the materials used is given. The experiment was approved by the Ethical commission of the KU Leuven (6/12/2012). Furthermore an overview of the different methods used for the analysis is presented in the second section.

2.1 Experimental Set-up

To test the hypothesis, an experiment was conducted

where the participants had to perform three sessions of penalties. In the last session the anxiety was induced. In the analysis the heart rate was compared before and after this induction. Based on the differences in heart rate a prediction was done concerning the penalty outcome and anxiety level in the different sessions.

2.1.1 Participants

Participants were chosen from the 3rd and 4th Belgian football division. In total, three clubs participated in the experiment. This resulted in a sample size of 54 male participants (age = 23±4,54 year). Before the experiment started personal data of all participants were collected, as well as their informed consent to participate on the experiment.

2.1.2 Sensors and Questionnaires

In total three questionnaires and two sensors were used. The first questionnaire is for personal data collection such as age, weight, etc. The second is the Borg rating scale for the measurement of perception of effort. The last is an adapted version of the STAI which was used as the gold standard to measure the level of anxiety. An adapted version was used since the full version consists of 20 questions. The questionnaire was used in between three stages of the experiment. To pose 20 questions each time would take too long and would possibly cause the induced anxiety to decrease. Therefore only five questions were retained. These were 'I feel tensed', 'I feel afraid', 'I feel certain', 'I feel calm' and 'I feel nervous'. These five specific questions were chosen based on the professional input of a sports psychologist.

For the measurement of heart rate and activity also two sensors were used. The heart rate was measured with a Zephyr HxM sensor (Zephyr™, Annapolis, Maryland, US) sampled at 1 Hz. For the measurement of activity the acceleration signal was used. This was measured with a Sony Xperia™ smartphone (Sony, Tokyo, Japan) sampled at 50 Hz. The acceleration was measured separately in three dimensions.

2.1.3 Set-up

All participants had to perform three penalty sessions. First 15 training penalties, then 5 control penalties and finally 5 induced anxiety penalties.

Before the experiment started the participant put on the Zephyr heart rate belt and the Sony smartphone for activity measurement.

First the participants had the chance to practice their penalty shooting. This was done with 15 penalties in the training session. The purpose was to shoot the penalty in one of the two holes in the net that was hung up at the goal. This set-up is preferable over the normal set-up with a goalkeeper, since now the effect of the goalkeeper on success is ruled out. These penalties could be taken freely, without any time constriction. The participant was told that the outcome was of no importance for the experiment, but that later on the outcome would become more important and he should utilise this practicing opportunity as well as possible. After this information was given, the adapted STAI and the Borg rating scale were filled in and the participant took the 15 training penalties.

After the training session a break of two minutes took place. During this break the participant got the information for the control condition. Here the participant was told that he would have to take 5 penalties with a time interval of 15 seconds. He was informed that this exercise session was important to test and calibrate the material of the heart rate belt. It was important that he would try to perform as well as possible, but the outcome of the shootout would remain confidential. The adapted STAI was filled out after this information was given, but before the exercise started.

After this control session a two minutes break took place. During this break the participant got the information for the induced anxiety condition. He was told that the results of this last test were the only results that would finally be of any importance. These would be passed on to the coach, who would use them to make a ranking for the penalty abilities of all the players of the team. The ranking would also be communicated later on with the other team players. After this information was given again the adapted STAI was filled in and, when the two minutes break had passed, the exercise started. Although the participants were told that their results would be made public, this did not really happen. The results remained confidential at any time, unless the participant gave his consent. After all participants had done the experiment they were debriefed about the real purpose and they were told that the results remain confidential.

2.2 Analysis

The goal was to predict, based on the information of acceleration and heart rate signal, whether or not a participant felt more anxious in the induced anxiety session compared with the control session and if he

would score more or less penalties. The reference level of anxiety and penalty performance for each participant was obtained based on the responses on the STAI and the amount of penalties scored. Participants could either be less anxious, no difference or more anxious in the anxiety condition compared with the control condition. For the penalty performance they could either score less penalties, no difference or more penalties in the anxiety condition compared with the control condition.

To achieve the goal different parameters were calculated from the heart rate signal. These parameters could then be used for prediction. To obtain these parameters two types of analyses were done, being a static and a dynamic analysis. Both are followed by a statistical analysis. These were all performed with the MATLAB R2011b (Mathworks, Natick, MA, USA) software.

