

FouSE: An Android Tool to Help in the Teaching of Fourier Series Expansions in Undergraduate Education

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Abstract: This paper presents an Android application to help in the teaching of Fourier series expansions in undergraduate Electrical Engineering. Consequently, it discusses the teaching of Fourier series concepts in connection with undergraduate Electrical Engineering education; some of the basic Fourier series theory is briefly reviewed. The presented Android application has been found useful in this context. As expected, the application has an easy-to-use, friendly interface, and can be viewed as a tool to help undergraduate students test and assess the Fourier series expansions on a typical set of signals, whose analytical Fourier series coefficients were found during the theoretical lectures. Additionally, some of its main characteristics include the ability for the students to control the total approximation error and the number of terms/harmonics used in the expansion.

1 INTRODUCTION

Among the most important studied topics in undergraduate Electrical Engineering education are Fourier series expansions (and Fourier theory in general). As a natural consequence, a large fraction of the signal processing and also mathematical literature is devoted to the teaching of Fourier series expansions (and Fourier theory in general). Having a solid understanding on the most basic principles of Fourier series expansions is of utmost importance, especially when dealing with problems which fall, for example, in the scope of multidimensional signal reconstruction, and which includes sampling theory, interpolation, extrapolation, signal and image conditioning, interactive image repair, deconvolution and other inverse problems, reconstruction in tomography, filter design, and much more.

Moreover, Clark Quinn (2000, <http://www.linezine.com/2.1/features/cqmmwiyp.htm>) defines Mobile Learning (*mLearning*) as “*eLearning* through mobile computational devices: Palms, Windows CE machines, even your digital cell phone”. Harris, (Harris, 2001), defines *mLearning* as the

intersection point between mobile computing and *eLearning*, producing a learning experience anytime and anywhere. *mLearning* is characterized by the capacity of accessing learning resources from anywhere, at any time, with high search capacities, high interaction, high support for an effective learning and a constant valorization based on the performance. So, one can see *mLearning* as an extension of *eLearning*, but characterized by its capacity of being independent in terms of space and time. The idea is to have small devices capable of being linked to the net, easy data input, and the ability to display high resolution images and also (very) good sound capacities. Hence, *mLearning* can benefit from all advantages of the web added to mobile technologies. In summary, we may say that *mLearning* relies on the utilization of mobile technologies at the service of the processes associated with teaching and learning. There is a huge number of mobile applications that have been recently developed with the aim of being used within the *mLearning* context. For example, in (Holzinger et al., 2012) the authors discuss the development of smart adaptive user interfaces for mobile *e-Business* applications, and the lessons learned. In (Mujacic

et al., 2012), Modelling, Design, Development and Evaluation of a Hypervideo Application for Digital Systems Teaching is presented and discussed. The process of life-long learning, in conjunction with three different models for mobile/ubiquitous applications is discussed in (Holzinger et al., 2010).

We believe that a first course focusing on Fourier series expansions would gratefully benefit from the fact of having a tool that let students test and assess Fourier series expansions, anytime and anywhere they want, and using their own Android powered devices (for example, during the lunch time conversations, using their own smartphones and tablets). Moreover, this application can be a welcome addition to the background of Electrical Engineering students, especially to those interested in telecommunications, information theory, or signal/ image processing in general.

Solving problems while experimenting and comparing the available algorithms is of fundamental importance in the learning process. The application presented in this paper was aimed at helping students do that.

Additionally, we felt a need for tools that could be easily used inside or outside the classroom, easily updated and/or maintained, and that the students could use with their own smartphones/tablets.

The development of the type of application proposed and described here is additionally justified by the increasing number of available mobile devices with the required functionalities and capacities, described above, and the number of students using them.

We believe that a small introduction to Fourier series will help to clarify the ideas that lead to the development of the application. The next section is dedicated to this.

