Fuzzy Similarity based Fuzzy TOPSIS with Multi-distances

Pasi Luukka[1], Mario Fedrizzi[2], Leoncie Niyigena[3] and Mikael Collan[1]
[1]*School of Business, Lappeenranta University of Technology, Lappeenranta, Finland*
[2]*Department of Industrial Engineering, University of Trento, Via Mesiano 77, I-38123 Trento, Italy*
[3]*Laboratory of Applied Mathematics, Lappeenranta University of Technology, Lappeenranta, Finland*

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Abstract: This article introduces a new extension to fuzzy similarity based fuzzy TOPSIS that uses multi-distance in ranking. OWA is used in the aggregation process. For the weight generation in OWA the O'Hagan's method is used to find optimal weights. Several different, predefined orness values are tested. The presented method is numerically applied to a research & development project selection problem.

1 INTRODUCTION

This paper investigates and presents a new extension of the fuzzy similarity based fuzzy Technique for Order Performance by Similarity to Ideal Solution (fuzzy TOPSIS). Fuzzy TOPSIS was originally introduced by Chen in (Chen, 2000) and later extended to include trapezoidal fuzzy numbers in (Chen et al., 2006). In these contributions a vertex based fuzzy distance method was used as a measure of distance from ("similarity to") the ideal solutions. A similarity measure based version of fuzzy TOPSIS was introduced in (Luukka, 2011), where the similarity (distance from) to the ideal solutions is calculated by using a fuzzy similarity measure. This strain of research was continued by (Niyigena et al., 2012), where two different fuzzy similarity measures were considered and by (Collan and Luukka, 2013), where four fuzzy similarity measure based fuzzy TOPSIS variants and a way of holistic overall ranking of projects was presented.

The fuzzy TOPSIS uses fuzzy numbers as inputs and is thus able to incorporate inaccurate and imprecise information in the analysis (not having to simplify reality by using crisp numbers). The main difference in using the fuzzy similarity measures and the (crisp) distance measures in the TOPSIS environment with fuzzy numbers is that fuzzy similarity measures can take into consideration more of the information that is stored in the fuzzy number, e.g. with regards to the perimeter and the area of the fuzzy number. The crisp distance measures defuzzify the fuzzy number in order to calculate a distance between the resulting crisp number and the ideal solution. Using a crisp distance based measure may cause a loss of relevant information. The fuzzy similarity measure used here is introduced in Hejazi et al (Hejazi et al., 2011) and can take into account the perimeter and area of fuzzy numbers. This similarity measure was previously studied in the context of fuzzy similarity based TOPSIS method in (Niyigena et al., 2012) and in (Collan and Luukka, 2013).

The new contribution of this paper concentrates on the application of multi-distances in creating additional information for project ranking by similarity coefficients, after they have been analyzed with fuzzy similarity measure based fuzzy TOPSIS. Multi-distances are used in analyzing the "level" of similarity between analyzed criteria. High level of similarity between criteria means a low multi-distance and can be interpreted as homogeneity or consistency of, e.g., performance or expectations. Such information may be valuable in the analysis and offers an additional differentiator between objects. Multi-distances were examined by Martin and Mayor (Martin and Mayor, 2010), and presented as a generalization of the notion of distance. Martin and Mayor proposed the construction of multi-distances by means of OWA functions in (Martin et al., 2011). Using the multi-distance in the aggregation will add a step of pairwise distance measurement of similarities between criteria (values) in the procedure.

Use of multi-distances with fuzzy TOPSIS is, to the best of our knowledge a new approach.

The remainder of the paper is organized as follows. In Section 2 the fuzzy similarity relation

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between fuzzy numbers, the OWA operator, multi-distances, and total ordering of fuzzy numbers are introduced. Section 3 is devoted to the description of the new approach to fuzzy TOPSIS based on fuzzy similarity and multi-distances. A numerical example is introduced in Section 4 and some conclusions in section 5 close the paper.

2 PRELIMINARIES

In this section some preliminary mathematical concepts, used in the MCDM method, are shortly introduced. They include: fuzzy similarity measures, the OWA-operator, and one often-used method to generate the weights for the OWA operator the O'Hagan's method. Multi-distances are defined, i.e. following the work of Martin and Mayor (Martin and Mayor, 2010) and the relationship between the OWA-operator and multi-distances is presented. Additionally, a way to find a total ordering for fuzzy numbers is shortly introduced.

