Context-aware Recommendation using Fuzzy Formal Concept Analysis

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Abstract: Most of the recommender systems are content-based: they provide the user a subset of items close to his interest by using the item features. In real recommender systems, the main problem is the big amount of items to be treated. In this work we propose to incorporate context information in a uniform way. We use fuzzy logic and formal concept analysis as a framework to combine context information and content-based recommender systems. Concretely, we specify the content by using fuzzy relations, the context by using fuzzy implications and Simplification Logic to develop an intelligent and linear pre-filtering process. We illustrate this method with an application to the tourism sector.

1 INTRODUCTION

In recent years, the use of recommender systems has become popular in many different applications to offer a personalized selection of products. The big amount of items to be recommended causes that in many cases users feel overwhelmed because they have to select from a wide range of alternatives (Lymberopoulos et al., 2011). In this work we focus on tourism recommender systems, that should implement filtering mechanisms to provide a set of points of interest (POIs) which are accurately adjusted to the real needs of the tourist.

(Leiva et al., 2012) presented a classification of the types of most commonly used recommender systems:

- Collaborative: it provides results obtained from the qualifications made by users. The user will be recommended items that people with similar tastes and preferences liked in the past.
- Content-based: it categorizes items and suggests products that have similar characteristics to those requested by the user or to those that he evaluated positively in the past.
- Demographics: it classifies users by different personal parameters, and recommendations are made taking into account the demographic group to which the user belongs.
- Knowledge-based: it has information about how an item satisfies a user, and establishes a relationship between need and recommendation.
- Utility-based: it recommends those items that maximize an utility function.
- Case-based: it uses information about resolving problems (cases) previous to the resolution of the present case. They can be viewed as a subtype of the knowledge-based and utility-based recommender systems.

In order to be used in tourism systems, a significant problem detected in previous models is not using context attributes (Adomavicius et al., 2010). The context is a multifaceted concept that has been studied in different disciplines, including Computer Science (mainly in Artificial Intelligence), Cognitive Science, Linguistics, Psychology and Organizational Science (Bazire and Brézillon, 2005). In order to improve the quality of recommendations, the system should not only use the qualifications and characteristics of different POIs, or tourist preferences. Systems need to handle information of different nature such as weather, company, schedules, location, time, etc. (Adomavicius and Tuzhilin, 2011). Some authors include the user's emotional status and expand the definition to any information that can be characterized and that is relevant to the interaction between a user and an application (Dey and Abowd, 2001).

The types of recommender systems (described above) that only consider items and users are called recommender systems in two dimensions (Adomavicius et al., 2010).

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Therefore, in order to improve the recommendations, we have to take into account the contextual information available as additional categories of data (Leiva et al., 2012). In (Adomavicius and Tuzhilin, 2011) the authors affirm that the recommender system should take into consideration three dimensions (users, items and context). They propose different paradigms of context-aware recommender systems:

- Contextual pre-filtering (or contextualization of recommendation input): contextual information drives data selection or data construction for that specific context. The selected data will be the input of a 2D recommender system.
- Contextual post-filtering (or contextualization of recommendation output): the ratings are predicted using any traditional 2D recommender system on the entire data. Afterwards, the resulting set of recommendations is adjusted (contextualized) for each user using the contextual information.
- Contextual modeling (or contextualization of recommendation functions). In this recommendation paradigm, contextual information is used directly in the modeling technique as part of rating estimation.

In our opinion, a recommender system for a consolidated tourist destination (probably with thousands of POIs) should apply the contextual pre-filtering paradigm. Thus, the recommender system works with a reduced number of POIs, decreasing the execution time. Another important advantage of this approach is that it can be combined with any existing 2D recommendation techniques.

The recommender system proposed in this paper uses a content-based contextual pre-filtering, based on contextual attributes and desirable characteristics of the POIs. Therefore, it is not necessary to have information about previous visits or qualifications of other tourists.

Some authors (Zenebe and Norcio, 2009) propose the use of fuzzy logic as a formal basis for recommender systems. Nevertheless we are looking for a new approach which allows us to also cover another question proposed in (Adomavicius and Tuzhilin, 2005): incorporation of diverse contextual information into the recommendation process. In this paper we tackle this issue by means of the Formal Concept Analysis (FCA).

