

Three Dimensional Localisation in Underwater Swarms through a Kalman Approach

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Abstract: A three dimensional localisation algorithm for a swarm of underwater vehicles is presented. The proposed approach is grounded on an extended Kalman filter (EKF) scheme used to fuse some proprioceptive data such as the vessel's speed and some exteroceptive measurement such as the time of flight (TOF) sonar distance of the companion vessels. The results of several simulations are presented. Some considerations about the available underwater bandwidth and the communication needed by the approach are discussed.

1 INTRODUCTION

The exploration of the oceans both for scientific and economic purposes is becoming more and more important. Out of the limitations of our biological characteristics, underwater robotics has gained an essential role in the study and exploitation of the seas. Its more promising branch is that of the autonomous underwater vehicles (AUV), i.e. those vehicles that are capable of performing the required tasks without human supervision, coping with the mission unknowns.

In the latest years the research on AUVs has broadened towards the simultaneous use of more vessels, i.e. the implementation of multi robot configurations all the way to full swarms of underwater vehicles.

Whether a single or more AUVs are considered, one of the focal points of autonomy is the reliable knowledge of the vessel position and orientation. Unfortunately an underwater system suffers because of the limiting characteristics of its environment. Water, especially salted one, blocks electromagnetic waves, inhibiting the use of positioning systems such as the GPS. At the same time this implies a difficult communication between an AUV and another or a remote operator. The available means to localise a single AUV are thus the exploitation of inertial sensors, velocity ones and/or gyroscopes combined in dead reckoning.

However, in the framework of an underwater swarm, the localisation of a single vehicle can profit from the gathering of information pertaining to the other fellow vessels. The key issue of all the swarm localisation methods is the best possible combination of proprioceptive measures (dead reckoning) and exteroceptive sensor readings, the main difference being the employed estimator. The localisation of swarms of robots has been extensively studied for packs of terrestrial surface robots. In this case it is possible to use GPS, if outdoors, or different methods for indoor teams. In addition the communication of information among the robots is unproblematic over the radio link.

An approach is based on the subdivision of the swarm in subgroups one of which, in turn, is kept at a fixed position and acts as a set of landmarks for the moving others (Kurazume et al., 1994, 1996) and (Rekleitis et al., 2002). (Fox et al., 1999; Fox et al., 2000) and (Thrun et al., 2000) have successfully employed belief functions combined with a Montecarlo approach and particle filtering optimization. The work of (Roumeliotis, 2000) and (Roumeliotis and Bekey, 2002) employs a Kalman filter where the proprioceptive measures are used to estimate the future state of the system and the exteroceptive ones are used to correct and update the estimate. In (Martinelli et al., 2005) this approach is extended by considering the most generic relative observations between two robots. More recently the work of Olfati-Saber, see e.g. (Olfati-Saber, 2007),

has addressed the problem of decentralized Kalman filtering in sensor networks through consensus algorithms for the swarm controlling strategies. (Huang et al., 2011) have investigated the consistency of EKF based cooperative localisation considering observability. The typical operative environment considered in these works is a two dimensional terrestrial one.

In the following a three dimensional algorithm for the global positioning of a swarm of underwater robots is presented, the problem addressed is not that of swarm control, but the mutual and absolute positioning of elements composing an underwater group of robots. A simple kinematic model of the AUV is considered, capable of measuring its own velocity and to communicate over an ultrasonic acoustic link with the other vessels. Through the measurement of the time of flight (TOF) of the ultrasound transmission the AUVs can measure one another their relative distance. All the available information is then combined with an extended Kalman filter distributed among the vessels.

In the second section of the paper the problem is stated in a three dimensional environment. In the third section the multi robot Kalman based algorithm is described. In the fourth section some experimental results are presented relative to different swarm trajectories. In the fifth section some consideration on the presented algorithm are discussed especially concerning the communication issues and the swarm size.

2 PROBLEM STATEMENT

The key point for a cooperative localisation in a swarm of robots is viewing the group as a single entity that can access the information of a large number of proprioceptive and exteroceptive sensors.

In the following all the vessels are described by the same motion equations and each robot possesses proprioceptive sensors for the motion estimate. Each AUV possesses also an ultrasound communication link, which can collect the information from the other vessels in the swarm (among these the transmitter ID and the communication starting time) and through the TOF measure the estimated inter vessel distance. The measurement noise is considered as Gaussian.