2.1 Fuzzy Similarity of Fuzzy Numbers

By focusing on uncertain objects like in fuzzy sets or fuzzy numbers, the notion of a fuzzy subset generalizes that of the classical subset, where the concept of similarity can be considered as a many-valued generalization of the classical notion of equivalence (Zadeh, 1971). As an equivalence relation is a familiar way to classify similar objects, fuzzy similarity is an equivalence relation that can be used to classify multi-valued objects (Niyigena et al., 2012). The similarity measures' concept is of high importance in this work, and it is defined as follow:

Definition 1. *For any fuzzy subset $F \neq \emptyset$ of $\mathbb{R}^n$, and for any elements $A,B \in F$ the similarity measure function is defined as (Shepard, 1987):*

$$s(A,B) : F \times F \to [0,1]$$

The defined similarity measures s satisfying the following properties for any $x,y,z \in F$,

- $s(x,x) = s(y,y)$, $\forall x,y \in F$ (*Reflexivity*)
- $s(x,y) \leq s(y,y)$, $\forall x,y \in F$ (*Minimality*)
- $s(x,y) = s(y,x)$ (*Symmetry*)
- If $s(x,y) = s(x,z)$ it implies that $s(x,y) = s(x,z) = s(y,z)$ (*Transitivity*)

Since fuzzy numbers can be considered to be a certain type of restricted fuzzy sets, the similarity measures for generalized fuzzy numbers come from similarity measures for fuzzy sets.

Represented by Chen (Chen, 1985), a generalized trapezoidal fuzzy number's notation is $\tilde{A} = (a,b,c,d;w)$, where a,b,c and d are real values and $0 < w \leq 1$. Its membership function $\mu_{\tilde{A}}$ satisfies the following conditions (Chen, 1985):

1. $\mu_{\tilde{A}}$ is a continuous mapping from the universe of discourse X to the closed interval in $[0,1]$
2. $\mu_{\tilde{A}} = 0$, where $-\infty < x \leq a$
3. $\mu_{\tilde{A}}$ is monotonically increasing in $[a,b]$
4. $\mu_{\tilde{A}} = w$, where $b \leq x \leq c$
5. $\mu_{\tilde{A}}$ is monotonically decreasing in $[c,d]$
6. $\mu_{\tilde{A}} = 0$, where $d \leq x < \infty$

Due to the fit and the applicability of similarity measures in the context of decision-making, various similarity measures have been proposed for the calculation the degree of similarity between fuzzy numbers of (Chen, 1985). In this work, a rather recently introduced similarity measure by Hejazi *et al* in (Hejazi et al., 2011) is used. The similarity measure takes in consideration the perimeter and area of fuzzy numbers. The similarity measure is denoted $s(M,N)$, and involves fuzzy numbers $M = (m_1,m_2,m_3,m_4;\omega_m)$ and $N = (n_1,n_2,n_3,n_4;\omega_n)$ with $0 \leq m_1 \leq m_2 \leq m_3 \leq m_4 \leq 1$, $0 \leq n_1 \leq n_2 \leq n_3 \leq n_4 \leq 1$, and $M(x_i)$ and $N(x_i)$ their corresponding membership functions with $i \in \{1,2,3,4\}$ for generalized trapezoidal fuzzy numbers, where ω_m and ω_n are their corresponding heights. The definition is as follows:

$$\begin{aligned} s(M,N) &= (1 - \frac{\sum_{i=1}^{4} |m_i - n_i|}{4}) \\ &\times \frac{min(p(m),p(n))}{max(p(m),p(n))} \\ &\times \frac{min(a(m),a(n)) + min(\omega_m,\omega_n)}{max(a(m),a(n)) + max(\omega_m,\omega_n)} \quad (1) \end{aligned}$$

Where the values $p(m)$ and $p(n)$ represent the perimeters of the trapezoidal fuzzy numbers M and N, and are defined as:

$$\begin{aligned} p(m) &= \sqrt{(m_1 - m_2)^2 + \omega_m^2} + \sqrt{(m_3 - m_4)^2 + \omega_m^2} \\ &+ (m_3 - m_2) + (m_4 - m_1) \end{aligned}$$

and

$$\begin{aligned} p(n) &= \sqrt{(n_1 - n_2)^2 + \omega_n^2} + \sqrt{(n_3 - n_4)^2 + \omega_n^2} \\ &+ (n_3 - n_2) + (n_4 - n_1) \end{aligned}$$

The values $a(m)$ and $a(n)$ represent the areas of the trapezoidal fuzzy numbers M and N, they are defined as:

$$a(m) = \frac{1}{2}\omega_m(m_3 - m_2 + m_4 - m_1),$$

and

$$a(m) = \frac{1}{2}\omega_n(n_3 - n_2 + n_4 - n_1).$$

Notice that the result of the above similarity measure $s(M,N)$ belongs to the unit interval $[0,1]$ and the larger the value of the similarity measure is, the stronger is the similarity between the fuzzy numbers M and N.

2.2 The OWA Operator

In 1988 Yager introduced a new aggregation operator, called ordered weighted averaging operator(OWA) (Yager, 1988) and formalized it as follows:

An ordered weighted averaging (OWA) operator of dimension m is a mapping $R^m \to R$ that has associated weighting vector $W = [w_1, w_2, ..., w_m]$ of dimension m with