2.2.1 Static Analysis

First for every participant the data were subdivided into three groups being the training, control and induced anxiety group. This subdivision was based on the beginning and end times of every session that were written down during the experiment. Then for each group five static parameters were calculated. These were the mean, the maximum, the minimum, the recovery slope and the increase slope of the heart rate signal during each session. The recovery slope was calculated based on the last 40 s, the increase slope on the first 20 s of every session. The calculation of these slopes was done using the beginning and end point of the time frames. This resulted in 15 values for each participant, coming from five static parameters with each three values for the three different sessions. These parameters did not provide any dynamics of the heart rate, but solely static information, hence the name of the static analysis. The parameters were subsequently compared among the participants. For this comparison not the absolute values of the parameters were used, but the pattern they followed. For each parameter the three different values, respectively from training, control and induced anxiety session, were placed next to each other. These values could increase, stay equal, form a maximum, form a minimum or decrease. These patterns all corresponded with a number from zero to four respectively. This resulted in five values for each participant, being the patterns for the five static parameters.

2.2.2 Dynamic Analysis

In this analysis the heart rate response of the participants was modelled, taking the dynamics of the heart rate into account. Both error and model parameters were then used as parameters to predict to which class a participant belongs. This analysis was done with the Captain toolbox (Taylor et al., 2007) in MATLAB.

For the modelling the Box-Jenkins methodology was used (Zhang et al., 2012). This system is based on the following equation

$$y(t) = \frac{B(z)}{A(z)} \times u(t) + \frac{D(z)}{C(z)} \times e(t) \quad (1)$$

In this equation $y(t)$ represents the system output vector, $u(t)$ is the system input vector and $e(t)$ is the noise vector. In this heart rate application the noise was assumed to be white, therefore it was not included in the model. $B(z)$ and $A(z)$ are defined by the following two equations

$$B(z) = B_1 z^{-1} + B_2 z^{-2} + \dots + B_n z^{-n} \quad (2)$$

$$A(z) = 1 + A_1 z^{-1} + A_2 z^{-2} + \dots + A_n z^{-n} \quad (3)$$

Where z^{-1} is the backwards shift operator with $z^{-1}y(t) = y(t-1)$.

In this case the system input is the acceleration signal, the output the heart rate signal. In MATLAB the *rivbjid* function was used to calculate a model. Both the order and the coefficients of the model could vary. In the analysis different models with different orders and time delays were calculated. All the possible combinations for orders in numerator and denominator going from one to three and for a time delay from one to five samples were calculated. This resulted in the calculation of 45 ($3 \times 3 \times 5$) different models.

To choose which model fits best on the data the Young identification criterion (YIC) was used. This criterion is interesting since it combines a measure of fit and parameter reliability. For a sample size N , the YIC is defined as follows (Young, 2011)

$$YIC = \ln \left\{ \frac{\sigma^2}{\sigma_y^2} \right\} + \ln \left\{ \frac{1}{np} \sum_{i=1}^{np} \frac{\hat{\sigma}^2 p_{ii}}{\hat{a}_i^2} \right\} \quad (4)$$

Where σ^2 is the variance of the model errors, σ_y^2 the variance of the data around the mean, np the number of parameters, $\hat{\sigma}^2 p_{ii}$ the value of uncertainty for the i^{th} parameter estimation and \hat{a}_i^2 the quadratic value of the i^{th} parameter. The best model is one with the most negative YIC value, if this value is highly positive it means the model is over-parameterised (Young, 2011).

Following this procedure, a model was calculated for the training, control and induced anxiety session. From this model different parameters could be calculated. The first group are the error parameters, the second the individual model parameters.

To calculate the error parameters the model that was calculated based on the training data, was fit on both the control and the induced anxiety data. Consequently the error for both control and induced anxiety session was calculated. This was done by subtracting the actual heart rate data from the simulated data. If it is assumed that the physical part of heart rate is modelled in the training session and that the metabolic part remains constant, then the errors represent the mental part of the heart rate. In this case a difference in anxiety level could become visible through a difference between errors of the control and induced anxiety session (Myrtek et al., 2004). From this analysis nine different error parameters were calculated.

The second group of parameters investigated the properties of the models calculated on the three sessions separately. These are the individual model parameters. First the model orders were calculated. This resulted in six parameters, being the order of the numerator and denominator for training, control and induced anxiety session. The time constants and the steady state gains of the three models were also calculated, which resulted in six additional parameters. The time constant represents the time that it takes for the model output to achieve 63,2% of its final value as a response to a step input (Lipták, 2006). The steady state gain is the ratio of the change in output to the change in input when the system has reached steady state as a response to a step input (Lipták, 2006).

