# A PSO/Snake Hybrid Algorithm for Determining Differential Rotation of Coronal Bright Points

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Keywords: Particle Swarm Optimization, Snake Model, Hybrid Algorithm, Image Processing, Tracking, Solar Images.

Abstract:

Particle swarm optimization (PSO) algorithm is a successful general problem solver, thanks to its computationally inexpensive mechanisms. On the other hand, snake model is a specialized image processing algorithm widely used in applications such as boundary delineation, image segmentation, and object tracking. In this paper we discuss the suitability of a hybrid PSO/Snake algorithm for determining the differential rotation of the Sun's coronal bright points. In the Snake/PSO hybrid algorithm each particle in the population represents only a portion of the solution and the whole population altogether will converge to the final complete solution. In this model a one-to-one relation between Snake model snaxels and PSO particles have been created and PSO's evolution equations have been modified with snake model concepts. This hybrid model is tested for tracking the coronal bright points (CBPs) along time, on a set of full-disc solar images obtained with the Atmospheric Imaging Assembly (AIA) instrument, onboard the Solar Dynamics Observatory (SDO) satellite. The algorithm results are then used for determining the differential rotation of CBPs. These final results are compared with those already reported in the literature, to assess the versatility of the PSO/Snake hybrid approach.

#### 1 INTRODUCTION

Particle swarm optimization (PSO), first introduced by (Kennedy and Eberhart, 1995), has become very popular as a general search strategy. It is computationally inexpensive to implement and yet it demonstrates a fast and reasonable performance. It is a stochastic algorithm based on the analogy of collective behavior of birds' swarms. PSO consists of a population of particles, each similar to a bird searching for the best place to find food. Each particle in PSO is a candidate solution. In PSO, particles are governed under their cognitive and social behaviors. These mechanisms make it possible for particles to communicate and diffuse their experience of explored space, and finally converge towards the optimum of search space, which is the solution to the formulated problem.

Image segmentation is one of the frequently addressed issues in digital image processing. Deformable contour was first used for object boundary delineation in the late 80s and its variations have been popular approaches ever since. Kass et al (1988) introduced a new kind of deformable contour called active contour model

(ACM), which deforms contours to lock onto features of interest within an image. Active contour model is also known as snake model, since the evolution of contour resembles snakes crawling. Snakes are widely used as an interesting approach in many applications, including image segmentation, stereo matching and object tracking (Ballerini and Bocchi 2003; Tsechpenakis et al., 2004; Kass et al., 1988).

Snake model is an energy minimization algorithm induced not only by low level image features such as image gradient or image intensity, but also with higher level information such as object shape, continuity of the contour and user interaction. Given an approximation of the object boundary, the snake model will be able to find the precise boundary of that object (Ballerini 1999; Ballerini and Bocchi, 2003). Snake model is in essence an optimization algorithm. Original snake model achieves this minimization by iteratively solving a pair of Euler equations on the discrete grid. Traditional active contour algorithms suffer from some limitations. One main drawback is the sensitivity of the initial contour; it must be within the vicinity of object boundary. An improper initialization may lead the algorithm to fail. A solution to this problem is to expand the search domain or increase the capture range of image force to guide the contour evolution (Leroy et al., 1996; Park et al., 2001; Cohen and Cohen, 1993). Another limitation is that it can hardly converge on concave object boundaries (Bresson et al., 2007; Davatzikos and Prince, 1994). There are a number of other problems associated with classical active contours such as sensitivity to noise, convergence to local minima, parameter selection and instability (Amini et al., 1988; Bresson et al., 2007; Mun et al., 2004). Several works addressed these problems but very few have produced satisfactory results. They either cannot address all the problems or if they can, they usually raise new drawbacks and impose further complexity and computational cost to the model. One successful approach is to minimize the snake energy by alternate strategies such as dynamic programming (Amini et al., 1988), greedy algorithms (Lam and Yan, 1994), genetic algorithms (Ballerini, 1999; Ballerini and Bocchi 2003; Mun et al., 2004) and swarm based optimization algorithms (Zeng and Zhou 2008; Nebti 2009; Tseng et al., 2009; Li et al., 2009; Shahamatnia and Ebadzadeh, 2011; Asl, 2006).

