

# Identification of Fuzzy Measures for Machinery Fault Diagnosis

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**Abstract:** This paper proposes an identification method of fuzzy measure for fault diagnosis of rotating machineries using vibration spectra method. The membership degrees for spectra in fuzzy set composed of vibration spectra are obtained from the optimized membership functions. The fuzzy measure is identified by the proposed method using the partial correlation coefficients between two spectra and the weight of each spectrum given by skilled engineers. The possibility of faults are determined by the fuzzy integral that is made by using the membership degrees and fuzzy measures for spectra. This paper also evaluates the method using field data.

## 1 INTRODUCTION

Diagnosis of faults in rotating machineries are made by applying prior knowledge in conjunction with diagnostic analysis techniques of diagnosing engineers. The need for diagnosing rotating machineries is rising due to the increased use of them in highly reliable systems such as aircrafts and nuclear power plants. Moreover, due to the increase of condition based maintenance (CBM) for highly dependable systems and for cost effective maintenance, many highly skilled engineers are required to make accurate diagnoses (Chen et al., 2002). However, it is difficult to satisfy the current need of skilled engineers because the requisite training is lengthy and very expensive.

Several diagnostic systems for rotating machineries have been developed to satisfy this need (Liu et al., 2007). Some of them use fuzzy measures and fuzzy integrals to encompass the existing knowledge of skilled engineers (Marinai and Singh, 2006). However, they still have several problems, such as difficulty in isolating faults generating similar vibration spectra.

This paper proposes an identification method of fuzzy measures using partial correlation coefficients of spectra used for fault diagnosis. The possibility of faults is determined by the fuzzy integral using the membership degree of spectra and fuzzy measure of the set of spectra. The membership degrees are

obtained by the optimized membership functions (Tsunoyama et al., 2010; Tsunoyama et al, 2012), and fuzzy measures are identified by the partial correlation coefficients of spectra and the weight of each spectrum given by skilled engineers.

This paper is organized as follows. The vibration spectra for faults, and fuzzy measure and fuzzy integral are described in Section 2. The identification method of fuzzy measure and variation of possibility are explained in Section 3. A sample diagnosis and evaluation of the proposed method are provided in Section 4. Our conclusions are presented in Section 5.

## 2 FAULT DIAGNOSIS OF ROTATING MACHINERIES

### 2.1 Faults and Vibration Spectra

Several kinds of faults occur in rotating machineries including abnormal vibration, oil or water leaks, and abnormal temperature. The proposed method diagnoses faults that produce abnormal vibration since a large number of faults in rotating machineries are accompanied by vibration. However, the presence of vibration is not necessarily indicative of a failure mode when the vibration power is low. The power level required for machinery failure is specified by ISO 2372. The

proposed method diagnoses faults generating vibrations larger than this level.

The vibration spectra vary depending on the type of fault. The vibration spectra method employed analyzes vibration at six locations specified by ISO2372 in a machinery using FFT (Fast Fourier Transform) and diagnoses faults using the spectra. An example of spectra for a faulty machinery is shown in Figure 1. In this case, engineers diagnose that the fault might be imbalance because the spectrum of fundamental frequency (60Hz) is high, the second harmonics (120Hz) is rather high, and more than third harmonics are very high from the figure.

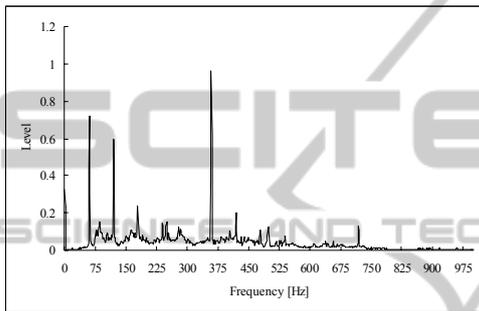


Figure 1: Vibration frequency spectra.