2.2.3 Statistical Analysis

The goal in the statistical analysis was to predict the class to which a participant belonged based on the parameter values calculated in the static and dynamic analysis. This was done with the multinomial logistic regression (MLR) model, which is an extension of the binary logistic regression model. It is used when the dependent variable exists of different categories. One of these categories is taken as the reference category. The probability of an observation to belong to this reference category is then compared with the probability to belong to one of the other categories (Prabhakar et al., 2013). In a first approach of the data, the classification was done with the whole dataset both as training and as test

set. This resulted in the most optimal classification results, but did not give an accurate representation of the model performance. Therefore also a fivefold cross-validation was performed. This means that 4/5th of the data was used as training set to create a model and the remaining 1/5th was used as test set to evaluate the performance of the model. This was done five times with each time a different training and test set. The division of the observations into training or test set was randomly chosen, but it was made sure that all the observations were exactly once used as test set. For the five different classifications each time the percentage with correctly classified observations was calculated. From these five results the mean was taken and this percentage represents the overall performance of the model.

3 RESULTS

The goal was to search for interesting parameters, static or dynamic, that could be used as predictors in the MLR model. This model could then be used to predict to which class a participant belonged. In the first section the results of the anxiety classification are presented, in the second those of the penalty performance classification.

3.1 Anxiety Classification

The result for the static analysis is presented in Figure 1. The rows represent the class to which the participant really belongs according to the reference, the columns the class to which he was classified according to the model or algorithm. This means that the diagonal of the figure represents the correct classified persons. The numbers represent the number of participants that are classified in a certain group. This classification however was done with the whole dataset as both training and test set. This resulted in the most optimal classification, but is not an accurate reflection of the model performance. Therefore also a five-fold cross-validation was performed. This showed that 36,67% of the participants were correctly classified using these static parameters.

A similar analysis was done for the dynamic parameters. There were however a total of 21 dynamic parameters, which is too much for an effective model. Therefore first a MLR model was calculated with all the 21 parameters. Afterwards only the significant parameters were retained and a new model with only these parameters was

calculated. The result of this model with only the significant parameters can be seen in Figure 2. In this case there were only three significant parameters present. These were the mean of the anxiety error, the time constant of the training session and the steady state gain of the induced anxiety session. After a five-fold cross-validation 47,11 % of the participants were correctly classified.

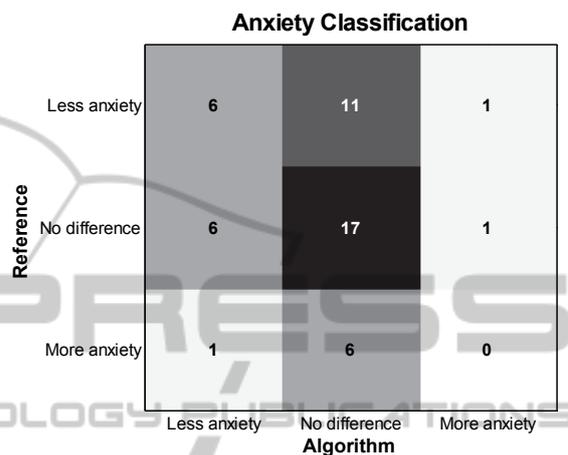


Figure 1: Anxiety classification with static parameters.

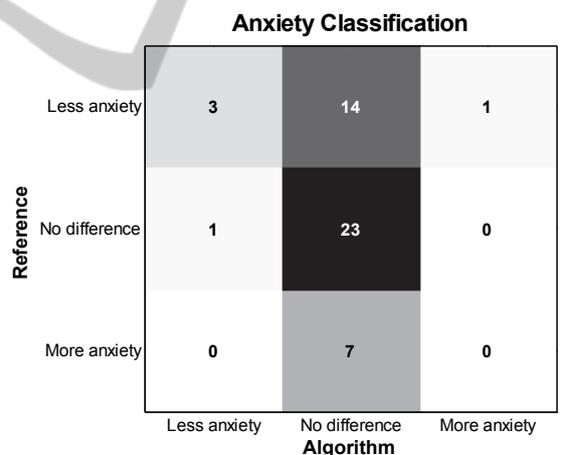


Figure 2: Anxiety classification with the significant dynamic parameters.

3.2 Penalty Classification

The result for the static analysis can be seen in Figure 3. A five-fold cross-validation indicated that 26,22% of the participants were correctly classified.