At first the localisation has been tackled considering as available the relative position and orientation measurements, this is helpful to check the overall reliability of the framework development. In a second phase only the relative distances and the

absolute values of depth and heading has been considered, this second case being similar to the actual underwater conditions. As above described the relative distance can be easily measured with the sonar communication while heading can be read from the compass and depth from a pressure gauge on board the vessels.

Let us consider the global dynamical state X of the whole swarm, it will be a vector composed of $M \times N$ items where M is the number of robots and N is the number of variables describing the single vehicle, i.e. a vector composed of the poses \bar{x}_i of all the robots.

$$\bar{X} = [\bar{x}_1, \bar{x}_2, \dots, \bar{x}_M] \quad (1)$$

The mathematical model describing the time evolution of the single vessel of the swarm is:

$$\bar{x}_i[k] = f(\bar{x}_i[k-1], \bar{u}_i[k-1], \bar{w}_i[k-1]) \quad i=1, \dots, M \quad (2)$$

where f is generally a non linear function of the state at the preceding time step $\bar{x}_i[k-1]$, of the input $\bar{u}_i[k]$ and of the noise $\bar{w}_i[k]$. The vessels can also measure all the other ones and this can be described by the:

$$z_i[k] = h(\bar{x}_i[k], \bar{x}_j[k], n_i[k]) \quad i=1, \dots, M, j \neq i \quad (3)$$

here h is the measurement function linking the state of the i -th robot with the state of the measured one (the j -th) and the measure noise $n_i[k]$.

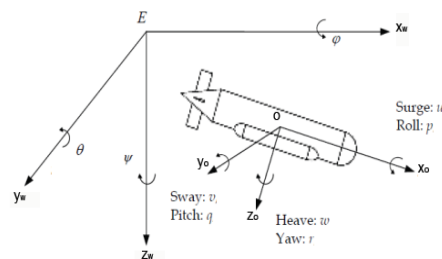


Figure 1: The coordinate system.

An extended Kalman filter estimates the state of this dynamical system fusing data coming from proprioceptive sensors and exteroceptive ones. The proprioceptive sensors are used to compute the kinematic time evolution of the system and the exteroceptive to reset periodically the time evolution estimate with an external *ground truth*.

The vessel coordinate system is centred in the centre of the vehicle and its x axis is longitudinally directed from stern to bow, the y axis is towards starboard and the z one downward, see Figure 1.

The kinematic model of the single robot uses a linear velocity parallel to the x axis (thrust) and the

possibility to change all the three Euler angles through appropriate angular velocities.

3 MULTI-ROBOT KALMAN LOCALIZATION

The Kalman filtering is a well known strategy that yields an estimate of a dynamical process using a feedback control. It foresees the process state at a given time and it employs a measurement feedback to update the state through a better estimate. It is an iterative process that loops through two different phases: on one side it predicts the state of the system and the error covariance, on the other it computes the so called Kalman gain to correct both the state estimate and the error covariance on the grounds of some kind of measure. Since the time evolution function (equation 2) may be not linear, an extended version of the filter has been here employed. The EKF basically behaves as the standard procedure but uses a local linearization of the functions. A very interesting characteristic of this filter is its iterative aspect. The results of a iteration of the filter is used as input for the successive step; in this way the filter retains memory of the history of the system.

Let us consider the whole swarm, the EKF cycles between the two phases of prediction and update.

3.1 Prediction

Each robot, at a given time step, estimates its state at the successive time step on the grounds of the kinematic model and the available proprioceptive measures (linear and angular velocities) and their null average Gaussian noise.

$$\hat{x}_i^-(k+1) = f(\hat{x}_i^+(k), u_i(k), w_i(k)) \quad (4)$$

$$\hat{P}_{ii}^-(k+1) = \Phi_i(k)\hat{P}_{ii}^+(k)\Phi_i^T(k) + G_i(k)W_i(k)G_i^T(k) \quad (5)$$

$$\hat{P}_{ij}^-(k+1) = \Phi_i(k)\hat{P}_{ij}^+(k)\Phi_j^T(k) \quad (6)$$

here equation (4) is the state time evolution and equations (5) and (6) describe the time evolution of the cross correlation matrix P in the diagonal and off diagonal terms; Φ is the system propagation matrix, G is the system noise input matrix and W is the noise input covariance. The minus sign stands for *a priori* and the plus one for *a posteriori*.

