

Observer-based Robust Fault Diagnosis

Logic-dynamic Approach

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Abstract: The problem of robust fault detection and isolation in robotic and mechatronic systems described by nonlinear models with non-smooth nonlinearities is considered. So-called logic-dynamic approach to construct the diagnostic observer with non-smooth nonlinearities by linear methods is considered. The method which allows obtaining full set of solutions with minimal sensitivity to the disturbance is suggested. This set of solutions can be used to choose the optimal solution with maximal sensitivity to the faults.

1 INTRODUCTION

There are many papers and books devoted to the problem of robust fault detection and isolation (FDI) in different technical systems (Blanke et al., 2006); (Chen and Patton, 1999); (Chen, 2008); (Li and Zhou, 2009); (Schreier et al., 1997). This problem is completely solved in the case when a residual generator is of the form of linear parity relations (Frank, 1990); (Low et al., 1984); (Patton et al., 2002); many papers consider this problem for diagnostic observer (Blanke et al., 2006); (Chen and Patton, 1999); (Chen, 2008); (Li and Zhou, 2009); (Schreier et al., 1997) in the case when the system under diagnosis is linear or nonlinear with smooth nonlinearities.

At the same time, many robotic and mechatronic systems are described by nonlinear models with non-smooth nonlinearities such as saturation, Coulomb friction, backlash and hysteresis. For such systems traditional methods of observer design (see Blanke et al., 2006; Frank, 1990) are not applicable, and special design methods must be used. One of these methods is the logic-dynamic approach suggested by (Zhirabok and Usoltsev, 2002).

In this paper we consider the problem of robust observer design for FDI in robotic and mechatronic systems with non-smooth nonlinearities. As usual, observer-based methods allow obtaining the single solution only whose robust properties have to be checked. If the result of this checking is not good, another solution is found and the robust properties

are checked again. To overcome this shortcoming, the new approach is suggested allowing obtaining a full set of solutions with given robust properties which can be used to choose the optimal solution.

2 SYSTEM TRANSFORMATIONS

Consider the system describe by the following equations

$$\dot{x}'(t) = f'(x'(t), u(t)), \quad y(t) = h'(x'(t)), \quad (1)$$

where $x'(t) \in X \subseteq R^n$, $u(t) \in U \subseteq R^m$, $y(t) \in Y \subseteq R^l$ are vectors of state, control, and output; f' and h' are nonlinear vector functions, the function f' may be non-smooth. It is supposed that the function h' satisfies the condition $\text{rank}\left(\frac{\partial h'}{\partial x'}\right) = l$ for all $x' \in R^n$

except on a set of measure zero.

To obtain a linear function of output, consider one-to-one transformation of the system (1). For this system, coordinate transformation

$$x = \Psi(x') = (h'_1(x') \quad \dots \quad h'_l(x') \quad x'_{i_1} \quad \dots \quad x'_{i_{n-l}})^T$$

is given by the function Ψ where $x'_{i_1}, \dots, x'_{i_{n-l}}$ are some state vector components functionally independent of the components of the function h' , h'_i is the i -th component of this function. Because of this choice, the function Ψ is invertible for all

$x' \in R^n$ except on a set of measure zero. In new coordinates the system takes the form

$$\begin{aligned} \dot{x}(t) &= \frac{\partial \Psi}{\partial x'} f'(x'(t), u(t)) = \\ & \frac{\partial \Psi}{\partial x'} f'(\Psi^{-1}(x(t)), u(t)) = f(x(t), u(t)), \quad (2) \\ y(t) &= h'(\Psi^{-1}(x(t))) = Hx(t), \end{aligned}$$

where $H = (I_{l \times l} \ 0)$, $I_{l \times l}$ is the identical $l \times l$ matrix.