PSO can be applied to drive control points on the snake, but particles must be prevented from convergence to the global best position experienced by the swarm. In the literature this is done by restraining the particle search space. (Tseng et al., 2009; Li et al. 2009) use multi-population PSO in which each control point is confined to a sub-swarm spatially distinct from other sub-swarms. In (Ballerini, 1999; Nebti, 2009), polar coordinate system is used to restrict the search space of each contour control point. In (Zeng and Zhou, 2008) an iterative method has been used to rank the best position set of particles at each epoch and according to some equations prevent particles from intersecting.

Most of the aforementioned methods act only as a general problem solver and take the approach of formulating the snake model calculations as a minimization problem and then just solving this optimization problem. In this paper, we take the hybrid PSO/Snake approach introduced in (Shahamatnia and Ebadzadeh, 2011) and show its versatility by further extending it to solve a real world problem from astrophysics domain. The method presented here customizes PSO algorithm to overcome snake model drawbacks including snake initialization, concave boundaries, sensitivity to noise and local minima. Yet, the simple structure of

PSO is preserved yielding to an algorithm with low order of complexity and hence good processing time. These factors are of utmost importance for precisely calculating the differential rotation of solar features.

Specifying the exact nature of the differential rotation of both the solar surface and the solar interior is a very important issue of solar physics. The solar surface rotates differentially. However, the differential rotation (DR) mechanism, most likely caused by interactions between convection and overall rotation, is not exactly known. DR plays an important role in generating solar activity (SA) every manifestation of SA is related to changes in the local magnetic field and local changes of the differential rotation. Rotational irregularities may also serve as indicators of hypothetical processes, going on beneath the solar surface. One example could be the location of a layer where rotational speed changes abruptly (the so-called jet stream). Sometimes it is called a layer of torsional oscillation (Ulrich and Boyden, 2005). The location of this layer (its heliographic latitude) is likely related with a phase of the solar activity cycle, therefore it would be rather useful to have a tool for regular determination of its location. During a series of consecutive days or within an interval of a few days, we plan also to trace a location of a jet stream in images obtained by the Atmospheric Imaging Assembly (AIA) instrument on board the space Solar Dynamics Observatory (SDO).

Coronal bright points (CBPs) or bright points, are small and bright structures observed in the extreme ultraviolet (EUV) and the X-ray part of the solar spectrum (Brajsa et al., 2001). They are known to have a mean lifetime of about 8 hours, a typical maximum area of  $2\times10^8$  Km<sup>2</sup>, but still they look like a tiny shape on the solar images. Figure 1 illustrates several CBPs. Bright points are associated with bipolar magnetic features and a large quantity of them (several thousands) emerge over the surface of the Sun per day and thereby in total they bring up huge magnetic fluxes. Precisely tracking the coronal bright points over extended periods of time will help solar physicists and space weather scientists to better understand this important solar feature. Such automatic tools will allow solar researchers to precisely process large amount of solar data and hence improve solar models. The aim of this paper is to present the result of applying a hybrid PSO/Snake algorithm for tracking coronal bright points. The result of tracking is then used for calculating the differential rotation of coronal bright points. Further, the result of PSO/Snake hybrid algorithm is cross

referenced with a state of the art study which entails manual procedure done by an expert (Lorenc et al., 2012).

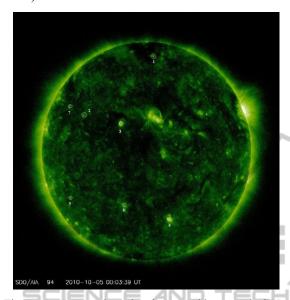


Figure 1: A sample solar image with several CBPs marked. (Image courtesy of NASAS/DO).

The rest of this paper is organized as follows: Snake model, PSO, and PSO/Snake algorithms are reviewed in section 2. Section 3 provides the experimental results and discussions. Finally, conclusions are provided in section 4.

## 2 PSO/SNAKE HYBRID ALGORITHM

The hybrid algorithm is a merge of the snake model and PSO. It integrates the active contour evolving paradigms with PSO dynamics. Snake model also known as Active Contour Model, is an energy minimization algorithm which takes into account both low level image features such as image gradient or image intensity and higher level information such as object shape, continuity of the contour and user interaction (Kass et al., 1988). After that whereabouts of the Region Of Interest (ROI) is approximated, the snake model will be able to find the precise boundary of that object. Due to their flexibility snakes are widely used in several applications such as image segmentation, shape modeling, stereo matching and object tracking (Niu, 2006; Ballerini and Bocchi, 2003; Wildenauer et al., 2006; Karlsson et al., 2003).