$\sum_{i=1}^{m} w_i = 1$, $w_i \in [0,1]$ and $1 \leq i \leq m$

such that:

$$OWA(a_1, a_2, ..., a_m) = \sum_{i=1}^{m} w_i b_i \qquad (2)$$

where b_i is the $i-$th largest element of the collection of objects $a_1, a_2, ..., a_m$. The function value $OWA(a_1, a_2, ..., a_m)$ determines the aggregated values of arguments $a_1, a_2, ..., a_m$. One of the measures related to the OWA is the so called "orness" measure. For a given weighting vector $W = [w_1, w_2, ..., w_m]^T$ the measure of orness of the OWA aggregation operator for W is given as

$$orness(W) = \frac{1}{m-1}\sum_{i=1}^{m}(m-i)w_i. \qquad (3)$$

It can be observed that the weighting vector has an important role in the OWA operator; next the O'Hagan's method for generating the weights is shortly presented. In 1988 O'Hagan (O'Hagan, 1988) introduced a technique for computing the weights in OWA. The procedure for obtaining the aggregation assumes a predefined degree of orness - the weights are obtained by maximizing the entropy $-\sum_{i=1}^{m} w_i ln(w_i)$. The solution is based on the constrained optimization problem

$$\begin{aligned} \text{maximize} \quad & -\sum_{i=1}^{m} w_i ln(w_i) \\ \text{subject to} \quad & \alpha = \frac{1}{m-1}\sum_{i=1}^{m}(m-1)w_i \\ & \sum_{i=1}^{m} w_i = 1 \text{ and } w_i \geq 0. \end{aligned}$$

The above constrained optimization problem can be solved by using different methods. Here an analytical solution introduced by (Fullér and Majlander, 2001) is used. Below this weighting scheme is presented:

a. if $m = 2$, implies that $w_1 = \alpha$ and $w_2 = 1 - \alpha$
b. if $\alpha = 0$ or $\alpha = 1$ implies that the corresponding weighting vectors are $w = (0, ...0, 1)$ or $w = (1, 0, ..., 0)$ respectively.
c. if $m \geq 3$ and $0 \leq \alpha \leq 1$ then, we have,

$$w_i = \left(w_1^{m-i} \cdot w_m^{i-1}\right)^{\frac{1}{m-1}}$$

$$w_m = \frac{((m-1)\cdot\alpha - m).w_1 + 1}{(m-1)\cdot\alpha + 1 - m\cdot w_1}$$

$$w_1[(m-1)\cdot\alpha + 1 - m\cdot w_1]^m = ((m-1)\cdot\alpha)^{m-1}\cdot[((m-1)\cdot\alpha - m)\cdot w_1 + 1]$$

For $m \geq 3$, the weights are computed by obtaining the first weight, followed by the last weight of the weighting vector, before other weights are computed.

2.3 Multi-distances

A multi-distance is a representation of the notion of multi-argument distances. The set X is a union of all m-dimensional lists of elements of X, multi-distance is defined as a function $D : X \to [0, \infty)$ on a non empty set X provided that the following properties are satisfied for all m and $x_1, x_2, ..., x_m, y \in X$

c1. $D(x_1, x_2, ..., x_m) = 0$ if and only if $x_i = x_j$ for all $i, j = 1, 2, ..., m$
c2. $D(x_1, x_2, ..., x_m) = D(x_{\sigma(1)}, x_{\sigma(2)}, ..., x_{\sigma(m)})$ for any permutation σ of $i, j = 1, 2, ..., m$
c3. $D(x_1, x_2, ..., x_m) \leq D(x_1, y) + D(x_2, y) + ... + D(x_m, y)$.

We say that D is a strong multi-distance if it satisfies $c1, c2$, and

$c3^\star$ $D(\vec{x}_1, \vec{x}_2, ..., \vec{x}_m) \leq D(\vec{x}_1, \vec{y}) + D(\vec{x}_2, \vec{y}) + ... + D(\vec{x}_m, \vec{y})$. for all $\vec{x}_1, \vec{x}_2, ..., \vec{x}_m, \vec{y} \in X$

In application contexts, the estimation of distances between more than two elements of the set X can be constructed using multi-distances by means of the OWA operator as suggested by Martin and Mayor (Martin and Mayor, 2010).

$$D_w(x_1, x_2, \cdots, x_m) = OWA_w(d(x_1, x_2), d(x_2, x_3), ..., d(x_{m-1}, x_m)) \qquad (4)$$

In this case, elements $x_1, x_2, \cdots, x_m$ are obtained from the similarity measure (1), and the distance applied is $d(x, y) = |x - y|$.

2.4 Total Ordering of Fuzzy Numbers

Set inclusion of fuzzy sets is only a partial order, where all fuzzy sets are not comparable. Kaufman and Gupta (Kaufman and Gupta, 1988) propose that when trying to find a *total order* or *linear order* for fuzzy numbers, where all fuzzy numbers and fuzzy intervals are comparable, we have to first check that it is possible to find a linear order by giving different emphases to different properties of fuzzy sets. To reach a *total order* or a *linear order* of fuzzy numbers, an importance order must be given to three criteria. If the first criterion does not give a unique linear order, then the second criterion should be used. One continues in this way as long as it is needed. The description of the three different criteria is given below:

1^{st} *The removal*: Let us consider an ordinary number $k \in \mathbb{R}$ and a fuzzy number A. The left side removal of A with respect to k, denoted by $R_l(A,k)$, is defined as the area bounded by k and the left side of the fuzzy number A. Similarly, the right side removal, $R_r(A,k)$ is defined. The removal of the fuzzy number A with respect to k is defined as the mean of $R_l(A,k)$ and $R_r(A,k)$. Thus,

$$R(A,k) = \frac{1}{2}(R_l(A,k) + R_r(A,k)). \tag{5}$$

The position of k can be located anywhere on the x-axis including $k = 0$. By definition, the areas are positive quantities, but here they are evaluated by integration taking into account the position (negative, zero, or positive) of the variable x; therefore, $R(A,k)$ can be positive, negative or null.