In order to perform a distributed EKF it is convenient to process the *a posteriori* estimated cross correlation matrix (equation 6) through a singular value decomposition (SVD). In this way each robot can compute its own term multiplying the

SVD term by its dynamical matrix, see (Roumeliotis and Bekey, 2002).

3.2 Update

Every time a robot measures something, an update can be performed. The here considered measures are the heading (compass) and the depth (pressure gauge) of the measuring vessel and the TOF distance of another vessel. The non linear measuring function h is shown in equation (3) and the noise is a null average Gaussian one. It is now possible to compute the *a posteriori* state estimate

$$\begin{aligned} \hat{x}_r^+(k) &= \hat{x}_r^-(k) + K_r(k)(z_i(k) - h(\hat{x}_i^-, \hat{x}_j^-)) \\ K_r(k) &= \hat{P}_{ri}^-(k)H_i^T(k) + \hat{P}_{rj}^-(k)H_j^T(k)S^{-1} \end{aligned} \quad (7)$$

where the index r describes the vessel $r=1, \dots, M$, $K_r(k)$ is the so called Kalman gain with S the residual covariance, and the last term is the residual. The H terms are the Jacobians of the measuring function h w.r.t. the two state vectors x_i and x_j :

$$H_i = \begin{bmatrix} -\frac{\Delta x}{\sqrt{\Delta r^2}} & -\frac{\Delta y}{\sqrt{\Delta r^2}} & -\frac{\Delta z}{\sqrt{\Delta r^2}} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$H_j = \begin{bmatrix} \frac{\Delta x}{\sqrt{\Delta r^2}} & \frac{\Delta y}{\sqrt{\Delta r^2}} & \frac{\Delta z}{\sqrt{\Delta r^2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

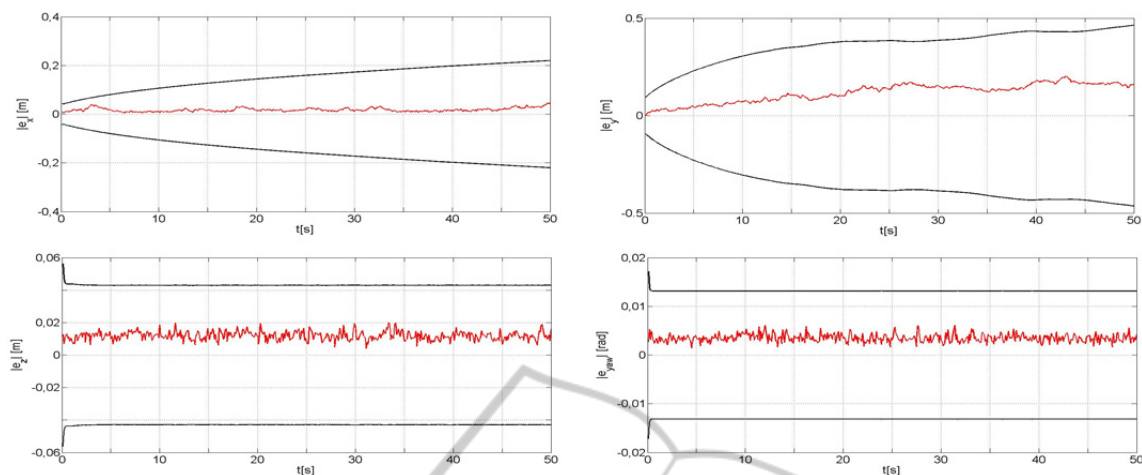
and $\Delta r^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$.

Finally the *a posteriori* covariance matrix estimate is:

$$\hat{P}_{rs}^+ = \hat{P}_{rs}^- - (\hat{P}_{ri}^-H_i^T + \hat{P}_{rj}^-H_j^T)S^{-1}(H_i\hat{P}_{is}^- + H_j\hat{P}_{js}^-) \quad (10)$$

with the indexes $r=1, \dots, M$ and $s=1, \dots, M$.

The problem of the localization of multiple robots can be approached in two different ways: in a centralised or in a distributed way. In the first there is a central supervisor collecting all the data from all the vehicles and performing the multi robot system state estimation. The second paradigm can be split into two further classes: uncooperative or cooperative algorithms. The first class simply tries to localise each robot as if it was alone in the world, i.e. counting on its own estimate and measures alone, without gathering further information from the others in the swarm. The second class can


 Figure 2: Average and 3σ error for x , y , z and yaw as a function of time, linear trajectories.

exploit the information coming from the companions and performing a local algorithm for the pose estimation.