## 2.2 Fault Diagnosis

### 2.2.1 Fuzzy Set of Spectra and Membership Functions

In the fault diagnosis, the possibility of forty three different faults such as imbalance, misalignment, looseness and so on are calculated using the vibration spectra collected from a faulty machinery, and membership degree and weight of each spectrum.

The fuzzy set of spectra  $\tilde{A}$  is represented by Eq.(1). The set is composed of vibration spectra and their membership degrees. The set  $X$  is the set of whole spectra used for calculating the possibility of a fault.

$$\tilde{A} = \sum_{i=1}^n h(s_i) / s_i \quad (1)$$

$$X = \{s_1, s_2, \dots, s_n\}.$$

where  $h(s_i)$ , ( $0 \leq h(s_i) \leq 1$ ) is the membership degree of spectrum  $s_i$  for a fault.

The intensity of spectra vary depending on the installation of the machinery or degree of damage by the fault, and its probability distribution can be approximated by the normal probability distribution.

We optimize membership functions based on the statistical properties of spectra (Tsunoyama et al., 2010).

In fault diagnosis, diagnosed results are classified into four cases (Table 1).

Table 1: Diagnosed Results.

Case	Cause of fault	Diagnosed result
1	$\alpha$	Not $\alpha$
2		$\alpha$
3	$\beta$	$\alpha$
4		Not $\alpha$

When the possibility of fault  $\alpha$  is calculated, the possibility for Cases 2 should be maximum and the possibility for Case 3 should be minimum, since Case 2 is correct but Case 3 is not. Moreover, Cases 1 and 2 are exclusive, as are Cases 3 and 4. Therefore, the membership function can be optimized by maximizing the mean value of the membership degree for Case 2 and minimizing that for Case 3. We call  $\alpha$  and  $\beta$  are a target fault and non-target faults, respectively.

Figure 2 shows a triangular membership function  $h(x)$  for a spectrum for diagnosing fault  $\alpha$ , and probability density function  $f_\alpha(x)$  ( $f_\beta(x)$ ) for intensity of the spectrum when fault  $\alpha$  ( $\beta$ ) occurs.

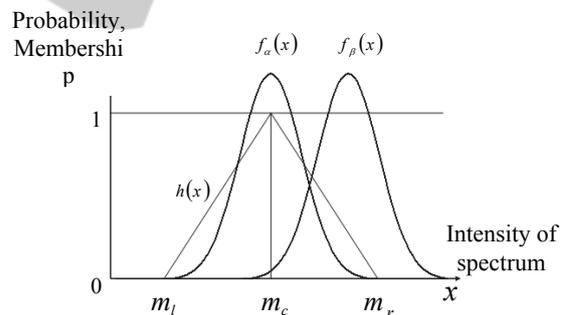


Figure 2: Membership function and probability density functions.

The integral of the probability distribution function  $F_a(x)$ , ( $a \in \{\alpha, \beta\}$ ) and the membership function  $h(x)$  for fault  $\alpha$  gives the average membership degree for fault  $\alpha$ . The optimization of the membership function for fault  $\alpha$  is performed by maximizing the average membership degrees for Case 2 and minimizing the average membership degrees for Case 3. Eq. (2) shows the average membership degrees for Case 2.

$$\begin{aligned}
 \int_{m_i}^{m_r} h(x) dF_{\alpha}(x) &= \frac{\mu_{\alpha} - m_i}{2(m_c - m_i)} \left( \sqrt{1 - e^{-\frac{2(m_i - \mu_{\alpha})^2}{\pi\sigma_{\alpha}^2}}} - \sqrt{1 - e^{-\frac{2(m_r - \mu_{\alpha})^2}{\pi\sigma_{\alpha}^2}}} \right) \\
 &+ \frac{\sigma_{\alpha}}{\sqrt{2\pi}(m_c - m_i)} \left( e^{-\frac{(m_i - \mu_{\alpha})^2}{2\sigma_{\alpha}^2}} - e^{-\frac{(m_r - \mu_{\alpha})^2}{2\sigma_{\alpha}^2}} \right) \\
 &+ \frac{m_r - \mu_{\alpha}}{2(m_r - m_c)} \left( \sqrt{1 - e^{-\frac{2(m_i - \mu_{\alpha})^2}{\pi\sigma_{\alpha}^2}}} - \sqrt{1 - e^{-\frac{2(m_r - \mu_{\alpha})^2}{\pi\sigma_{\alpha}^2}}} \right) \\
 &- \frac{\sigma_{\alpha}}{\sqrt{2\pi}(m_r - m_c)} \left( e^{-\frac{(m_i - \mu_{\alpha})^2}{2\sigma_{\alpha}^2}} - e^{-\frac{(m_r - \mu_{\alpha})^2}{2\sigma_{\alpha}^2}} \right)
 \end{aligned} \quad (2)$$