Our first criterion, therefore, will be this removal with respect to k. However, two different fuzzy numbers can have the same removal with respect to the same k. In fact, this criterion decomposes a set of fuzzy numbers into classes having the same removal number.

The removal number $R(A,k)$ defined in this criterion, relocated to $k = 0$ is equivalent to an "ordinary representative" of the fuzzy number. In the case of a triangular fuzzy number this ordinary representative is given by:

$$\widehat{A} = \frac{a_1 + 2a_2 + a_3}{4}, \tag{6}$$

where $A = (a_1, a_2, a_3)$.

2^{nd} *The mode*: In each class of fuzzy numbers, one should look for the mode, and these modes will generate sub-classes. If the fuzzy numbers under consideration have a non-unique mode, one takes the mean position of the modal values. It must be noted that this is only one way of obtaining sub-classes, and one may need the following third divergence criterion for further sub-classification.

3^{rd} *The divergence*: The consideration of the divergence around the mode in each sub-class leads to the sub-sub-classes, and this criterion may be sufficient to obtain the final linear ordering of fuzzy numbers.

When one orders fuzzy numbers to size order, one proceeds as follows. Apply the above presented criteria in the exact given order, such that if the unique linear order is not obtained then move to the next criterion.

3 FUZZY SIMILARITY BASED FUZZY TOPSIS WITH MULTI-DISTANCES

Fuzzy extension to the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was presented by Chen (Chen, 2000) and it has been extended to solve problems involving trapezoidal fuzzy numbers and applied, e.g., to solving the supplier selection problems (Chen et al., 2006). It is a Multiple Criteria Decision Making (MCDM) method (Chen et al., 2006) (Socorro and Lamata, 2007) useful in ranking objects, based on the similarity of the object characteristics to the characteristics of an ideal object (ideal solution). The method is based on the idea that objects are ranked higher the shorter their distance is from the *Fuzzy Positive Ideal Solution (**FPIS**)* and the further away they are from the *Fuzzy Negative Ideal Solution (**FNIS**)*. One advantage of having extended the TOPSIS method to the fuzzy environment is that a linguistic assessment can be properly used, instead of being constrained into using only numerical values; linguistic variables can be mapped to corresponding fuzzy numbers (Chen, 2000),(Chen et al., 2006).

Solution to the project selection problem, when using the $TOPSIS$ approach, can be presented by considering a situation of a finite set of projects $P = \{P_i | i = 1,2,...,m\}$ which need to be evaluated by a committee of decision-makers $D = \{D_l | l = 1,2,...,k\}$, by considering a finite set of given criteria $C = \{C_j | j = 1,2,...,n\}$.
Let us consider a decision matrix representing a set of performance ratings of each alternative project $P_i, i = 1,2,...,m$ with respect to each criterion $C_j, j = 1,2,...,n$, as follows (Cui et al., 2011):

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

Let us also assume the weight w_j of the $j-$th criterion C_j, such that the weight vector is represented as follows:

$$\mathbf{W} = [\ w_1, \quad w_2, \quad \ldots, \quad w_n\ ]$$

Where m rows represent m possible alternatives, n columns represent n relevant criteria, and x_{ij} represent the performance rating of the $i-$th project P_i with respect to the $j-$th criterion C_j. The above fuzzy ratings for each decision-maker $D_l, l = 1,2,...,k$ are represented by positive trapezoidal fuzzy numbers $\hat{R}_l = (a_l, b_l, c_l, d_l), l = 1,2,...,k$ with the respective membership function $\mu_{\hat{R}_l}(x)$. As the rating $\hat{R}_l = (a_l, b_l, c_l, d_l)$ is for the $l-$th decision-maker, the aggregated fuzzy number that can stand for all decision-makers' rating is:

$$\hat{R} = (a,b,c,d)$$

with:
$a = \min_l\{a_l\}$, $b = \frac{1}{k}\sum_{l=1}^{k} b_l$, $c = \frac{1}{k}\sum_{l=1}^{k} c_l$, $d = \max_l\{d_l\}$. The fuzzy rating and importance weight of the $l-$th decision-maker can respectively be represented by $x_{ijl} = (a_{ijl}, b_{ijl}, c_{ijl}, d_{ijl})$ and $\hat{w} = (w_{jl1}, w_{jl2}, w_{jl3}, w_{jl4})$ with $i = 1,2,...,m; j = 1,2,...,n$. Then, the aggregated fuzzy ratings x_{ij} of alternatives with respect to each criterion are:

$$x_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}),$$

calculated as: $a_{ij} = \min_l\{a_{ijl}\}$, $b_{ij} = \frac{1}{k}\sum_{l=1}^{k} b_{ijl}$, $c_{ij} = \frac{1}{k}\sum_{l=1}^{k} c_{ijl}$, $d_{ij} = \max_l\{d_{ijl}\}$. The aggregated fuzzy weight $\hat{w}_j$ of each criterion can be calculated as :

$$\hat{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$$

with $w_{j1} = \min_l\{w_{jl1}\}$, $w_{j2} = \frac{1}{k}\sum_{l=1}^{k} w_{jl2}$, $w_{j3} = \frac{1}{k}\sum_{l=1}^{k} w_{jl3}$, $w_{j4} = \max_l\{w_{jl4}\}$. After aggregation the decision matrix and the weight vector are of the following form $X = \{x_{ij}\}_{m\times n}$ and $W = \{w_j\}_{1\times n}$, where $i = 1,2,...,m$ and $j = 1,2,...,n$.

These matrices' elements are given by positive trapezoidal fuzzy numbers as :

$x_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ and $w_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$.

The linear scale transformation is used to transform the various criteria scales into comparable scales in order to avert overly complex mathematical operations in a decision process. By dividing the set of criteria into *benefit criteria B*, where the larger the rating, the greater the preference and *cost criteria C*, where the smaller the rating, the greater the preference. A normalization method designed to preserve the property in which the elements are normalized trapezoidal fuzzy numbers is used. The normalized value of x_{ij} is r_{ij}, and the normalized fuzzy decision matrix is then represented as:

$$R = [r_{ij}]_{m\times n} \tag{7}$$

with

$$r_{ij} = (\frac{a_{ij}}{d_j^+}, \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+}), j \in B$$

$$r_{ij} = (\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}), j \in C$$

where $d_j^+ = \max_i\{d_{ij}\}, j \in B$ and $a_j^- = \min_i\{a_{ij}\}, j \in C$. The weighted normalized value of r_{ij} is called v_{ij}, and by considering the importance of each criterion, the weighted normalized fuzzy decision matrix is represented as:

$$V = [v_{ij}]_{m\times n} \tag{8}$$

where $v_{ij} = r_{ij} \cdot w_j$. For all i, j, the elements v_{ij} are now normalized positive trapezoidal fuzzy numbers.

Next, the ideal solutions must be determined and taken from the given criteria which are linguistically expressed, they are commonly referred to as *Fuzzy Positive Ideal Solution (**FPIS**)* and *Fuzzy Negative Ideal Solution (**FNIS**)*. By considering a finite set of given criteria $C = \{C_j | j = 1,2,...,n\}$, the ways to select the *FPIS*(P^+) and the *FNIS*(P^-) come from the weighted normalized decision matrix $V = (v_{ij})_{m\times n}$, where the obtained weighted normalized values v_{ij} are fuzzy numbers expressed as:

$$v_{ij} = (v_{ij1}, v_{ij2}, v_{ij3}, v_{ij4})$$

The fuzzy positive-ideal solution P^+ and the fuzzy negative-ideal solution P^-, respectively are:

$$P^+ = [v_1^+, v_2^+, ..., v_n^+] \tag{9}$$

$$P^- = [v_1^-, v_2^-, ..., v_n^-] \tag{10}$$

One way for choosing the *FPIS* (P^+) and the *FNIS* (P^-) have been explained in (Luukka, 2011), and is given as follows:

Every element of P^+ is the maximum for all i weighted normalized value , and every element of P^- is the minimum for all i weighted normalized value:

$$v_j^+ = (\max_i v_{ij1}, \max_i v_{ij2}, \max_i v_{ij3}, \max_i v_{ij4}) \tag{11}$$

$$v_j^- = (\min_i v_{ij1}, \min_i v_{ij2}, \min_i v_{ij3}, \min_i v_{ij4}) \tag{12}$$

This was also considered in our approach. The similarity measure between each project and the ideal solutions P^+ and P^- will be needed later, when calculating the closeness coefficients to determine the ranking order of all possible alternative projects.

The similarities s_i^+ from the positive and negative ideal solution are calculated as:

$$s_i^+ = \{s_{i1}(v_{i1}, v_1^+), s_{i2}(v_{i2}, v_2^+), \cdots, s_{in}(v_{in}, v_n^+)\} \tag{13}$$

$$s_i^- = \{s_{i1}(v_{i1}, v_1^-), s_{i2}(v_{i2}, v_2^-), \cdots, s_{in}(v_{in}, v_n^+)\} \tag{14}$$

where for similarity we used the similarity measure from equation (1).

Table 1: Evaluation of R & D projects.