From the point of view of Philosophy, a *concept* is a general idea that corresponds to some kind of entity and that may be characterized by some essential features of the class. When B. Ganter and R. Wille (Wille, 1982; Ganter and Wille, 1999) conceive a framework inside the lattice theory to *formalize concepts*, they probably do not guess the wide diffusion of their original work.

Nowadays, FCA has become an useful framework both in the theoretical and in the applied areas. The works related to FCA cover from data analysis, information retrieval, knowledge representation, etc. It is considered an outstanding tool in emergent environments like data mining, semantic web, etc.

The main goal of Formal Concept Analysis (FCA) is to identify in a binary table the relationships between set of objects and set of attributes. These relationships establish a Gallois Connection which allows us to identify the concepts using a formal framework inside the lattice theory. Apart from building the concept lattice itself, one of the key problems is to extract the set of attribute implications which hold in the concept lattice. Implications constitute important information that is extracted in a separate stage from data and constitute a dual representation of the lattice itself. One of the most important advantages in the use of implications is that they may be managed using Functional Dependencies Logics (Armstrong, 1974).

Another novelty in this work is the integration of the context into the FCA method by means of set of implications. We propose the generation of a set of fuzzy implications which corresponds with a given context. Thus, when the user identifies his/her context (company, weather, etc), the system enriches the specification by adding a set of new implications which corresponds with this context. The new information is treated with our fuzzy logic to automatically reduce the specification by removing redundancy. The reduction in the set of implications allows a more efficient validation process which prune the original set of POIs, and therefore the content-based 2D recommender works with a smaller set of POIs. In figure 1 the system architecture of our proposal is depicted.

The paper is organized as follows: in the next section we analyze some related works. Section 3 introduces the theoretical background of our work and Section 4 describes an executable logic to manage fuzzy implications, named $\mathbb{FSL}$. It will be used in section 5 to introduce a context-aware recommender system with a solid base. Finally some conclusions and future works are presented.

2 RELATED WORKS

In (Zenebe and Norcio, 2009) fuzzy logic is presented as a proper framework for tourist recommenders, addressing the problems described in (Adomavicius and Tuzhilin, 2005). Particularly, their approach uses fea-

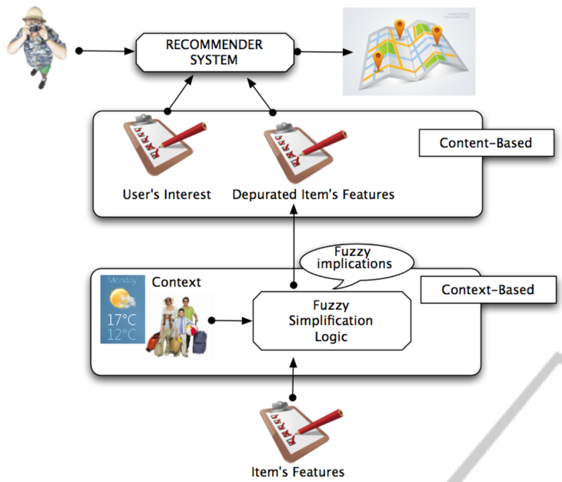

Figure 1: Context-based recommender System.

tures of items as background data and users feedback such as ratings of items as input. That paper provides a solid and well-founded method to incorporate the subjectiveness, imprecision and vagueness that usually appear in items features and user feedback. One outstanding result of the paper is that, despite of the flexible and enriched language to specify user interest and item features, they develop a method to infer recommendations which shows an improvement in precision without loss of recall.

Some authors have used FCA methods as a interesting approach in recommender systems. In (du Boucher-Ryan and Bridge, 2006) the authors propose FCA as an approach to group items and users into concepts. That work may be considered a collaborative recommender system and it shows how FCA may be used to find neighbours in a efficient and accurate way. A similar and recent approach to the same problem with similar results may be found in (Li and Murata, 2010). These works shows that FCA may be successfully used in collaborative recommenders.

In this paper we work in this line and enrich the previous results in some points. First, we aim to add a more flexible specification by considering fuzzy relations in FCA. This extension was first introduced in (Belohlavek, 1999). The problems that arise are related with the development of new methods to infer the concepts and manage implications in fuzzy relations. We apply our previous theoretical results presented in (Belohlavek et al., 2012) to provide a sound and complete fuzzy logic for functional dependencies as a framework for the efficient management of implications.