Regardless of the employed approach the basic issue is the extremely limited bandwidth that is available underwater. Two or more AUV can communicate one another through different means, but the only one that allows long distance data exchange is the ultrasound transducer. Naturally there is a trade off between bandwidth and distance travelled by the ultrasound wave. All this points to a severe limitation on data circulation among the swarm vessels. In other words transmissions should be kept to a minimum with respect to the algorithm performance.

Unfortunately the Kalman approach needs to distribute a large quantity of data. In order to limit the communication problem in the swarm a distributed-centralized approach has been tested. The idea is as follows. Whenever a measurement is done, the measuring robot performs the EKF for the whole swarm, gathering the states from all the vessels and broadcasting the new matrices to the companions. It is obviously a centralised algorithm but it is distributed in time, at each time step only the measuring robot is computing, and at the next time step it will probably be a different one.

This scheme may limit the communication among the swarm members. The amount of exchanged data will be the same of a fully centralized approach, but since only one robot is actually broadcasting its results, there will be much less problems arising from the communication overheads and possible multipaths deriving from multiple robots trying to communicate all at the same time.

4 EXPERIMENTAL RESULTS

In the experiments, the vehicles are considered as kinematic objects, i.e. without the computation of their dynamics, and are able to exchange information instantaneously. All the simulations have been performed under Matlab. A first series of 2D simulations have been carried out to assess the overall correctness of the algorithm implementation.

In the 3D algorithm version the due kinematic model has been considered and a more realistic suite of sensors for the single vessel has been considered as well. As said, each AUV is equipped, besides the sonar for communication and distance estimation, with a compass for the absolute orientation and a pressure gauge for the depth measure (z coordinate). The introduction of these sensors and data are of basic importance since they improve the *observability* of the system. In a three dimensional environment each vessel possesses six degrees of freedom, thus the overall system can be considered as unbalanced towards non observability.

Let us consider a set of $M=10$ vessels moving in parallel along a straight trajectory. The standard deviation on linear and angular speed is 0.1 m/s and 0.05 rad/s respectively. The standard deviation on the TOF distance measure is 0.05 m , on the heading is 1° and 0.07 m on the depth. All these values are consistent with low cost sensors commonly available on the market.

In Figure 2 are shown the average error (on the whole swarm of ten) and the 3σ error relative to the x , y , z coordinates and the yaw angle. For clarity the roll and pitch ones are not showed. The average position error after 50 meters is of about 0.35%,

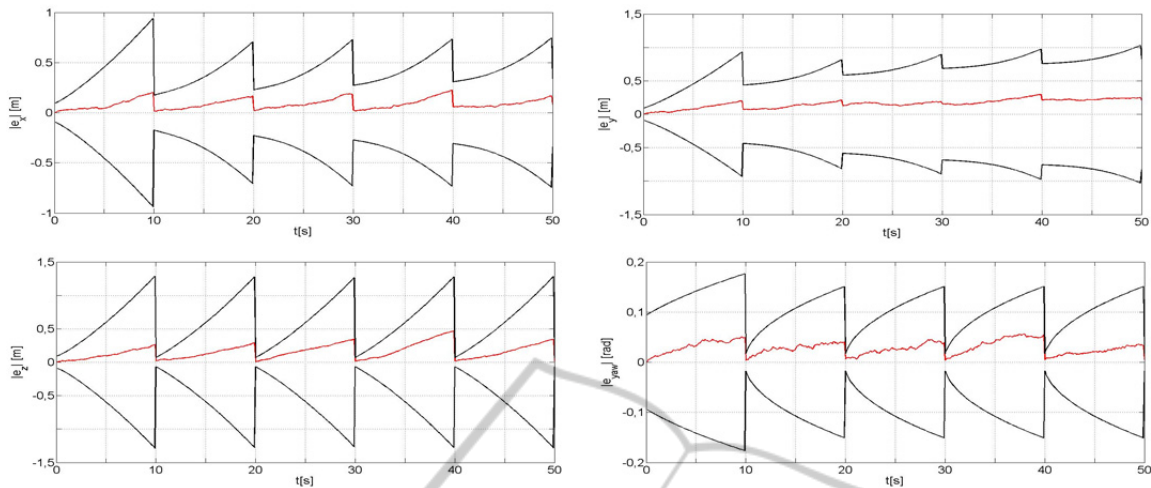


Figure 3: Average and 3σ error for x , y , z and yaw for the periodical exteroceptive measures.