In our model, contour or snake has an energy associated with it, which correlates with the location

of the snake in the image and its geometrical characteristics. The idea is to minimize the integral measure which represents the total snake energy, by evolving the snake over time. Original snake model achieves this minimization by iteratively solving a pair of Euler equations on the discrete grid, resulting in a computationally expensive algorithm (Karlsson et al., 2003). Two main approaches for snake presentation are Geometric and Parametric representations. Geometric models use an implicit presentation based on the curve evolution theory and are usually implemented with level-set techniques. Effectively handling multiple objects and topology alteration is the advantage of this approach, with the cost of being computationally more complex. On the other hand, the parametric approach computationally efficient and easy to interact with users (Horng et al., 2010). In the parametric implementations, snake is defined as curve p(s) =(x(s),y(s)), having arc length s. As it is shown in equation (1), a number of discrete points called control points or snaxels characterize the snake (Kass et al., 1988). PSO/Snake hybrid uses this presentation since it well matches the snaxels and particles. The parametric implementation is as follows:

$$p(s,t) = (x(s,t), y(s,t)), \ s \in [0,1]$$
 (1)

where *t* is the time step. Total snake energy is the sum of its internal (spatial) and external (geometrical) integrals as shown in equation (2). In the PSO/Snake hybrid algorithm, the objective function calculates the total snake energy. Since in this implementation the whole population altogether represents one candidate solution to the problem, the objective is to find the contour with the least total snake energy. The lesser the total snake energy, the better it matches the ROI or moves towards it.

$$E_{snake} = \int_0^1 E_{int}(p(s))ds + \int_0^1 E_{ext}(p(s))ds \qquad (2)$$

The snake model is considered to be a controlled continuity spline under the influence of internal and external forces, which induce the snake energy. Internal energy consists of two terms which are first and second derivatives of the snake with respect to s. First term coerces the spline to act like a membrane and the second term makes the snake act like a thin plate (Kass et al., 1988). The external energy determines the snake relationship to the image. It is formulated in a way that its local minima corresponds the image features of interest. Various external energies can be employed such as image intensity, image gradient, object size or shape. One common definition used for gray-level images is the

gradient of Gaussian.

The leading part of PSO/Snake hybrid algorithm, is its PSO component. PSO is a population based evolutionary optimization algorithm. The population in the PSO is called swarm and consists of a number of particles; each potentially can be a solution to the optimization problem. Each particle has a position and a speed which are initialized with random values. Over a set of iterations, each particle's position on the search space is updated by revising its velocity according to its best experience and also its neighbors' experiences. Particle position and its corresponding fitness value are stored as personal best experience and form the cognitive aspect of particle evolution. Other aspect of the particle position update is called the social behavior and shows particles influence from its neighbors. The neighborhood can be defined with various topologies such as ring, star, Von Neumann and random. If the particle neighborhood is restricted to a subset of swarm it is called local best (lbest) PSO, while if the neighborhood equals whole swarm it is called global best (gbest) PSO. PSO/Snake hybrid used *lbest* with ring structure and radius of 3. The following equations show the dynamics of the canonical PSO algorithm for updating particle velocity and position:

$$v_{i}(t+1) = \omega(t)v_{i}(t) + c_{1}r_{1}(y_{i}(t) - x_{i}(t)) + c_{2}r_{2}(\hat{y}_{i}(t) - x_{i}(t))$$
(3)

$$x_i(t+1) = x_i(t) + v_i(t+1)$$
 (4)

Where  $x_i(t)$  and  $v_i(t)$  are position and velocity of *i-th* particle at time t,  $y_i(t)$  and  $\hat{y}_i(t)$  denote the best positions discovered by the i-th particle and its neighborhood up to the time t, i.e. pbest and lbest respectively.  $\omega(t)$  is the inertia weight which controls the impact of the previous velocity and prevents radical changes. Usually inertia weight is decreased dynamically during the run time to balance between exploration in the early iterations and exploitation in the later iterations. Coefficients r1 and r2 are random numbers. Weights of cognitive and social aspects of the algorithm are represented by acceleration factors c1 and c2 respectively. As it is shown in (Van den Bergh, 2002) regulated values for inertia and acceleration weights can be used to achieve guaranteed convergence.