For the dynamic analysis again only the significant parameters were retained. The result can be seen in Figure 4. In this case there are nine significant parameters. These are the standard deviation and the cumulative sum of the last part of

the control error, the model orders of both numerator and denominator of the training session, the denominator of the anxiety session and the time constant and steady state gain of training and anxiety session. The five-fold cross-validation indicated a correct classification of 55,11%.

Penalty Classification

Reference	Less penalties	6	5	6
	No difference	2	9	3
	More penalties	4	5	9
		Less penalties	No difference	More penalties

Algorithm

Figure 3: Penalty classification with static parameters.

Penalty Classification

Reference	Less penalties	12	3	2
	No difference	2	9	3
	More penalties	1	5	12
		Less penalties	No difference	More penalties

Algorithm

Figure 4: Penalty classification with the significant dynamic parameters.

4 DISCUSSION

This discussion consists of three sections. In the first the classification results are discussed. The goal is to investigate whether the hypothesis can be confirmed, meaning that heart rate in combination with activity, in this case represented by the acceleration signal, can predict for both anxiety level and penalty performance. In the second section a discussion of the biological meaning of the different heart rate parameters is presented. Finally some suggestions

for future work are given.

4.1 Classification Results

The most important observation is that generally the classification percentages were not as high as hoped for. This implicates that still some improvements are necessary for the approach to be used in practice. The best classification was obtained with nine parameters from the dynamic analysis for the penalty classification. Here a correct classification of 55,11 % was reached with a five-fold cross-validation. An interesting solution to increase the classification performance could be the use of a non-linear classifier instead of the MLR model which is linear. A possible method is by using support vector machines. This is a statistical classification method which was originally designed for binary classification, but can be broadened for classification into three groups as is the case in this experiment. The method provides an optimal hyper plane that separates the different classes (Bosch et al., 2013). The advantage of this method is that it can use a non-linear approach when necessary and therefore in this situation can be more interesting for classification than MLR. A second solution could be to search for some additional parameters. For example next to the time constant and steady state gain there are still other dynamic properties of a model such as the overshoot, rise time and settling time. Furthermore also the results of the dynamic approach can be improved by using a combination of the R^2 and YIC value to select the best model. Finally it needs to be mentioned that in general the parameters of the dynamic approach performed better than those of the static. Also the outcome of the penalty in this experiment depends on the skill of the player. The higher the skill level the more consistent the player can be and this has an influence on the penalty outcome. If someone is not consistent then this can be the cause of the difference in penalty outcome instead of the anxiety level. Therefore it is suggested to focus in the future on the dynamic approach and players with a similar, high skill level.

It can be concluded that qualitatively the hypothesis could be confirmed. This means that heart rate in combination with activity could serve as a predictor for both anxiety level and penalty outcome. However, future research, with the previous suggestions in mind, is necessary to improve the quantitative results of the classification.

4.2 Biological Interpretation

Different parameters were used as predictors in the MLR model for classification. It is not only the goal to find these parameters, but also to search for an explanation why exactly these parameters can make the connection between heart rate and the mental state. Especially the error parameters have attracted researchers' attention over the last years.

The effect of the mental activation on the additional heart rate has already been investigated (Myrtek et al., 2005). It is defined as the increase in heart rate without a corresponding increase in activity (Myrtek et al., 2005). Since the heart rate dynamics due to activity were modelled, the error reflects this additional heart rate (Jansen et al., 2009). The hypothesis therefore is that when a mental activation is present, the error should increase. An important remark to keep in mind in this context is that additional heart rate is also influenced by parameters such as cardiac drift, fatigue, etc. In this research this hypothesis is not confirmed. Therefore it is suggested that future research focuses more on this topic.

4.3 Future Work

The goal of this research was to predict the outcome of a penalty shootout based on the measurements of heart rate and activity. The underlying goal was to find a physiological variable that could measure the level of anxiety. The results have indicated that heart rate has potential as predictor for anxiety. However, the results are not yet good enough for practical applications. In the previous section already some possible improvements were listed. If these suggestions are taken into account in future research better results will become possible.

Furthermore it needs to be said that generally research on the biological interpretation of model parameters should increase. It is important to know not only that some parameters could predict the mental state, but also why this would be the case.

Finally, this research has only focused on football. However, the need for a physiological variable to measure anxiety or the mental state in general is not restricted to this sport only. Future research should test whether the algorithms developed in this research are also applicable in other sports. It is also important to broaden the investigation further than only heart rate analysis. As presented earlier also blood pressure and biochemical variables such as epinephrine can predict for anxiety. The downside of these variables

is that these cannot be used online which is an important factor in the penalty shootout. However, this is not equally important in every application. Therefore these parameters should not be excluded and more research should be dedicated to them.