Eq. (3) shows the average membership degree for Case 3.

$$\begin{aligned}
 \int_{m_i}^{m_r} h(x) dF_{\beta}(x) &= \frac{\mu_{\beta} - m_i}{2(m_c - m_i)} \left( \sqrt{1 - e^{-\frac{2(m_i - \mu_{\beta})^2}{\pi\sigma_{\beta}^2}}} + \sqrt{1 - e^{-\frac{2(m_r - \mu_{\beta})^2}{\pi\sigma_{\beta}^2}}} \right) \\
 &+ \frac{\sigma_{\beta}}{\sqrt{2\pi}(m_c - m_i)} \left( e^{-\frac{(m_i - \mu_{\beta})^2}{2\sigma_{\beta}^2}} - e^{-\frac{(m_r - \mu_{\beta})^2}{2\sigma_{\beta}^2}} \right) \\
 &+ \frac{m_r - \mu_{\beta}}{2(m_r - m_c)} \left( \sqrt{1 - e^{-\frac{2(m_i - \mu_{\beta})^2}{\pi\sigma_{\beta}^2}}} + \sqrt{1 - e^{-\frac{2(m_r - \mu_{\beta})^2}{\pi\sigma_{\beta}^2}}} \right) \\
 &- \frac{\sigma_{\beta}}{\sqrt{2\pi}(m_r - m_c)} \left( e^{-\frac{(m_i - \mu_{\beta})^2}{2\sigma_{\beta}^2}} - e^{-\frac{(m_r - \mu_{\beta})^2}{2\sigma_{\beta}^2}} \right)
 \end{aligned} \quad (3)$$

Membership functions can be optimized by maximizing Eq. (2) and minimizing Eq. (3).

### 2.2.2 Fuzzy Measure and Fuzzy Integral

A fuzzy measure  $g$  is a set function on  $X$  satisfying the following conditions:

$$g: 2^X \rightarrow [0,1] \quad (4)$$

$$(C1) \quad g(\emptyset) = 0$$

$$(C2) \quad g(X) = 1$$

$$(C3) \quad A \subset B \subset X \Rightarrow g(A) \leq g(B)$$

The fuzzy measure can cope with the following three interactions between the functions on sets A and B depending on the additivity of fuzzy measures (Wang and Klir, 1992).

(I1) No interaction between A and B.

(I2) Positive synergy between A and B.

(I3) Negative synergy between A and B.

Several fuzzy integrals have been proposed such as Sugeno's and Choquet integrals (Grabisch 2000). In this paper, the Choquet integral is used. The Choquet integral of a non-negative function  $h$  on  $X$  with respect to fuzzy measure  $g$  is defined:

$$(C) \int_X h(s) dg = \sum_{i=1}^n [h(s_i) - h(s_{i-1})] \cdot g(A_i) \quad (5)$$

where  $A_i = \{s_i, s_{i+1}, \dots, s_n\}$ ,  $h(s_i) = 0$  when  $i=0$ , and

the order of  $h(s_i)$ ,  $(1 \leq i \leq n-1)$  is assumed to be  $h(s_n) \leq h(s_{n-1}) \leq \dots \leq h(s_1)$ .