To take into account faults and disturbances and to apply the logic-dynamic approach (Zhirabok and Usoltsev, 2002), the model (2) has to be transformed into the following form:

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) + C \cdot \begin{pmatrix} \varphi_1(A_1x(t), u(t)) \\ \dots \\ \varphi_p(A_px(t), u(t)) \end{pmatrix} + \\ & Dd(t) + L\rho(t), \quad y(t) = Hx(t), \end{aligned}$$

where F and G are matrices describing linear dynamics; A_1, \dots, A_p are matrices-rows; L and D are known constant matrices, the term $L\rho(t)$ models unknown parameters and unknown inputs to the actuator and to the dynamic process, the evaluation of the vector function $\rho(t)$ must generally be considered unknown; the term $Dd(t)$ models the faults: if there are no faults, then $d(t) = 0$, if a fault occurs, $d(t)$ becomes an unknown function; C is $n \times p$ matrix: is the right-hand side of the equation for the i -th component of the state vector of the system (2) contains nonlinearity $\varphi_j(A_jx, u)$, then $C(i, j) \neq 0$, otherwise $C(i, j) = 0$. Generally, the function φ_j has several terms of the form A_jx .

3 PRELIMINARY RESULTS

Firstly, consider the linear case when $C = 0$ and the system is described by the equations

$$\dot{x}(t) = Fx(t) + Gu(t) + Dd(t) + L\rho(t), \quad y(t) = Hx(t).$$

Description of the linear observer is found in the following form:

$$\begin{aligned} \dot{x}_*(t) &= F_*x_*(t) + G_*u(t) + Jy(t) + Kr(t), \\ y_*(t) &= H_*x_*(t), \end{aligned} \quad (3)$$

where K is the feedback gain matrix; F_* , G_* , J , and

H_* are matrices describing the observer; $x_*(t)$ is the state vector of the observer, $r(t)$ is a residual generated as $r(t) = Ry(t) - y_*(t)$ for some matrix R . If there are no faults and $\rho(t) = 0$, then $r(t) = 0$, if a fault occurs, $r(t) \neq 0$. The problem of the matrix K choice is considered in (Schreier *et al*, 1997).

It is supposed that for the healthy system the vectors $x_*(t)$ and $x(t)$ satisfy the equality $x_*(t) = \Phi x(t)$ for some matrix Φ satisfying the equations (Chen and Patton, 1999; Frank, 1990):

$$\Phi F = F_*\Phi + JH, \quad RH = H_*\Phi, \quad G_* = \Phi G \quad (4)$$

To ensure the reliable fault detection, the residual $r(t)$ has to be sensitive to the faults and invariant with respect to the unknown inputs $\rho(t)$, that is $\Phi L = 0$, $\Phi D \neq 0$ (Frank, 1990). Notice that in the case when $\Phi L = 0$ one says about full decoupling with respect to unknown inputs.

To design an observer in the linear case, there are a number of approaches, e.g., the eigenstructure assignment, the approach based on the Kronecker canonical form (Frank, 1990). Another linear procedure suggested in (Zhirabok *et al.*, 2010) also is based on the Kronecker canonical form. According to this approach, the matrices F_* and H_* describing the observer are represented in the canonical form

$$F_* = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}, \quad H_* = (0 \ 0 \ \dots \ 1).$$

In this case (4) may be presented in the form of the set of k equations:

$$\begin{aligned} RH &= \Phi_k, \quad \Phi_i F = \Phi_{i-1} + J_i H, \\ & i = 2, \dots, k, \\ \Phi_1 F &= J_1 H, \end{aligned} \quad (5)$$

where Φ_i and J_i are the i -th rows of the matrices Φ and J , $i = 1, \dots, k$, k is the dimension of the observer. It is shown in (Zhirabok *et al.*, 2010) that (5) can be transformed into the single equation

$$RHF^k = J_k HF^{k-1} + J_{k-1} HF^{k-2} + \dots + J_1 H. \quad (6)$$

The solution of this equation gives the minimal integer k and the matrices R and J ; then the rows of the matrix Φ are obtained from (5). This matrix is used to find the matrix $G_* = \Phi G$ and to check the conditions $\Phi L = 0$ and $\Phi D \neq 0$.