5 CONCLUSIONS

As a general conclusion of this research it can be said that the analysis of heart rate offers some interesting perspectives for the future concerning the measurement of anxiety. A follow-up study should indicate whether better classification results can be obtained when the different proposed adjustments are implemented. Furthermore, more research should be focused on finding a biological interpretation of the parameters. Finally it is important to broaden the research to other sports and other variables.

REFERENCES

- Borg, G., (1998). Borg's perceived exertion and pain scales. *Human kinetics publishers*, Champaign. 104 p.
- Bosch, P., López, J., Ramírez, H., Robotham, H., (2013). Support vector machine under uncertainty: an application for hydroacoustic classification of fish-schools in Chile. *Expert systems and applications*, 40: 4029-4034.
- Chen, M. J., Fan, X., Moe, S. T., (2002). Criterion-related validity of the Borg ratings of perceived exertion scale in healthy individuals: a meta-analysis. *Journal of sports sciences*, 20: 873-899.
- De Morree, H. M., Klein, C., Marcora, S. M. (2012). Perception of effort reflects central motor command during movement execution. *Psychophysiology*, 49: 1242-1253.
- Frazier, S. K., Moser, D. K., Riegel, B., McKinley, S., Blakely, W., Kim, K. A., Garvin, B. J., (2002). Critical care nurses' assessment of patients' anxiety: reliance on physiological and behavioural parameters. *American journal of critical care*, 11: 57-64.
- Hoehn, T., Braune, S., Scheibe, G., Albus, M., (1997). Physiological, biochemical and subjective parameters in anxiety patients with panic disorder during stress exposure. *European archives of psychiatry and clinical neuroscience*, 247: 264-274.
- Horikawa, M., Yagi, A., (2012). The relationships among trait anxiety, state anxiety and the goal performance of penalty shoot-out by university soccer players. *Plos one*, 7(4).
- Jansen, F., Van Der Krogt, J., Van Loon, K., Avezzù, V., Guarino, M., Quanten, S., Berckmans, D., (2009). Online detection of an emotional response of a horse during physical activity. *The veterinary journal*, 181, 38-42.

- Jordet, G., (2009). When superstars flop: public status and choking under pressure in international soccer penalty shootouts. *Journal of applied sport psychology*, 21, 125-130.
- Knicker, A. J., Renshaw, I., Oldham, A. R., Cairns, S. P., (2011). Interactive processes link the multiple symptoms of fatigue in sport competition. *Sports Medicine*, 41: 307-328.
- Lipták, B. G., (2006). Process control and optimization. *CRC Press*, Florida, US. 67 p.
- Marcora, S. M., (2009). Perception of effort during exercise is independent of afferent feedback from skeletal muscles, heart and lungs. *Journal of applied physiology*, 106: 2060-2062.
- Mateo, M., Blasco-Lafarga, C., Martínez-Navarro, I., Guzmán, J. F., Zabala, M., (2012). Heart rate variability and pre-competitive anxiety in BMX discipline. *European journal of applied physiology*, 112: 113-123.
- Myrtek, M., Aschenbrenner, E., Brügger, G., (2005). Emotions in everyday life: an ambulatory monitoring study with female students. *Biological Psychology*, 68: 237-255.
- Prabhakar, M., Prasad, Y. G., Desai, S., Thirupathi, M., Gopika, K., Rao, G. R., Venkateswarlu, B., (2013). Hyperspectral remote sensing of yellow mosaic severity and associated pigment losses in Vigna mungo using multinomial logistic regression models. *Crop protection*, 45: 132-140.
- Taylor, C. J., Pedregal, D. J., Young, P. C., (2007). System identification, time series analysis and forecasting. The captain toolbox. *Centre for environmental systems and statistics*, Lancaster, England. 278 p.
- Young, P. C., (2011). Recursive estimation and time-series analysis. Springer, Berlin, Germany. 443 p.
- Zamunér, A. R., Moreno, M. A., Camargo, T. M., Graetz, J. P., Rebelo, A. C. S., Tamburús, N. Y., Da Silva, E., (2011). Assessment of subjective perceived exertion at the anaerobic threshold with the Borg CR-10 scale. *Journal of sports and medicine*, 10: 130-136.
- Zhang, Z., Jia, J., Ding, R., (2012). Hierarchical least squares based iterative estimation algorithm for multivariable Box-Jenkins-like systems using the auxiliary model. *Applied mathematics and computation*, 218: 5580-5587.