Project	C_1	C_2	C_3	C_4
P_1	$(53,62,68,78)$	$(43,50,55,63)$	$(115,128,128,141)$	0.06
P_2	$(83,98,108,123)$	$(85,100,110,125)$	$(126,140,140,154)$	0.0594
P_3	$(157,185,204,231)$	$(170,200,220,250)$	$(170,189,189,208)$	18
P_4	$(204,240,268,300)$	$(170,200,220,250)$	$(164,182,182,200)$	0.54
P_5	$(259,305,336,381)$	$(510,600,660,750)$	$(209,232,232,255)$	3.10
P_6	$(85,100,110,125)$	$(85,100,110,125)$	$(185,206,206,227)$	5
P_7	$(259,305,336,381)$	$(510,600,660,750)$	$(209,232,232,255)$	3.10
P_8	$(94,110,121,138)$	$(85,100,110,125)$	$(177,197,197,217)$	1.58
P_9	$(140,165,182,206)$	$(153,180,198,225)$	$(238,264,264,290)$	17.15
P_{10}	$(190,223,245,279)$	$(323,380,418,475)$	$(257,285,285,314)$	1.65
P_{11}	$(60,70,77,88)$	$(68,80,88,100)$	$(148,164,164,180)$	10.03
P_{12}	$(91,107,118,134)$	$(85,100,110,125)$	$(144,160,160,176)$	2.39
P_{13}	$(247,290,319,363)$	$(34,40,44,50)$	$(297,330,330,363)$	0
P_{14}	$(370,435,479,544)$	$(595,700,770,875)$	$(338,375,375,413)$	278.25
P_{15}	$(166,195,215,244)$	$(425,500,550,625)$	$(279,310,310,341)$	320.25
P_{16}	$(221,260,286,325)$	$(255,300,330,375)$	$(315,350,350,385)$	39.66
P_{17}	$(235,277,305,346)$	$(298,350,385,438)$	$(311,346,346,381)$	72.48
P_{18}	$(281,330,363,413)$	$(468,550,605,688)$	$(331,368,368,405)$	231
P_{19}	$(344,405,446,506)$	$(680,800,880,1000)$	$(365,406,406,447)$	414.75
P_{20}	$(451,530,583,663)$	$(978,1150,1265,1438)$	$(394,438,438,482)$	651

These similarity vectors are then aggregated using OWA, as follows:

$$S^+_{iw} = OWA_w(s^+_{i1}, s^+_{i2}, \cdots, s^+_{in}) \quad (15)$$

$$S^-_{iw} = OWA_w(s^-_{i1}, s^-_{i2}, \cdots, s^-_{in}) \quad (16)$$

Besides this we also aggregate s^+_i vector by using multi-distance as

$$D^+_{iw}(s^+_{i1}, s^+_{i2}, \cdots, s^+_{in}) = OWA_w(d(s^+_{i1}, s^+_{i2}), d(s^+_{i2}, s^+_{i3}), ..., d(s^+_{i(n-1)}, s^+_{in})) \quad (17)$$

In the closeness coefficient we now want to take both into account, the similarities from the positive and the negative ideal solution but also the multi-distance. This is now done by modifying the closeness coefficient in form given in equation (18). The closeness coefficients of the alternative project P_i with respect to the positive ideal solution by using the distance matrix (CC_i) are defined as:

$$CC_i = \frac{S^-_{iw} + D^+_{iw}}{D^+_{iw} + S^+_{iw} + S^-_{iw}}, i = 1,2,...,m \quad (18)$$

Next we rank the projects by closeness coefficients, now using ascending order.

For all $i = 1,2,...,m$ and $j = 1,2,...,n$. Different steps for the given $TOPSIS$ algorithm can be presented as follows:

Step1: Form a decision-makers' committee, and identify the evaluation criteria.

Step2: Choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternative projects.

Step3: Aggregate the weight of criteria to get the aggregated fuzzy weight $\hat{w}_j$ of the criterion C_j and join the decision-makers' ratings to get the aggregated fuzzy rating x_{ij} of the project P_i in consideration of the criterion C_j.

Step4: Construct the fuzzy decision matrix and the normalized fuzzy decision matrix.

Step5: Construct the weighted normalized fuzzy decision matrix.

Step6: Determine the fuzzy positive (and negative) ideal solution *FPIS* (and *FNIS*).

Step7: Construct the similarity matrix by calculating the similarity measure of each project from the *FPIS* (and *FNIS*).

Step8: Calculate aggregated similarity values for each project wrt. *FPIS* and *FNIS* by using OWA.

Step9: Calculate multi-distance value for each project wrt. *FPIS*.

Step10: Calculate the closeness coefficient for each project in order to determine the projects' ranking order.

4 NUMERICAL EXAMPLE

This numerical example uses the same data that is also used in (Hassanzadeh et al., 2012). A pharmaceutical company can select a certain number of projects to invest in from among 20 R & D projects. Criteria that are used in the example come from costs, revenues, budget constraints, and real option values (ROV) calculated for each project by using the pay-off method for real option valuation (Collan et al., 2009); these are represented as trapezoidal fuzzy numbers. First and third criteria are cost criterias and second and fourth, benefit criterias.

In Table 1 one can see evaluations of the different criteria by using trapezoidal fuzzy numbers. The fourth (ROV) criterion is carried out in computations as a fuzzy number of form $A = (a_1, a_2, a_3, a_4)$, where $a_1 = a_2 = a_3 = a_4$.