Shortcoming of this approach is that it does not allow to take immediately into consideration the condition $\Phi L = 0$, therefore one has to check whether or not the solution of (6) satisfies this condition. If not, then another solution must be found and the condition $\Phi L = 0$ must be checked again. To overcome this shortcoming, the new approach is suggested which allows to include the condition $\Phi L = 0$ in (5) and to obtain a full set of solutions with specified robust properties.

4 PROBLEM SOLUTION

4.1 Main Relationships

Introduce the matrix L_* of full rank such that $L_* L = 0$. This choice allows rewriting the condition $\Phi L = 0$ in the form $\Phi = M L_*$ for some matrix M . Replace in (5) the row Φ_i of the matrix Φ with $M_i L_*$ that gives the equations $RH = M_k L_*$, $M_i L_* F = M_{i-1} L_* + J_i H$, $i = 2, \dots, k$, $M_1 L_* F = J_1 H$, where M_i is the i -th row of the matrix M . Rewrite these equations as follows:

$$(R \quad -M_k)(H^T \quad L_*^T)^T = 0, \quad (7)$$

$$(M_i \quad -M_{i-1} \quad -J_i)((L_* F)^T \quad L_*^T \quad H^T)^T = 0, \quad (8)$$

$$(M_1 \quad -J_1)((L_* F)^T \quad H^T)^T = 0. \quad (9)$$

We begin to solve equations (7)-(9) from the last one finding at every step all linearly independent solutions. The result of each step is a conclusion about possibility to construct the observer satisfying the condition $\Phi L = 0$; if it is possible, then the observer is constructed, otherwise the dimension k is increased and the next step is fulfilled. Consider these steps in detail.

4.2 The First Step

Equation (9) has a solution in the case when rows of the matrices $L_* F$ and H are linearly dependent, this can be checked by the criterion

$$\text{rank} \begin{pmatrix} L_* F \\ H \end{pmatrix} < \text{rank}(L_* F) + \text{rank}(H). \quad (10)$$

If (10) is not valid, then full decoupling is impossible, and one has to use the robust methods (Frank, 1990); (Low et al., 1986); (Patton, 1994). Suppose that condition (10) is valid.

Let the matrix $(N_1 \quad -P_1)$ contains all linearly independent solutions of (9), then one can set $M_1 = W_1 N_1$ for some matrix W_1 . To check possibility to construct the observer of dimension $k = 1$, consider (7) replacing M_1 with $W_1 N_1$ and rewriting the result in the form

$$(R \quad -W_1) \begin{pmatrix} H \\ N_1 L_* \end{pmatrix} = 0. \quad (11)$$

Criterion of existence of this equation solution is the condition

$$\text{rank} \begin{pmatrix} H \\ N_1 L_* \end{pmatrix} < \text{rank}(H) + \text{rank}(N_1 L_*). \quad (12)$$

If it is valid, the observer of dimension 1 exists, it can be constructed as follows. Let the matrix $(R_0 \quad -P_0)$ contains all linearly independent solutions of (11), then the equality $R_0 H = P_0 N_1 L_*$ is valid, and one can set $R = W_0 R_0$ for some matrix W_0 . Notice that the matrix R_0 describes the set of all linearly independent solutions guarantees full unknown inputs decoupling for $k = 1$.

Choosing the certain matrix W_0 , one obtains $RH = W_0 R_0 H = W_0 P_0 N_1 L_*$. Comparing this equation with $RH = M_k L$ for $k = 1$, one concludes that $M_1 = W_0 P_0 N_1$. Then relation $N_1 L_* F = P_1 H$ obtained from (9) gives $W_0 P_0 N_1 L_* F = W_0 P_0 P_1 H$. This means that $\Phi_1 = W_0 P_0 N_1 L_*$, $J_1 = W_0 P_0 P_1$; set $G_* = \Phi_1 G$, and the observer has been constructed.