## 3 IDENTIFICATION OF FUZZY MEASURES

### 3.1 Fuzzy Measure based on Partial Correlation Coefficient

Several methods for identifying fuzzy measures have been proposed (Wang and Klir, 1992). However, they are difficult to apply to fault diagnosis of rotating machinery, since several parameters must be assigned experimentally before identification or they are difficult to differentiate the possibility of target fault from non-target faults. In this paper, the fuzzy measure based on partial correlation coefficients is defined. This fuzzy measure is the extension of the fuzzy measure defined by (Taya and Murofushi, 2006). In the definition,  $w_i$  ( $1 \leq i \leq n$ ) is called a weight of spectrum  $s_i$ .

$$g(A_i) = \omega_0 (\alpha_{A_i} + \beta_{A_i}), 1 \leq i \leq n, \quad A_i \subset X \quad (6)$$

$$\alpha_{A_i} = \sum_{t=1}^{n-i+1} w_t \quad w_i \in W$$

$$\beta_{A_i} = \sum_{r=1}^{n-i} \sum_{s=r+1}^{n-i+1} \frac{k_{r,s}}{1 - k_{r,s}} (w_r + w_s)$$

where coefficient  $k_{r,s}$  is the absolute value of partial correlation coefficient between spectra  $s_r$  and  $s_s$  (Siple, 2000). The set of weights of spectra are represented as follows, and given by skilled engineers.

$$W = \{w_1, w_2, \dots, w_n\}, \quad \sum_{i=1}^n w_i = 1$$

The membership degree of spectra are assumed to be

$$h(s_n) \leq h(s_{n-1}) \leq \dots \leq h(s_1)$$

without loss of generality. The factor  $\omega_0$  is determined from the following equation to satisfy the condition (C2) of fuzzy measure shown in 2.2.2.

$$g(A_1) = \omega_0 \left\{ \sum_{i=1}^n w_i + \sum_{r=1}^{n-1} \sum_{s=r+1}^n \frac{k_{r,s}}{1 - k_{r,s}} (w_r + w_s) \right\} = 1$$

Thus the factor is given by the following equation.

$$\omega_0 = \frac{1}{1 + \sum_{r=1}^{n-1} \sum_{s=r+1}^n \frac{k_{r,s}}{1 - k_{r,s}} (w_r + w_s)} \quad (7)$$

### 3.2 Fuzzy Integral and Variation of Possibilities

Fuzzy integral defined by Eq.(5) can be rewritten by Eq.(8) when fuzzy measure is additive, since every partial correlation coefficient is zero and the fuzzy measure  $g(A_i)$  is given by Eq.(9).

$$(C)\int_X h(s)dg = \sum_{i=1}^n w_i \bullet h(s_i) \tag{8}$$

$$k_{r,s} = 0(1 \leq r, s \leq n)$$

$$g(A_i) = \omega_0(\alpha_{A_i} + \beta_{A_i}) = \sum_{i=1}^{n-i+1} w_i, \quad 1 \leq i \leq n \tag{9}$$

$$\omega_0 = 1$$

$$\beta_{A_i} = 0$$

The above fuzzy integral is called a weighted average and has been commonly used for calculating the possibility of faults in fault diagnosis.

When fuzzy measure is not additive, fuzzy integral is represented by the following equation by substituting Eq.(7) to Eq.(6).

$$\begin{aligned} (C)\int_X h(s)dg &= \sum_{i=1}^{n-1} (h(s_i) - h(s_{i+1})) \bullet g(A_{n-i+1}) + h(s_n) \bullet g(A_1) \\ &= \sum_{i=2}^n h(s_i)(g(A_{n-i+1}) - g(A_{n-i+2})) + h(s_1) \bullet g(A_n) \tag{10} \\ &= \sum_{i=2}^n h(s_i) \bullet \omega_0 \bullet (w_i + \sum_{j=1}^{i-1} \frac{k_{i,j}}{1-k_{i,j}}(w_i + w_j)) \end{aligned}$$

In fault diagnosis, we can distinguish faults correctly when possibility of target fault(Case 2 in 2.2.1) is higher than the possibilities of non-target faults(Case 3 in 2.2.1). In order to evaluate the proposed fuzzy measure, we compare the difference of possibilities between target fault and non-target faults. The difference is obtained from the variation of fuzzy integral when membership degree of spectra changes.