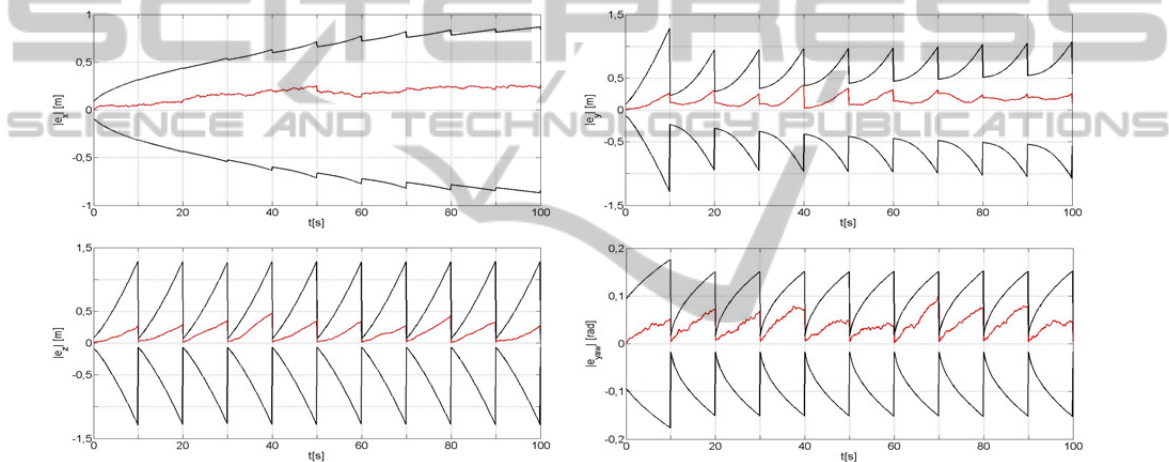


Figure 4: Average and 3σ error for x , y , z and yaw for the sinusoidal trajectory test case.

while the 3σ error is of about 1%. The same quantities considering dead reckoning alone are 8% and 30%.

As above mentioned the underwater realm is quite a difficult one and the communications should be kept at a minimum, hence it has been experimented a situation in which the vessels do not measure continuously the distance of fellow robots but perform an exteroceptive measure once every 100 time steps. The results are presented in Figure 3.

It is evident the saw tooth shape due to the periodical correction of data. In this case after 50 m the average and 3σ errors are respectively 0.46% and 1.8%, slightly larger than the continuous case, as it could have been expected.

In Figure 4 is shown the algorithm performance in the case of sinusoidal trajectories, i.e. with a

constant linear velocity but an angular one slowly varying in time as a sinusoid. Also in this case the measures are periodic as in the previous experiment and the number of time steps is doubled and the average and 3σ errors are 0.28% and 1%.

Figure 5 and 6 present the trial in which the vessels are made follow a circular trajectory for 1300 m, still with periodic exteroceptive measures. In this case the two errors are respectively 0.05% and 0.2%. In Figure 5 only x and y are shown for better intelligibility.

In Figure 7 it is shown the dependence of the average position error as a function of the number of vehicles in the swarm. It is clear that “union is strength”: the more the vessels the better the estimate, until an asymptote is reached.

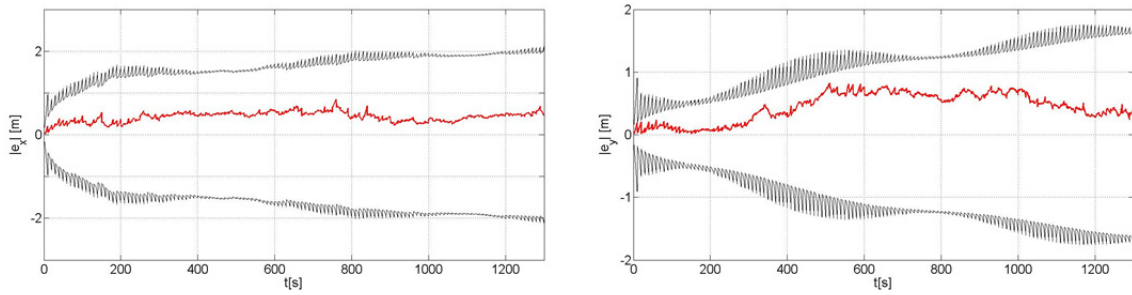


Figure 5: Average and 3σ error for x and y in the circular test case.