4.3 The Second Step

If (12) is not valid, it is necessary to find the observer of higher dimension. Consider (8) with $i = 2$, replace M_1 with $W_1 N_1$ and rewrite the result:

$$(M_2 \quad -W_1 N_1 \quad -J_2)((L_* F)^T \quad L_*^T \quad H^T)^T. \quad (13)$$

Since (13) contains additional addend $W_1 N_1 L_*$ in comparison with (9) which by assumption has a solution, then (13) has a solution as well.

Let the matrix $(N_2 \quad -Q_1 \quad -P_2)$ contains all linearly independent solutions of (13), then $M_2 = W_2 N_2$ for some matrix W_2 . To check possibility to construct the observer of dimension $k = 2$, consider (7) after replacing M_2 with $W_2 N_2$. It can be shown that such checking reduces to (11)

and (12) after replacing N_1 with N_2 and W_1 with W_2 .

It follows from (9) and (13) that all rows of the matrix N_1 are contained in the matrix N_2 , therefore one obtains additional possibility to satisfy the condition (12) and to solve (11) at the second step.

If (12) is valid, denote the solution of (11) by $(R_0 \ -P_0)$, set $R = W_0R_0$ that gives $RH = W_0P_0N_2L^*$ and $M_2 = W_0P_0N_2$. Then the relation $N_2L^*F = Q_1N_1L^* + P_2H$ obtained as a solution of (13), implies $W_0P_0N_2L^*F = W_0P_0Q_1N_1L^* + W_0P_0P_2H$, i.e. one can let $\Phi_2 = W_0P_0N_2L^*$, $J_2 = W_0P_0P_2$, and $M_1 = W_0P_0Q_1N_1$. Multiplying the equation $N_1L^*F = P_1H$ by $W_0P_0Q_1$, one obtains $W_0P_0Q_1N_1L^*F = W_0P_0Q_1P_1H$ and $\Phi_1 = W_0P_0Q_1N_1L^*$, $J_1 = W_0P_0Q_1P_1$.

Calculating the matrix $G_* = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} G$, one obtains the linear observer of dimension 2. Notice that one can solve (6) with $R = W_0R_0$ and obtain rows of the matrix Φ from (5).

If (12) is not valid, consider (8) with $i = 3$, replace M_2 with W_2N_2 and continue similar analysis of (13) after replacing N_1 with N_2 . Such a procedure continues as long as either the observer is constructed at some step or the condition $N_i = N_{i-1}$ is fulfilled for some i . The latter means that possibility to solve (11) does not improve, and absence of a solution at step $i - 1$ implies absence of that at next steps. In this case, one has to use the robust methods (Frank, 1990; Low *et al*, 1986).

4.4 Nonlinear Case

According to the logic-dynamic approach suggested in (Zhirabok and Usoltsev, 2002) for nonlinear systems, the nonlinear observer is based on the linear one constructing above. The nonlinear term for the model (3) is obtained as follows. Calculate the product and combine the similar terms, for

$$\Phi C \begin{pmatrix} \varphi_1(A_1x(t), u(t)) \\ \dots \\ \varphi_p(A_px(t), u(t)) \end{pmatrix} \quad (14)$$

example the sum $A_i x u_k + A_j x u_k$ is rewritten as $(A_i + A_j)x u_k = A_{ij} x u_k$; notice that this operation

allows to minimize the dimension of the observer. Then the matrix A is built up from the matrices-rows A_i and A_{ij} contained in the product (14) and the condition

$$\text{rank}(\Phi^T \ H^T) = \text{rank}(\Phi^T \ H^T \ A^T) \quad (15)$$

is checked. If it is valid, the equation

$$A = A_* \begin{pmatrix} \Phi \\ H \end{pmatrix} \quad (16)$$

is solved and the matrices $A_{*i_1}, \dots, A_{*i_d}$ are found, where d is the number of the matrix A rows. These matrices are used to form an argument of the nonlinear term $\varphi_*(x_*, y, u)$ by replacing the term

$A_{ij}x$ in (14) with $A_{*i} \begin{pmatrix} x_* \\ y \end{pmatrix}$ according to (16). As a result, the nonlinear observer takes the form

$$\dot{x}_*(t) = F_*x_*(t) + G_*u(t) + Jy(t) + \varphi_*(x_*(t), y(t), u(t))$$

If the condition (15) is not valid, one has to construct another linear observer of bigger dimension or to use the additional observer considered below.