3 BACKGROUND AND THEORETICAL FRAMEWORK

We incorporate to the specification some degree of imprecision and uncertainty by means of Fuzzy Logic. Since Lofti Zadeh introduced Fuzzy Set Theory, the most usual approach is to replace the truthfulness value set $\{0,1\}$ (false and true) for an arbitrary residuated lattice. Our proposal uses an extension of the residuated lattice, specifically $([0,1],\vee,\wedge,0,1,\otimes,\rightarrow,^*,\smallsetminus)$ in which the unit interval $[0,1]$ is endowed with the following operations: the infimum (denoted by $\wedge$) that plays the role of universal quantifier, the supremum ($\vee$) as the existential quantifier, an arbitrary left-continuous t-norm ($\otimes$) as the conjunction, the residuum defined by $a \rightarrow b = \sup\{x \in [0,1] \mid x \otimes a \leq b\}$, a truth-stressing hedge operation * and the difference $\smallsetminus$. The most used t-norms are (standard) product, Łukasiewitz product and Gödel product. Truth-stressing hedges (Hájek, 2001) are a class of unary truth functions that captures the semantic of the "very true" notion. The two boundary cases of hedges are identity and so-called globalization (i.e. $1^* = 1$ and $x^* = 0$ for all $1 \neq x \in L$). Finally, the difference operation is given by: $x \smallsetminus y = x$ if $y < x$ and 0 otherwise.

A fuzzy set in an universal set U is a mapping $A\colon U \rightarrow [0,1]$ and the set operations are defined pointwise as follows: for $A,B\colon U \rightarrow [0,1]$, for all $u \in U$, $(A\cup B)(u) = A(u) \vee B(u)$, $(A\cap B)(u) = A(u)\wedge B(u)$, $(A\otimes B)(u) = A(u)\otimes B(u)$, $(A \rightarrow B)(u) = A(u) \rightarrow B(u)$, $(A\smallsetminus B)(u) = A(u)\smallsetminus B(u)$, and $A^*(u) = (A(u))^*$. Moreover, $\varnothing$ and U are the fuzzy sets in which, for all $u \in U$, $\varnothing(u) = 0$ and $U(u) = 1$.

The set inclusion can be extended as follows: for $A,B\colon U \rightarrow [0,1]$, the grade in which A is a subset of B is

$$S(A,B) = \bigwedge_{u\in U} (A(u) \rightarrow B(u))$$

Particularly, if $S(A,B) = 1$ we write $A \subseteq B$ and, in this case, $A(u) \leq B(u)$ for all $u \in U$.

We are going to work with finite fuzzy sets, that is, fuzzy sets in which at most a finite number of elements has non-zero values. In the notation that we are going to use, zero-valued elements does not appear and grade 1 is omitted. So, for example, $A = \{b/_{0.4}, d/_{0.1}, f\}$ denotes that $A(b) = 0.4$, $A(d) = 0.1$, $A(f) = 1$ and $A(x) = 0$ otherwise.

As we have presented before, we organize the information of the recommender system using the fuzzy extension of Formal Concept Analysis (FCA) introduced in (Belohlavek, 1999) that may be consider the most current trend in this area.

The starting point in fuzzy FCA is the fuzzy relation[1] that captures the degree in which a given attribute holds on an object. Specifically, given a finite set of objects X and a finite set of attributes Y, fuzzy FCA extracts knowledge from a fuzzy relation $I\colon X \times Y \to [0,1]$ where $I(x,y) = \vartheta$ means that ϑ is the degree in which the object x has the attribute y. Usually, the fuzzy relation I is showed in a table in which rows represents objects, columns corresponds to attributes and in position (x,y) on the table appears the degree $I(x,y)$.

An important information that can be extracted from the fuzzy relation is given in terms of attribute implications. They are formulas of the form $A \Rightarrow B$ where A and B are fuzzy sets of attributes. The grade in which this attribute implication is satisfied by a fuzzy relation I is given by

$$||A \Rightarrow B||_I = \bigwedge_{x\in X} (S(A,I_x)^* \to S(B,I_x))$$

where I_x denotes the fuzzy set in which $I_x(y) = I(x,y)$ for all $y \in Y$. So, for example,

$$\left\{b/_{0.2}, d\right\} \Rightarrow \left\{c/_{0.8}\right\}$$

means that every object that has attribute b to degree at least 0.2 and attribute d to degree 1, has attribute c to degree at least 0.8.