2 FOURIER SERIES DEFINITION

A signal (function) $s(t)$ is said to have period T if $s(t+T) = s(t)$ for all t . If the signal is piecewise continuous in a period, with finitely many points of discontinuity and finitely many maxima or minima (also known as Dirichlet's conditions), then the Fourier series converges to $(s(t+) + s(t-))/2$ (see, for example (Tolstov, 1976), for details on existence and convergence of the Fourier series). In that case, the Fourier series of the signal $s(t)$ is given by

$$s(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t),$$

with

$$A_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} s(t) dt,$$

$$A_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} s(t) \cos(n\omega_0 t) dt$$

and

$$B_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} s(t) \sin(n\omega_0 t) dt,$$

where $\omega_0 = 2\pi f$, and $f = \frac{1}{T}$, the fundamental frequency of $s(t)$.

Fourier series may come in different flavors and tastes. Here, we are interested in the different connections and interpretations it may have from a future electrical engineer point of view. For example, see chapter 5 of (Brigham, 1974) or (Amardar, 1995) for the connections between the complex and trigonometric forms of the Fourier series, the Fourier series as a special case of the Fourier integral, waveform sampling, and sampling theorems.

2.1 Fourier Series of Even, Odd and Half-wave Symmetric Signals

The Fourier series expansion of an even signal $s_e(t)$ with the period of T does not involve the terms with sin and has the form

$$s_e(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t),$$

where the Fourier coefficients are given by

$$A_0 = \frac{2}{T} \int_0^{+\frac{T}{2}} s_e(t) dt,$$

and

$$A_n = \frac{4}{T} \int_0^{+\frac{T}{2}} s_e(t) \cos(n\omega_0 t) dt.$$

Accordingly, the Fourier series expansion of an odd T -periodic signal $s_o(t)$ consists of sine terms only and has the form

$$s_o(t) = \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t),$$

where the coefficients B_n are

$$B_n = \frac{4}{T} \int_0^{+\frac{T}{2}} s_o(t) \sin(n\omega_0 t) dt.$$

A T -periodic signal $s_h(t)$ is said to have half-wave symmetry if $s_h(t \pm \frac{T}{2}) = -s_h(t)$. In that case, the Fourier series expansion coefficients will be

$$A_0 = 0,$$

$$A_n = \begin{cases} 0, & n \text{ even,} \\ \frac{4}{T} \int_0^{+\frac{T}{2}} s_h(t) \cos(n\omega_0 t) dt, & n \text{ odd,} \end{cases}$$

and

$$B_n = \begin{cases} 0, & n \text{ even,} \\ \frac{4}{T} \int_0^{+\frac{T}{2}} s_h(t) \sin(n\omega_0 t) dt, & n \text{ odd.} \end{cases}$$

2.2 A Typical Example—Square Wave

The next example shows a typical calculation of the Fourier series expansion coefficients, that we usually solve during the theoretical-practical lessons.

Let the periodic signal $s(t)$ be defined by

$$s(t) = \begin{cases} -1, & -2 < t < -1, \\ 1, & -1 < t < 1, \\ -1, & 1 < t < 2. \end{cases} \quad (1)$$

Typically, to find the coefficients of its Fourier series expansion we begin by plotting the signal, like the one presented in figure 1. From this figure (or from equation 1) we can easily find that the period is $T = 4$, leading to $\omega_0 = 2\pi/T = \pi/2$, and that $s(t) = -s(t)$, leading to $B_n = 0$. Additionally, we may also note that $s(t + \frac{T}{2}) = -s(t)$ (half-wave symmetry), leading to $A_n = 0$ for all n even. In conclusion, we only need to find the coefficients A_n for n odd, and so the Fourier series expansion will be of the form

$$s(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} A_n \cos\left(n\frac{\pi}{2}t\right).$$