For different α values, the following Table 2 shows the computed closeness coefficients and the rankings for each of the twenty projects for three different orness values α. In computation of fuzzy numbers a larger set of values of α was used. There α values were $\alpha = 0.1, 0.2, \cdots, 0.9, 1$.

Table 2: Projects' closeness coefficients values and rankings, for $\alpha = 0.1, 0.5, 1$.

Project	$CC_{(\alpha=0.1)}$	Rank	$CC_{(\alpha=0.5)}$	Rank	$CC_{(\alpha=1)}$	Rank
P_1	0.83	19	0.710	8	0.709	8
P_2	0.813	13	0.728	11	0.728	11
P_3	0.79	10	0.741	15	0.741	15
P_4	0.80	11	0.751	19	0.751	19
P_5	0.61	2	0.614	2	0.615	2
P_6	0.821	17	0.74	14	0.739	14
P_7	0.819	15	0.717	10	0.716	10
P_8	0.824	18	0.744	17	0.743	17
P_9	0.809	12	0.751	18	0.75	18
P_{10}	0.68	6	0.681	7	0.682	7
P_{11}	0.81	14	0.716	9	0.715	9
P_{12}	0.82	16	0.737	13	0.736	13
P_{13}	0.85	20	0.798	20	0.797	20
P_{14}	0.64	4	0.674	6	0.674	6
P_{15}	0.58	1	0.612	1	0.613	1
P_{16}	0.77	9	0.742	16	0.743	16
P_{17}	0.75	8	0.729	12	0.729	12
P_{18}	0.64	3	0.672	5	0.673	5
P_{19}	0.68	5	0.661	4	0.662	4
P_{20}	0.71	7	0.652	3	0.651	3

In Table 3, the minimum, mean, and the maximum rankings from our experimental setup are summarized. These are then used in the formation of triangular fuzzy numbers for each project.

Table 3: The minimum, mean, and the maximum rankings.

Project	Minimum	Mean	Maximum
P_1	8	10.3	19
P_2	11	11.9	14
P_3	10	13.7	15
P_4	11	17.1	19
P_5	2	2	2
P_6	14	15.2	18
P_7	10	11	15
P_8	17	17.5	19
P_9	12	17.1	18
P_{10}	6	6.8	7
P_{11}	9	10	14
P_{12}	13	14.1	17
P_{13}	20	20	20
P_{14}	4	5.4	6
P_{15}	1	1	1
P_{16}	9	13.6	16
P_{17}	8	10.5	12
P_{18}	3	4.4	5
P_{19}	4	4.3	5
P_{20}	3	4.1	7

Total ordering is found for fuzzy numbers presented in Table 3 by using the method introduced by Kaufman and Gupta (Kaufman and Gupta, 1988). For this purpose removal number, dispersion, and modal value are calculated in a way presented above - Table 4 presents the resulting overall ranking. According to the result the top five projects are $15, 5, 18, 19$, and 20.

Table 4: Overall rankings of the R & D projects using removal number, dispersion, and modal value.

Project	Rank	Removal no	div	mode
P_{15}	1	1	0	1
P_5	2	2	0	2
P_{18}	3	4.2	2	4.4
P_{19}	4	4.4	1	4.3
P_{20}	5	4.55	4	4.1
P_{14}	6	5.2	2	5.4
P_{10}	7	6.65	1	6.8
P_{17}	8	10.25	4	10.5
P_{11}	9	10.75	5	10
P_7	10	11.75	5	11
P_1	11	11.9	11	10.3
P_2	12	12.2	3	11.9
P_{16}	13	13.05	7	13.6
P_3	14	13.1	5	13.7
P_{12}	15	14.55	4	14.1
P_6	16	15.6	4	15.2
P_9	17	16.05	6	17.1
P_4	18	16.05	8	17.1
P_8	19	17.75	2	17.5
P_{13}	20	20	0	20

5 CONCLUSIONS

A new multiple-criteria decision making approach was presented; it is an extension for the fuzzy similarity based fuzzy TOPSIS. OWA was used for aggregating similarity to fuzzy negative and positive ideal solutions for each criterion and multi-distance was used in collecting information about the "similarity of these similarities" that can be understood as a measure of homogeneity or consistency of a given project. This has allowed the inclusion of more relevant information than the use of a simple defuzzification. The method was applied to a R & D project selection problem. The results are dependent on the proper selection of α, when the weights are generated for the OWA operator. This weight generation was done by using O'Hagan's method that finds the weights as an optimal solution for a predefined (given) orness value (α). We examined the effect of the pre-selection by testing with a number of orness values. We presented a way to take in to consideration the "created" information with different orness values by forming fuzzy numbers from the different rankings of the projects created in this way. By using multidistances a measure of homogeneity of similarity of the different criteria of each project to the fuzzy positive ideal solution was calculated. This was done to include information about the consistency of the level of goodness of projects (by the selected criteria). This information was included in the closeness coefficient that was used in the ranking of the projects. The final ranking thus includes information about the goodness of each project (as ranked by TOPSIS) and about the "stability" of the level of goodness of each of the criteria of each project. The top five projects from the numerical example were found to be $15, 5, 18, 19$, and 20. Notable from the results is that projects 15 and 5 were always top 2 choices, but project 20 varied between rankings 3 to 7 so that with lower values of orness ranking was lower and after orness value 0.6 it was always the third best choice. Forming a fuzzy number from different rankings allows one to include different points of view and creating an intelligent overall ranking. Furthermore, more relevant information is carried along in the analysis, until the ranking stage, enabling the ranking to take more things into consideration and thus being based on a more holistic view of the problem.