Observe that the left and right hand side of the implications (the A and B sets) may be empty. If B if the empty set, the implication captures an information which always valid and it has not to be considered in the inference process. Nevertheless, if the A set is empty the implication provides a relevant information, particularly in the application we are working with. For instance, the implication $\varnothing \Rightarrow \left\{c/_{0.8}\right\}$ is interpreted as follows: the c attribute must have a degree at least 0.8.

Given a fuzzy relation I, $A \overset{\vartheta}{\Longrightarrow} B$ denotes that the implication $A \Rightarrow B$ holds to degree at least ϑ and it is equivalent to ensure that $A \Rightarrow \vartheta \otimes B$ is satisfied to degree 1 (see (Belohlavek et al., 2012) for further details). We use $\vartheta \otimes B$ to denote so-called ϑ-multiple of B which is a fuzzy set such that $(\vartheta \otimes B)(y) = \vartheta \otimes B(y)$ for all $y \in Y$. The cited result ensures that the user may specify implications with degrees and they can be translated to implications without degrees to get a simpler management in the automated methods.

[1]In FCA literature, this fuzzy relation is usually called "context" but we omit this denomination to avoid confusion with the term context used in recommender systems.

4 A LOGIC FOR THE MANAGEMENT OF FUZZY ATTRIBUTE IMPLICATIONS

For the management of the information given in terms of implications, an automated method to infer new information from a set of implications is required. Although there exist in the literature several axiomatic systems, they have not been developed for the design of such automated methods. In (Belohlavek et al., 2012), the authors presented the Fuzzy Attribute Simplification Logic, $\mathbb{FSL}$, providing a sound and complete axiomatic system for reasoning with implications and, for first time, an automated deduction method. The system consists of three deduction rules,

[Ax] $\vdash AB \Rightarrow A$ (Axiom)

[Sim] $A \Rightarrow B, C \Rightarrow D \vdash A(C \smallsetminus B) \Rightarrow D$ (Simplification)

[Mul] $A \Rightarrow B \vdash \vartheta^* \otimes A \Rightarrow \vartheta^* \otimes B$ (Multiplication)

where A,B,C,D are fuzzy sets and $\vartheta \in [0,1]$. Here, we use the convention of writing AB instead of $A \cup B$.

The most relevant characteristic of this axiomatic system is that the inference rules can be seen as equivalence rules that allows us to remove (simplify) redundant information. This simplification is applied both to implications and to attributes inside of implications. It is specially appropriated for heterogeneous environment where different users provide several implications which may cause specifications with a hide degree of redundancy. These equivalencies are the following

1. $\{A \Rightarrow B\} \equiv \{A \Rightarrow B \smallsetminus A\}$;
2. $\{A \Rightarrow B, A \Rightarrow C\} \equiv \{A \Rightarrow BC\}$;
3. $\{A \Rightarrow B, C \Rightarrow D\} \equiv \{A \Rightarrow B, A(C \smallsetminus B) \Rightarrow D \smallsetminus B\}$ when $A \subseteq C$.

where A, B, C and D are fuzzy sets.

5 APPLICATION OF $\mathbb{FSL}$ TO A CONTEXT RECOMENDER SYSTEM

Up to now, we have presented all the theoretical foundations that we combine to incorporate the context into a recommender system. Our proposal is based on fuzzy attribute implications and it has the following characteristics:

- Fuzzy Logic and fuzzy multivalued FCA have been shown to be sound formalisms to specify and reasoning with uncertainty.

- We propose a unified combination of context-based reasoning inside a content-based recommendation framework.
- We associate each context with a set of fuzzy implications defined on the items characteristics.
- The user introduces in the system all the characteristics of his/her context and then the reasoning methods depurate the set of all associated implications to get an equivalent and simpler set of fuzzy implications.
- The final set of implications is used to validate the items to be recommended. Thus, the original set of items is pre-filtered and only a subset of them will be the input of the recommender system.

Now, we detail how context information is represented by means of fuzzy implications. POIs are represented by a set of attributes which describes its features (it is cheap or expensive, its atmosphere is romantic or cheerful, etc.). Real world data are often complex and difficult to be labelled with a binary domain without loss of information. For instance a restaurant may have a very beautiful garden with some tables where children may enjoy and also an intimate hall with quiet music for a romantic dinner. We propose to store these items features by using a fuzzy relation, as example 1 shows. Thus, each row corresponds with an object and each column with an attribute.