The PSO/Snake hybrid algorithm integrates the snake model mechanisms with PSO dynamics.

While most of swarm intelligence approaches in the literature used in conjunction with snake model try to optimize the snake model equations, PSO/Snake hybrid does not employ PSO algorithm only as a general problem solver to optimize snake energy minimization, but it also customizes the standard PSO to better solve this specific type of image processing problems. Early experiments on medical image segmentation (Shahamatnia & Ebadzadeh 2011) and sunspot tracking (Shahamatnia et al. 2012) reported promising results. The hybrid model helps to overcome the major drawbacks of traditional snakes; initialization and convergence to the boundary concavities, while benefitting from PSO robustness and simplicity. In the Hybrid PSO/Snake model we use a population of particles where each particle is a snaxel of the contour. All particles together form the contour and hence the population is the final solution. As the algorithm runs, each particle updates its position and its velocity according to its personal best experience. local best experience, and also according to the internal force of the snake and external force of the image. This gives the PSO/Snake dynamics a wider range of informative guides to update the particle position so that it converges to the ROI.

PSO/Snake hybrid explores the search space according to PSO trajectory disciplines. This eliminates the need to have a separate searching window around each particle as many swarm based snake optimization algorithms do (Nebti 2009; Horng et al. 2010; Tseng et al. 2009). These methods consider a searching window around each particle and evaluate every position inside that window to determine the snaxels' next position. Since this local search is performed for each particle per iteration, it is a computationally expensive operation that is avoided in the PSO/hybrid model. The velocity update equation in PSO/Snake is as follows:

$$v_{i}(t+1) = \omega v_{i}(t)$$

$$+c_{1}r_{1}(pbest_{i}(t) - x_{i}(t))$$

$$+c_{2}r_{2}(lbest_{i}(t) - x_{i}(t))$$

$$+c_{3}r_{3}(\bar{x}(t) - x_{i}(t))$$

$$+c_{4}(f.Image_{i})$$
(5)

where  $pbest_i(t)$  and  $lbest_i(t)$  are personal best velocity and local best velocity terms respectively.  $\bar{x}(t)$  is the average of positions at time step t, approximating center of mass of particles. This term pushes the snake to contract or expand with respect to the sign of its weighting factor,  $r_3$ . This term speeds up the algorithm and is particularly useful when the snake is stagnated and there is no other compelling force. If the snake is initialized far from the ROI, this term allows the snake to either expand or shrink towards the ROI and hence it increases the



Figure 2: Red circle denotes the initial snake around a CBP on the first image (16 June 2010).

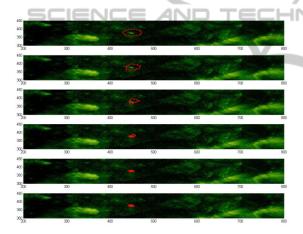


Figure 3: Detection and tracking process of the selected CBP (16 June 2010). In this figure the evolution of the snake to detect CBP boundaries can be seen.

convergence rate and speed.  $f.Image_i$  is the normalized image force corresponding to external energy from snake model principles. For particle i,  $f.Image_i$  gives the image force at the position specified by that particle. Image force can be any arbitrary function depending on the application, but generally external energies such as image gradient and gradient of Gaussian functional are enough for satisfactory performance. It must also be noted that image force does not vary by time and it is calculated only once for an image pixel.  $c_4$  is the weighting factor to control the effect of image force. Inertia weight,  $\omega$ , is taken to be a relatively small constant and  $r_1$ ,  $r_2$  and  $r_3$  denote random numbers.

Coefficients  $c_1$ ,  $c_2$ ,  $c_3$  and  $c_4$  are determined dynamically in a way that if there is a higher image force  $c_4$  always gets a higher value. It ensures that if a snaxel is next to the object boundary, it will latch to the object of interest. The whole process can be summarized as:

Step 1. Initialization. A pre-processing of images is done if required, i.e. normalizing the size of images, correcting the orientation and contrast of images, etc.