The partial differential of the fuzzy integral with respect to membership degree  $h(s_i)$  of spectrum  $s_i$  is given by the following equation when fuzzy measure is additive.

$$\frac{\partial(C)\int_X h(s)dg}{\partial h(s_i)} = w_i \tag{11}$$

The total variation of fuzzy integral is given by the following equation from the above partial differential.

$$\sum_{i=2}^n \frac{\partial(C)\int_X h(s)dg}{\partial h(s_i)} = \sum_{i=2}^n w_i = 1 - w_1 \tag{12}$$

On the other hand, when fuzzy measure is not additive, the partial differential is given by the following equation.

$$\frac{\partial(C)\int_X h(s)dg}{\partial h(s_i)} = \frac{w_i + \sum_{j=1}^{i-1} \frac{k_{i,j}}{1-k_{i,j}}(w_i + w_j)}{1 + \sum_{r=1}^{n-1} \sum_{s=r+1}^n \frac{k_{r,s}}{1-k_{r,s}}(w_r + w_s)} \tag{13}$$

Thus the total variation is given by the following equation.

$$\begin{aligned} \sum_{i=2}^n \frac{\partial(C)\int_X h(s)dg}{\partial h(s_i)} &= \sum_{i=2}^n \left\{ w_i + \sum_{j=1}^{i-1} \frac{k_{i,j}}{1-k_{i,j}}(w_i + w_j) \right\} \\ &= \frac{\sum_{i=2}^n \left\{ w_i + \sum_{j=1}^{i-1} \frac{k_{i,j}}{1-k_{i,j}}(w_i + w_j) \right\}}{1 + \sum_{r=1}^{n-1} \sum_{s=r+1}^n \frac{k_{r,s}}{1-k_{r,s}}(w_r + w_s)} \tag{14} \\ &= \frac{1 - w_1 + \sum_{i=2}^n \sum_{j=1}^{i-1} \frac{k_{i,j}}{1-k_{i,j}}(w_i + w_j)}{1 + \sum_{r=1}^{n-1} \sum_{s=r+1}^n \frac{k_{r,s}}{1-k_{r,s}}(w_r + w_s)} \end{aligned}$$

We evaluate the proposed fuzzy measure by comparing the above two total variations. The ratio of the above two variations is given by the following equation and is called the ratio of improvement.

$$\begin{aligned} K &= \frac{\sum_{i=2}^n \frac{\partial(C)\int_X h(s)dg}{\partial h(s_i)}}{\sum_{i=2}^n w_i} = \frac{1}{1 - w_1} \cdot \frac{1 - w_1 + \sum_{i=2}^n \sum_{j=1}^{i-1} \frac{k_{i,j}}{1-k_{i,j}}(w_i + w_j)}{1 + \sum_{r=1}^{n-1} \sum_{s=r+1}^n \frac{k_{r,s}}{1-k_{r,s}}(w_r + w_s)} \\ &= \frac{1 + \frac{1}{1 - w_1} \sum_{i=2}^n \sum_{j=1}^{i-1} \frac{k_{i,j}}{1-k_{i,j}}(w_i + w_j)}{1 + \sum_{r=1}^{n-1} \sum_{s=r+1}^n \frac{k_{r,s}}{1-k_{r,s}}(w_r + w_s)} \tag{15} \end{aligned}$$

The ratio of improvement shows that when proposed fuzzy measure is used, we can expect K times larger difference of possibilities between target fault (Case 2) and non-target faults (Case 3) than weighted average is used.

## 4 EXAMPLE OF DIAGNOSIS

In this example, we compare the possibility of looseness fault (target fault) with the possibilities of imbalance and misalignment faults (non-target

faults) using field data.