4.5 Additional Observer Design

If (15) is not valid for all possible linear observers, the additional observer estimating some rows of the matrix A has to be used. Denote by A_0 the row of the matrix A for which the condition (15) is not valid. By analogy with (4), one can write the equations

$$A_0 = H_0\Phi_0, \quad \Phi_0F = F_0\Phi_0 + J_0H, \quad (17)$$

where the index “0” corresponds to matrices describing the additional observer. By analogy with the matrix Φ , the matrix Φ_0 has to satisfy the condition $\Phi_0 = M_0L^*$ for some matrix M_0 . Replace the matrix Φ_0 in (17) with M_0L^* and rewrite the equations obtained as follows:

$$A_0 = H_0M_0L^*,$$

$$(M_0 \ -F_0M_0 \ -J_0)(L^*F)^T \ L_*^T \ H^T)^T = 0.$$

Denote the solution of these equations by D_0 and $(D_1 \ D_2 \ D_3)$, respectively. It follows from the above equations that $D_0 = H_0M_0$, $D_1 = M_0$, $D_2 = -F_0M_0$, and $D_3 = -J_0$. The first three equalities show that rows of the matrices D_0 and D_2 must be linearly depended on rows of the matrix

D_1 . Therefore, if some row of the matrix D_2 is independent of the rows of the matrix D_1 , then this row must be removed from the matrix $(D_1 \ D_2 \ D_3)$. Such a procedure has to be applied to every row of the matrix D_1 .

Denote the result of this analysis as $(D'_1 \ D'_2 \ D'_3)$. Then the procedure similar to the one suggested above is applied to the matrices D_0 and D'_2 with removing the appropriate rows of the matrix D_0 . If the resulting matrix D'_0 is not equal to zero, the additional observer exists. To construct this observer, solve the algebraic equation $D'_2 = -F_0 D'_1$ for F_0 , set $J_0 = -D'_3$, $\Phi = D'_1 L^*$, $G_0 = \Phi G$, and $A_0 = D'_0 L^*$. If $D'_0 = 0$, then the additional observer invariant with respect to the unknown inputs does not exist and the robust methods must be used.

5 PRACTICAL EXAMPLE

Consider the general electric servoactuator of manipulation robots studied in (Zhirabok et al., 2010). The servoactuator dynamic, with the backlash and elasticity taken into account, may be described by the following nonlinear equations:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= (1/W)(-K_r + w)x_2 - M_r + C_r i_r \mathbf{BI}(x_3 - i_r x_1), \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{1}{J_M}(-K_d x_4 + K_M x_5 - M_d - C_r \mathbf{BI}(x_3 - i_r x_1)), \\ \dot{x}_5 &= (1/L)(-K_\omega x_4 - R x_5 + u). \end{aligned}$$

Here x_1 and x_2 are the output rotation angle and velocity at the reducer output shaft, respectively; x_3 and x_4 are the output rotation angle and velocity at the motor output shaft, respectively; x_5 is the current through the servoactuator windings; W and w are the components of the inertia and velocity, respectively; M_d and M_r are the moments of the Coulomb friction at the motor and reducer shaft output, respectively: $M_d = M_{d0} \text{sign}(x_4)$, $M_r = M_{r0} \text{sign}(x_2)$; K_d and K_r are the respective coefficient of viscous friction of the motor and reducer output shaft; i_r is the reducing ratio of the reducer; C_r is the rigidity coefficient of the mechanical reducer; J_M is the moment of inertia of

the electric servoactuator and of the rotating parts of the reducer; K_ω and K_M are the respective coefficients of the counter EMF and of the torque; R and L are the active and inductive resistances of the electric servoactuator windings, respectively; the function \mathbf{BI} describes the backlash:

$$\mathbf{BI}(z) = 0.5(|z| - \sigma)(\text{sign}(z + \sigma) + \text{sign}(z - \sigma)),$$

2σ is the backlash span, $z = x_3 - i_r x_1$.