Step 2. Initial Contour. The ROI is chosen by the operator. This is the initial snake. For most cases a rough estimation of the initial contour is enough. This step is done only once when the coronal bright point appears.

Step 3. Internal parameters set-up. The weight parameters for the PSO/Snake hybrid algorithm are initialized in this step.

Step 4. Snake force calculation. The external force (image force) is calculated, once for every image.

Step 5. Calculation of social and cognitive parts. In this step we update the *pbest* value (the best velocity the snaxel ever experienced) and the *lbest* value as average of velocities of neighboring particles.

Step 6. Moving snaxels. For each snaxel its velocity is evaluated and then each snaxel velocity and position are updated.

Step 7. Snake detection. This step checks the convergence of snake contour to the coronal bright point outline, i.e. choosing the snake with the lowest total energy calculated. If the results are not satisfactory, algorithm goes back to step 5. The outcome of this step is the CBP contour for an image frame.

Step 8. Tracking CBPs. This step tracks the same CBP in the next image by feeding the subsequent image frame to the system as input. The algorithm loops back to step 4, and passes the specifications of the detected SBP.

Step 9. Stopping tracking. Tracking a CBP stops when it reaches the solar limb and disappears into the other side of the Sun, or when the CBP shrinks to a size smaller than a predefined threshold, according to the size and resolution of image.

### 3 RESULTS AND DISCUSSIONS

Our benchmark data are corona images at 9.4 nm. This line is emitted by the FeXVIII ion. We have

used selected JPEG images taken between 14 September 2010 and 20 October 2010 downloaded from a freely accessible database at internet site http://sdo.gsfc.nasa.gov/ data/aiahmi/ browse.php. 256 gray levels per pixel and image force is calculated by a gradient of Gaussian functional with  $\sigma$ =3. Images are resized to 512x512 resolutions. In

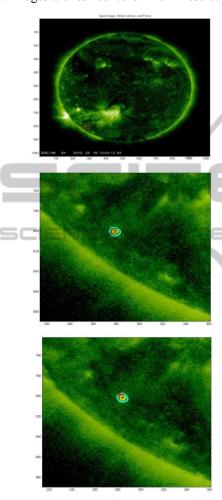


Figure 4: Initial snake on first image (top panel, 16 June 2010) and tracking process of the selected CBP during time (middle and bottom panels). The cyan contour is the boundary of tracked CBP, red square is the experts manual CBP positioning result and the yellow circle is the PSO/Snake hybrid algorithms automated tracking result for CBP's center of mass.

this automated process, the CBP to be tracked is chosen by an operator. For test purposes we've chosen the same CBPs for which we have the benchmark data available from the expert's manual CBP positioning. It should be noted that in the automated process, after that each CBP is chosen (only once), the tracking process is automatic during

life span of that CBP. Figure 2 shows a screen shot PSO/Snake hybrid algorithm tracking tool for a test image. The red circle is the initial snake around a CBP chosen by an operator. Figure 3 shows how the initial snake is evolved under PSO/Snake algorithm and the CBP boundary is detected. After CBP is detected, its characteristics including heliographic coordinates of its center of mass are calculated and are stored. Then the next frame in the sequence is fed into the system. Detected CBP contour from previous frame is used as a baseline to automatically track the CBP in the new frame. Figure 4 shows a closer look on a tracked CBP. The results show that due to the dynamic nature of PSO/Snake hybrid algorithm, detected contours are flexible and can conform to the changes in shape and size of the deformable objects like CBPs.

Altogether we have observed motion of 69 moreor-less point-like structures in 674 images (4998 measurements). In manual procedure (Lorenc et al. 2012), the CBP structures were observed directly on a PC monitor in an interactive session. Figure 5 shows latitudinal dependence of sidereal angular speed of coronal rotation obtained in this study in comparison with other authors. Further details can

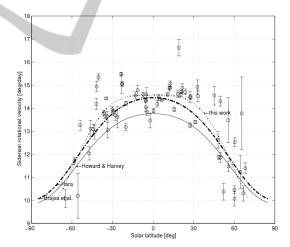


Figure 5: Derived values of the rotational speed with error bars showing the 95% confidence level intervals for individual point-like structures. The dotted curve shows the fit to the mean  $\omega(b)$  values as a function of latitude b. Over-plotted are the results of Howard and Harvey (1970), in solid line and Hara (2009) and Brajsa et al. (2004) both in the dashed-dotted curve because they are almost identical.