The spectra used to diagnose looseness fault are  $s_{1N}$ ,  $s_{2N}$ , and  $s_{3MN}$ , where 1N, 2N, and 3MN are fundamental frequency, second harmonics and over third harmonics, respectively. The weights of spectra given by skilled engineers are shown in Table 2.

Table 2: Weights of spectra.

$w_{1N}$	$w_{2N}$	$w_{3MN}$
0.6	0.2	0.2

The membership degrees of spectra of field data for three faults obtained from the optimized membership functions are shown in Table 3.

Table 3: Membership degrees of spectra.

Fault	$h(s_{1N})$	$h(s_{2N})$	$h(s_{3MN})$
Looseness	0.864	0.809	0.790
Imbalance	0.839	0.810	0.084
Misalignment	0.875	0.534	0.029

The partial correlation coefficients between two spectra of field data for looseness fault are shown in Table 4.

Table 4: Partial correlation coefficients.

$k_{1N,2N}$	$k_{1N,3MN}$	$k_{2N,3MN}$
0.572	0.843	0.330

The ratio of improvement given by Eq.(15) is obtained as follows:

$$K = (\xi_1 \cdot \xi_2 + 1) / \xi_3 \tag{16}$$

Where

$$\xi_1 = \frac{1}{1 - w_{1N}}$$

$$\xi_2 = \frac{k_{1N,2N}}{1 - k_{1N,2N}}(w_{1N} + w_{2N}) + \frac{k_{2N,3MN}}{1 - k_{2N,3MN}}(w_{2N} + w_{3MN}) + \frac{k_{1N,3MN}}{1 - k_{1N,3MN}}(w_{1N} + w_{3MN})$$

$$\xi_3 = 1 + \frac{k_{1N,2N}}{1 - k_{1N,2N}}(w_{1N} + w_{2N}) + \frac{k_{2N,3MN}}{1 - k_{2N,3MN}}(w_{2N} + w_{3MN}) + \frac{k_{1N,3MN}}{1 - k_{1N,3MN}}(w_{1N} + w_{3MN})$$

The ratio is calculated as  $K = 2.27$  by using the values in the above tables.

The possibilities for field data obtained from the fuzzy integral and from weighted average are shown in the following figure.

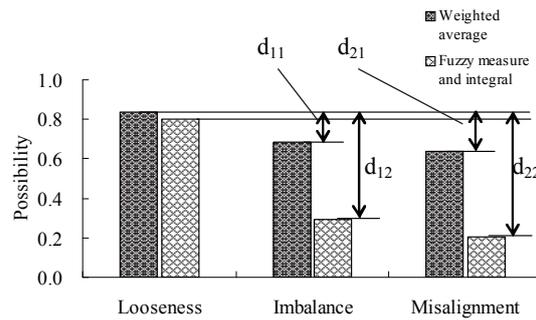


Figure 3: Possibility of faults.

From the figure, we can see that the ratio of improvement ( $d_{12}/d_{11}$ ) is 3.25 when imbalance data is used, and the ratio ( $d_{22}/d_{21}$ ) is 2.97 when misalignment data is used. These values are larger than that of the ratio of improvement given by Eq.(16). We can say that the possibility of target fault (Looseness) is more differentiated than that of non-target fault (Imbalance and Misalignment).

## 5 CONCLUSIONS

Herein, an identification method of fuzzy measure for diagnosing faults in rotating machinery is proposed. The fuzzy measures are determined by using partial correlation coefficients between vibration spectra and the weights of spectra given by skilled engineers.

The fuzzy measure is evaluated by comparing the possibility obtained by using the proposed fuzzy measure and the possibility obtained by using weighted average. The ratio of improvement K is introduced to compare the difference of the above two possibilities, and the equation for K is derived using the partial differentials of fuzzy integral.

The evaluation is also made using field data. The results show that the ratio of improvement obtained from field data are around three and higher than the value obtained from the equation in the paper.

In future work, we will improve the accuracy of the ratio of improvement, and apply this method to other fault diagnoses and evaluate the method using extensive field data.

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