Suppose that $y_1(t) = x_1(t)$, $y_2(t) = x_2(t)$, $y_3(t) = x_4(t)$. According to the logic-dynamic approach, this system has the following matrix description:

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{K_r + w}{W} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -K_d/J_M & K_M/J_M \\ 0 & 0 & 0 & -K_\omega/L & -R/L \end{pmatrix},$$

$$G = (0 \ 0 \ 0 \ 0 \ 1/L)^T, \quad H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Nonlinear term is described as follows:

$$\begin{pmatrix} \varphi_1(A_1 x(t), u(t)) \\ \dots \\ \varphi_p(A_p x(t), u(t)) \end{pmatrix} = \begin{pmatrix} \text{sign}(A_1 x) \\ \text{sign}(A_2 x) \\ \mathbf{BI}(A_3 x) \end{pmatrix},$$

$$A_1 = (0 \ 1 \ 0 \ 0 \ 0), \quad A_2 = (0 \ 0 \ 0 \ 1 \ 0), \\ A_3 = (-i_r \ 0 \ 1 \ 0 \ 0),$$

$$C = \begin{pmatrix} 0 & 0 & 0 \\ -M_{r0}/W & 0 & C_r i_r / W \\ 0 & 0 & 0 \\ 0 & -M_{d0}/J_M & -C_r / J_M \\ 0 & 0 & 0 \end{pmatrix}.$$

Suppose that $D = (0 \ 1 \ 0 \ 0 \ 0)^T$ and $L = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}^T$. Obviously,

$$L^* = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

It can be shown that (10) and (12) are valid therefore one can find from (11) $R_0 = (1 \ -1 \ 0)$; set $W_0 = 1$. Equation (6) is solvable for $k = 1$:

$$RHF = (0 \ \frac{K_r + w}{W} + 1 \ 0 \ 0 \ 0) = (\frac{K_r + w}{W} + 1)H_2.$$

Since condition (15) is not valid for the matrices A_3 , the solution must be improved. An analysis shows that increase of the observer dimension cannot overcome this difficulty therefore the additional observer estimating the variable A_3x must be used. It can be shown that the main and additional observers are described as follows:

$$\begin{aligned}\dot{x}_{*1} &= 11.01y_2 + 10\text{sign}(y_2) - 200.00011\mathbf{BI}(z_*), \\ \dot{x}_{*2} &= -100x_{*2} - 5x_{*3} + 0.2478u_1 + 90.09\mathbf{BI}(z_*), \\ \dot{x}_{*3} &= 45.05u_1 - y_3 - 1802\mathbf{BI}(z_*), \\ r &= y_1 + y_2 - x_{*1},\end{aligned}$$

where $z_* = 200x_{*2} - 1.1x_{*3} - 100y_1 + y_3$. Numerical values of the electrical servoactuator parameters can be found in (Zhirabok et al., 2010).

The residual $r(t)$ time behavior is shown in Figure 1, the fault occurred at $t = 30$ s; obviously, the disturbance does not influence on the residual.

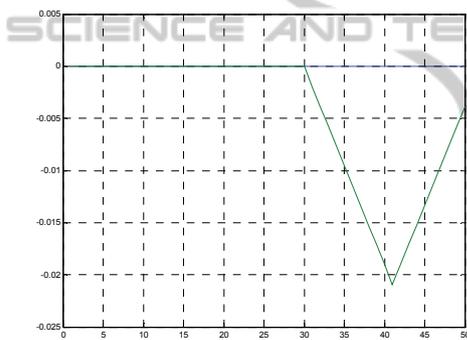


Figure 1: Simulation results.

6 CONCLUSIONS

The problem of robust fault detection and isolation in mechatronic systems described by nonlinear models with non-smooth nonlinearities has been considered. The logic-dynamic approach suggested in the paper allows solving this problem by linear methods. The method which allows obtaining a full set of solutions with full decoupling with respect to unknown inputs has been suggested.

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