Example 1. *We consider a group of POIs of a tourism destination with some attributes to describe them (Design, Atmosphere, Price and Facilities). Each attribute has a set of finite possible values and let us suppose that a destination expert manages the system by giving a degree to each value in the domain. Thus, we get a table of objects with grades by flattening the information to obtain a fuzzy relation (see table 1).*

The context of the system is represented by a set of discrete domains $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$. Each domain is associated with a dimension of the context (for instance weather, company, time of the day, etc.) and it has a finite set of values: $\mathcal{C}_i = \{v_1^i, \ldots, v_n^i\}$. We define the user context, named *state*, as a n-tuple of pairs *(value of the domain, degree)*.

Example 2. *Let Weather, Company and Time of the day be three context dimensions with the following domains: Weather=*{*hot, warn, cloudy, rainy*}*, Company=*{*alone, friends, couple, family, large group*} *and Time=*{*morning, afternoon, evening, night*}*. A user may specify his context by means of the state: [(Hot, 0.8), (afternoon,0.8), (family,0.7)].*

We define a context segment to be an specific value of a domain and its associated degree. We provide a framework where each context segment is associated with a set of fuzzy implications. As we presented in section 3, implications may be labelled with a degree to express the truthfulness of the implication itself. Thus, the degree of the context segment is inherited by all its implications.

Example 3. *The implications associated with each context segment are introduced as follows (observe that the degree of the context is transferred to the implication):*

- *Context segment: Hot/$_{0.8}$. Implications:*
  Expensive/$_{0.8}$,ClosedSpace/$_{0.8}$ $\stackrel{0.8}{\Rightarrow}$ Air Cond./$_{0.8}$, Views/$_{0.9}$, Picturesque/$_{0.2}$
  OpenSpace/$_{0.8}$ $\stackrel{0.8}{\Rightarrow}$ Inexpensive/$_{0.7}$
- *Context segment: Afternoon/$_{0.8}$. Implications:*
  OpenSpace/$_{0.8}$ $\stackrel{0.8}{\Rightarrow}$ Terrace/$_{0.6}$, Inexpensive/$_{0.9}$
- *Context segment: Family/$_{0.7}$. Implications:*
  $\emptyset \stackrel{0.7}{\Rightarrow}$ Inexpensive/$_{0.6}$
  ClosedSpace/$_{0.8}$ $\stackrel{0.7}{\Rightarrow}$ Air Cond./$_{0.9}$

As we mention in Section 4, the implication $\emptyset \stackrel{0.7}{\Rightarrow}$ Inexpensive/$_{0.6}$ in the last context segment indicates that if the user is accompanied by his family with degree 0.7 *then the restaurant need to be inexpensive with a degree greater than* 0.6*.*

When a system manages a certain amount of information, we have to provide an automatic way to analyze and extract the important information to reduce the computation cost. In our approach we propose to use the automatic methods developed over $\mathbb{FSL}$ to depurate the specification of the context and obtain a canonical set of implications. There is a lot of works related with the search of basis in FCA. An up to date and complete work is (Bertet and Monjardet, 2010) where the authors identify a set of properties that may be cover by different basis definitions (minimal, direct, canonical, etc). These characteristics may be combined providing a diferent notion of basis. The work of Bertet and Monjardet is focussed on crisp FCA and it is still an open problem the definition of suitable definitions for fuzzy implications basis for fuzzy FCA.

Nevertheless, as example 4 shows, it is possible to illustrate the benefits of using $\mathbb{FSL}$ to get an equivalent and simpler set of implications.

Example 4. *From the specification of the above example, if we have that the context provided by the user is* {*Hot/$_{0.8}$, Afternoon/$_{0.8}$, Family/$_{0.7}$*} *then the set of implication is built by adding all the above implication in a unified set:*

Table 1: FCA representation of POIs.