REFERENCES

Niyigena, L., Luukka, P., Collan, M., 2013. Supplier evaluation with fuzzy similarity based fuzzy TOPSIS with new fuzzy similarity measure, *13th IEEE International symposium on computational intelligence and informatics.* Budapest, November, 2012.

Zadeh, L. 1971., Similarity relations and fuzzy orderings. *Inform sci, 3, pp*. 177-200.

Shepard, R. N., 1987. Toward a universal law of generalization for psychological science. *Science, 237, pp*. 1317-1323.

Chen, S. H., 1985. Operations on fuzzy numbers with function principal, *Tamkang journal of management sciences*, 6,1, pp. 13-25.

Wei S.-H., Chen, S.-M., 2009, A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers, *Expert systems with applications* 36. pp. 589-598.

Chen C. T., 2000. Extensions of the TOPSIS for group decision-making under fuzzy environment, *Fuzzy sets and systems, Vol.* 114, pp. 1-9.

Chen, C.-T., Lin, C.-T., Huang, S.-F., 2006. A fuzzy approach for supplier evaluation and selection in supply chain management, *International journal of production economics, Vol. 10*2, Issue 2, pp. 289-301.

Collan, M., Luukka, P., 2013. Evaluating R & D projects as investments by using an overall ranking from four new fuzzy similarity measure based TOPSIS variants, *IEEE Transactions on fuzzy systems, Vol 26, Issue* 6, pp. 1-11.

Hejazi, S.R., Doostparast, A., Hosseini, S.M., 2011, An improved fuzzy risk analysis based on a new similarity measures of generalized fuzzy numbers, *Expert systems with applications, Vol. 38,* Issue 8, pp. 1-7.

Kaufmann, M., Gupta, M., 1988. Fuzzy mathematical models in engineering and management science: *Elsevier science publishers B. V.*

Luukka, P., 2011. Fuzzy similarity in multicriteria decision-making problem applied to supplier evaluation and selection in supply chain management, *Hindawi publishing corporation, Advances in artificial Intelligence,* pp. 1-9.

Socorro, M., García-Cascales, M., Lamata, T., 2007. Solving a decision problem with linguistic information, *Pattern recognition letters, Elsevier, Vol.* 28, No. 16, pp. 2284-2294.

Cui, Z. X., Yoo, H. K., Choi, J. Y., Youn, H. Y., 2011. Multi-criteria group decision making with fuzzy logic and entropy based weighting, *Proceedings of the 5th ICUIMC'11,* Feb. 2011, pp. 1-7.

Martin, J., Mayor, G., 2010. Some properties of multi-argument distances and Fermat multidistance, In: *proceedings of IPMU 2010,* pp. 703-711.

Martin, J., Mayor, G., Valero, O., 2011. Multi-argument distances, Fuzzy sets and systems, 167, pp. 92-100.

Mahmoodzadeh, S., Shahrabi, J., Pariazar, M., Zaeri, M.S., 2007. Project selection by using Fuzzy AHP and TOPSIS technique, *International journal of human and social sciences*, Vol. 1, No. 3, pp. 135-140.

O'Hagan, M., 1988. Aggregating template or rule antecedents in real time expert systems with fuzzy set logic, In *proceedings of 22nd Annual IEEE asilomar confernence on signals,systems, computers, Pacific Grove*, California, pp. 681-689.

Fullér, R., Majlander, P., 2001. An analytical approach for obtaining maximal entropy OWA operator weights, *Fuzzy sets and systems*, 124, pp. 53-57.

Yager, R. R., 1988. On ordered weighted averaging operators in multicriteria decision making, *IEEE Transactions on systems, man, cybernetics, 18*, pp. 183-190

Brunelli,M., Fedrizzi,M., Fedrizzi,M., Molinari,F., 2012. On some connections between multidistances and valued *m*-ary adjancency relations, Advances on computational intelligence, *14th International conference on information processing and management of uncertainty in knowledge-based systems, IPMU 2012,* Catania, Italy, July 9-13, 2012. Proceedings, Part I. Vol.297, pp. 201-207

Molinari, F. , 2012. About a new family of multi-distances, *Fuzzy sets and systems, Vol.* 195, pp. 118-122.

Hassanzadeh F., Collan, M., Modarres, M., 2012. A practical approach to R & D portfolio selection using fuzzy set theory, *IEEE transactions on fuzzy systems vol. 20*, pp. 615-622.

Collan, M., Fullér, R. and Mezei, J., 2009. A fuzzy pay-off method for real option valuation, *Journal of applied mathematics and decision science*s, Online, Available: http://ideas.repec.org/p/pra/mprapa/13601.html., Accessed 20, march, 2012.