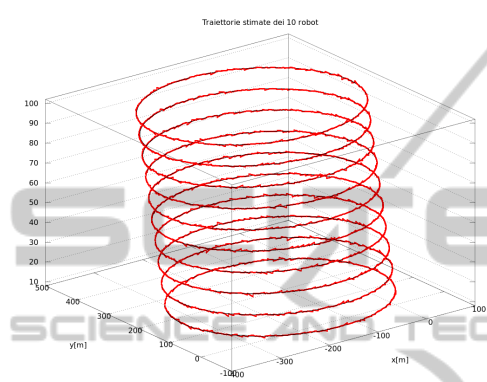


Figure 6: Vessels trajectories in the circular case.

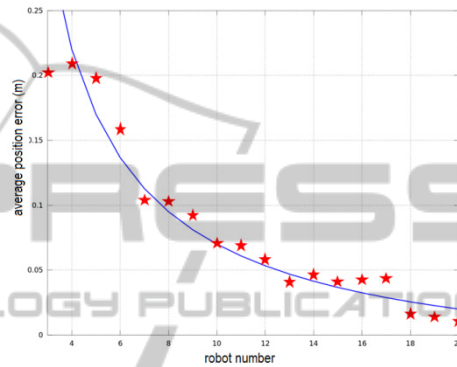


Figure 7: Average position error versus the swarm numerosity.

5 DISCUSSION AND CONCLUSIONS

This work has presented the results of a three dimensional Kalman based localisation algorithm for a swarm of underwater vehicles.

In a three dimensional environment each vessel possesses six degrees of freedom, thus the overall system is heavily undetermined, i.e. the covariance on the system state quickly diverges. The introduction of real world measures such as the yaw angle (compass) and the z coordinate (pressure gauge) greatly improves the Kalman filter performance, enhancing the system observability.

It is here important to recall that the presented scheme greatly relies on communication among the members of the swarm. During the Kalman computation the various vehicles must distribute to the others their own estimates and covariance and all the cross correlations. This heavy communication scheme suggested the periodical exteroceptive measures in order to reduce the number of Kalman updates. Notwithstanding a reduced set of measures, the system is able to assure a good localisation.

In the actual physical swarm, presently under development, there will be two possible sonar

communication channels, one around 300 Kbit/s and a second at a higher frequency but at a smaller range, around 1 Mbit/s. Let us now assume for the sake of simplicity that all the transmission band can be allocated to the Kalman 3D localisation. It is possible to compute the total number of bytes to be transmitted if only one robot makes a periodical observation (one every 100 time steps as above) and consider this as a lower bound. If all the vehicles measure, this quantity should be multiplied by the number of robots. These two functions are plotted in Figure 8 where the two bounds are shown together with the two possible transmission rates.

The diagram should be read as follows. If the available transmission link is the lower, this localisation system may work for a swarm smaller than 25 members, if only one observer is allowed at a time or with less than 13 if everybody can measure. With the higher throughput these figures rise to 47 and 19.

The devised algorithm strategy is based on a mixed distributed-centralised approach. Each robot computes the Kalman filter for all of the system elements and it distributes its results to all the community, since a different robot will be the next to observe and compute the system state.

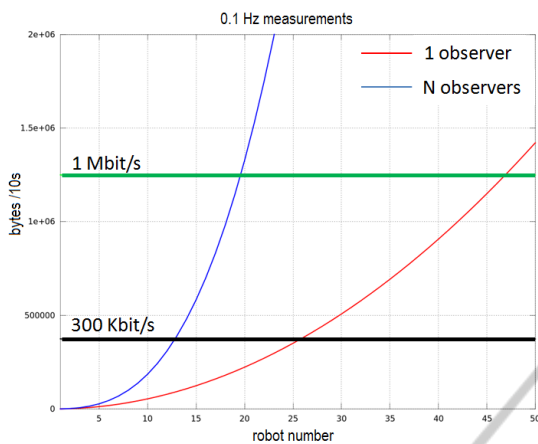


Figure 8: Needed throughput as a function of the swarm robot number.

In conclusion it is possible to affirm that the presented 3D Kalman based localisation system can be employed for a swarm of underwater robots, yielding accuracy in the computed positions, but with a limited swarm numerosity. Nevertheless further work is needed in order to reduce the so precious communication bandwidth in underwater environments and or allow more numerous swarms.

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