	Design		Atmosphere			Price			Facilities		
	OpenSpace	ClosedSpace	Quiet	Lively	Picturesque	Inexpensive	Moderate	Expensive	Air Cond.	Views	Terrace
Standard Restaurant	0.3	0.8	0.8	0.5	0.2	0.7	0.3	0.3	0.3	0.3	0.3
Michelin Star	0.1	0.8	0.9	0.2	0.1	0	0.1	0.9	0.9	0.5	0.1
Burger	0.3	0.8	0.3	0.8	0.1	0.9	0.3	0.1	0.8	0.1	0.4
Tapas Bar	0.3	0.8	0.2	0.8	0.9	0.9	0.5	0.1	0.5	0.1	0.1
Pizzeria	0.1	0.9	0.3	0.8	0.7	0.9	0.5	0.3	0.8	0.3	0.5
Beach Fresh Fish	0.9	0.2	0.3	0.8	0.8	0.5	0.7	0.8	0.3	0.9	0.9

$\{$*Expensive/*$_{0.8}$,*ClosedSpace/*$_{0.8} \stackrel{0.8}{\Rightarrow}$ *Air Cond./*$_{0.8}$, *Views/*$_{0.9}$, *Picturesque/*$_{0.2}$;

OpenSpace/$_{0.8} \stackrel{0.8}{\Rightarrow}$ *Inexpensive/*$_{0.7}$;

OpenSpace/$_{0.8} \stackrel{0.8}{\Rightarrow}$ *Terrace/*$_{0.6}$, *Inexpensive/*$_{0.9}$;

$\emptyset \stackrel{0.7}{\Rightarrow}$ *Inexpensive/*$_{0.6}$;

ClosedSpace/$_{0.8} \stackrel{0.7}{\Rightarrow}$ *Air Cond./*$_{0.9}$ $\}$

Using the inference rules of $\mathbb{FSL}$ *we may remove redundant information and we obtain an equivalent and simpler set of implications:*

$\{$*expensive/*$_{0.8}$,*ClosedSpace/*$_{0.8} \stackrel{0.8}{\Rightarrow}$ *Views/*$_{0.9}$, *Picturesque/*$_{0.2}$;

OpenSpace/$_{0.8} \stackrel{0.8}{\Rightarrow}$ *Terrace/*$_{0.6}$; $\emptyset \stackrel{0.7}{\Rightarrow}$ *Inexpensive/*$_{0.6}$;

ClosedSpace/$_{0.8} \stackrel{0.7}{\Rightarrow}$ *Air Cond./*$_{0.9}$ $\}$

It should be noted that the redundancy removal algorithm has a quadratic complexity with respect to the number of implications. This number is much lower than the number of POIs (usually several thousands) in any touristic destination. Finally, our system make use of the information associated with the user context, provided by the unified and depurated set of implications, to stretch the set of POIs to be recommended to the user. For each POI in the FCA table, we validate the set of implications, removing all the POIS that does not satisfied them. The complexity of this last step is $O(n)$ where n is the number of POIs. This way, we have designed a linear contextual pre-filtering process.

Example 5. *As in example 4, if the user context is the afternoon of a hot day, traveling with his family, our contextual pre-filtering process reduces the list of restaurants of table 1 to Burger and Pizzeria, since:*

- *Michelin star does not satisfy Expensive/*$_{0.8}$,*ClosedSpace/*$_{0.8} \stackrel{0.8}{\Rightarrow}$ *Views/*$_{0.9}$, *Picturesque/*$_{0.2}$
- *Beach Fresh Fish does not satisfy* $\emptyset \stackrel{0.7}{\Rightarrow}$ *Inexpensive/*$_{0.6}$
- *Standard restaurant and Tapas bar do not satisfy ClosedSpace/*$_{0.8} \stackrel{0.7}{\Rightarrow}$ *Air Cond.*$_{0.9}$

This way, the set of POIs to be managed by the content-based recommender is significatively reduced.

6 CONCLUSIONS AND FUTURE WORKS

Content-based recommender systems may be significatively improved by including contextual information. To achieve this goal, we use fuzzy logic and formal concept analysis as a solid framework to combine context information and content-based recommenders. More specifically, we use Simplification Logic to develop an intelligent and linear pre-filtering process. This process generates a set of implications which captures the context information and that it is used to validate the items to be recommended. The method is applied in two steps: in the first one we translate the context information provided by a user as an *state*, i.e. a simplified set of fuzzy implications, and in the second step, the implications are used to filter the items which fulfills them.

This work may be extended by considering two future works related with the two steps of the pre-filtering process. First, the implications induced by the context may be enriched with implications automatically extracted from the user interests stored in the content. We propose to use formal concept analysis to extract this information. As a second trend, we propose to substitute the recommender algorithms by formal concept analysis techniques.

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