be found in (Lorenc et al., 2012). In that paper, an expert operator manually determines the 4998 CBPs positions. Then, we run our PSO/Snake hybrid algorithm on the images. Input images are converted to grayscale color map with

To compare the precision of the algorithm, we used several parameters that were reported in (Lorenc et al., 2012). In that paper, after an expert manually determined positions of CBPs on the solar images, the following measurements were calculated (reported in Table 1 of the referenced paper): angular rotation velocity denoted by  $\omega$ , and measurement error at 95% confidence level denoted by  $\Delta\omega$ . Tables 1 and 2 show the result obtained with manual CBP tracking and result obtained by PSO/Snake hybrid algorithm for some structures. In these tables, the structure is the identifier of CBP, b is the heliographic latitude of CBP.  $\omega_E$  is the orbital angular rotation velocity of the Earth which can be looked up from solar almanacs. Figure 6 illustrates the difference between our calculated values and the benchmark values for all 69 CBPs.

Table 2 and Figure 6 show that the obtained results are very close to the result of manual CBP tracking. Computed angular rotation velocity is

Table 1: Results reported in (Lorenc et al., 2012).

Structure	n	В	ω	Δω	$\omega_E$
xy0510.01	86	66.7	10.295	±0.327	0.986
xy0510.03	86	20.5	14.586	±0.099	0.986
xy0510.04	86	33.8	13.648	±0.209	0.986
xy0510.07	86	27.8	14.478	±0.116	0.986

Table 2: Results obtained by PSO/Snake hybrid algorithm.

Structure	n	b	ω	Δω	$\omega_E$
xy0510.01	8	67.103	11.213	±0.642	0.985
xy0510.03	8	21.057	14.387	±0.303	0.9
xy0510.04	8	-32.27	13.803	±0.342	0.985
xy0510.07	8	28.170	15.112	±0.439	0.985

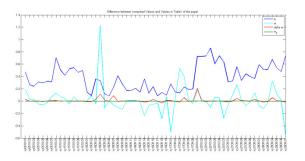


Figure 6: Deviation of PSO/Snake hybrid result from benchmark data. The differences in b,  $\omega$ ,  $\Delta\omega$ , and  $\omega_E$  are presented with blue, cyan, red and green lines.

within  $\pm 0.2$  of the benchmark data most of the time. However, it should be noted that part of this deviation is due to code implementation differences, which, in precise calculations, impose a minute variation. It is also worth mentioning, that in several cases, results displayed bigger differences, and by further investigation by a solar physicist expert (coauthor), we found out that PSO/Snake hybrid algorithm behaves consistently and the user-error is the main cause.

### 4 CONCLUSIONS

In this paper the PSO/Snake hybrid algorithm has been used to solve a real solar physic/space weather problem. By tracking CBPs over time, the angular rotational velocity in the Sun can be automatically calculated.

Based on the results analysis and comparison with a manual method the obtained values of rotational speed are reliable. We also observed that the manual method is laborious and with a large number of images becomes unworkable for practical reasons. Therefore, we developed an automatic image-processing tool (with a hybrid Snake/PSO algorithm) capable of providing the same precision. Here we discussed the suitability of using a computer aided tool for tracking coronal bright points, which includes a combined optimization process, based on a Snake model and the PSO evolutionary algorithm.

The combination of PSO dynamics with snake model kinematics makes it possible to successfully overcome active contour difficulties, preserving the simplicity of PSO. By adding two new terms to the PSO velocity update equations, PSO/Snake model still can evolve even if some of the components are missing or misleading. The PSO/Snake model can be used for different applications in image processing for object detection, image segmentation or tracking. It is especially suitable for object tracking, since the particle/snaxels have embedded velocity information, which adapts itself to the movement of the object in the images.

#### **ACKNOWLEDGEMENTS**

We would like to thank the SDO (NASA) and AIA science team for the provided observational material. This work was partially supported by grant SFRH/BPD/44018/2008 (I.D.) and SFRH/BD/62249/2009 (E.S.) from Fundação para a Ciência e

Tecnologia, MCTES, Lisbon, Portugal.

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