By the definition above,

$$\begin{aligned} A_n &= \frac{4}{T} \int_0^{+\frac{T}{2}} s(t) \cos(n\omega_0 t) dt, \\ &= \frac{4}{T} \left\{ \int_0^1 \cos(n\omega_0 t) dt + \int_1^2 -\cos(n\omega_0 t) dt \right\}, \\ &= \frac{4}{T} \left\{ \left[\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_0^1 - \left[\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_1^2 \right\}, \\ &= \frac{4}{\frac{2\pi}{\omega_0} n\omega_0} \left[\sin\left(n\frac{\pi}{2}\right) - \sin(n\pi) + \sin\left(n\frac{\pi}{2}\right) \right], \\ &= \frac{4}{2n\pi} \left[2\sin\left(n\frac{\pi}{2}\right) \right], \end{aligned}$$

i.e.,

$$A_n = \begin{cases} -\frac{4}{n\pi}, & n = 3, 7, 11, \dots, \\ \frac{4}{n\pi}, & n = 1, 5, 9, \dots. \end{cases}$$

Note that the signal plotted in figure 1 and the corresponding expansion coefficients are the same used in the “square”-like wave form used in the available signals of the application (also illustrated in figure 3).

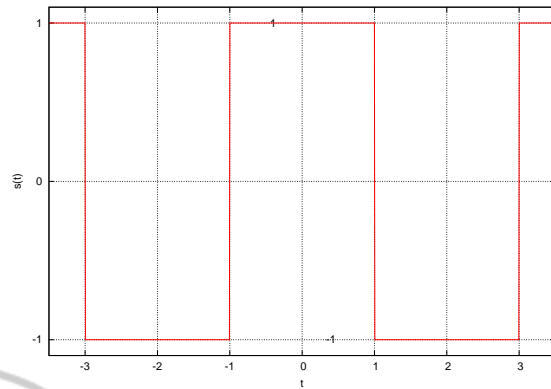


Figure 1: Plot of the square-like wave signal defined by equation 1.



Figure 2: Result of shifting and translating by one the signal presented in equation 1.

Now, suppose that we shift the signal up and translate it right by one. The resulting signal, $s_s(t)$, is presented in figure 2. As a consequence, the signal will no longer be even, and will also lose its half-wave symmetry. A common mistake our students sometimes do is to assume that the translation of the signal converts it into an odd one, but the shift up also removes this symmetry (despite of the signal shape, which still is a square-like one). In this particular case the new coefficients will be

$$\begin{aligned} A_0 &= 1, \\ A_n &= 0, \\ B_n &= \frac{8}{n\pi}, \quad n \text{ odd.} \end{aligned}$$

This means that

$$\begin{aligned} s_s(t) &= 1 + \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{8}{n\pi} \sin\left(n\frac{\pi}{2}t\right), \\ &= 1 + \frac{8}{\pi} \sin\left(\frac{\pi}{2}t\right) + \frac{8}{3\pi} \sin\left(\frac{3\pi}{2}t\right) + \dots \end{aligned}$$

This type of manipulations (translations and shifts) are made so the students can gain some insight

on the changes introduced in the coefficients of the Fourier series expansion, and actually realize that the new values of the coefficients may become very different.

3 THE APPLICATION

The Android application, named “FouSE—Fourier Series Expansions”, is freely available for download from “Google Play” (www.google.com). Android was our first choice, because the majority of our students have Android powered devices; still, we want to produce an iOS equivalent application. No particular concerns have directed the design of the user interface, besides the basic and obvious ones, and the major guidelines, as discussed for example in (Holzinger et al., 2012; Mujacic et al., 2012). Figure 3 shows a screenshot of the application. When no signal is selected, the spinner on the upper left corner will show the word “none” and the plotting spaces of the original and approximation signals, as well as the next harmonic plot, will be empty, because they have nothing to show.

The user interaction with the application is very straightforward. The first thing a user must do is to choose a signal from the list (by pressing the spinner below the text “Signal”). Then, by simply pressing the “Add” button, the user refines the approximation to the desired accuracy.

The list of available signals include:

- Sawtooth;
- $|\sin|$ (full wave rectified);
- Half sin (half wave rectified);
- Square;
- Triangle;
- Parabolic (t^2).

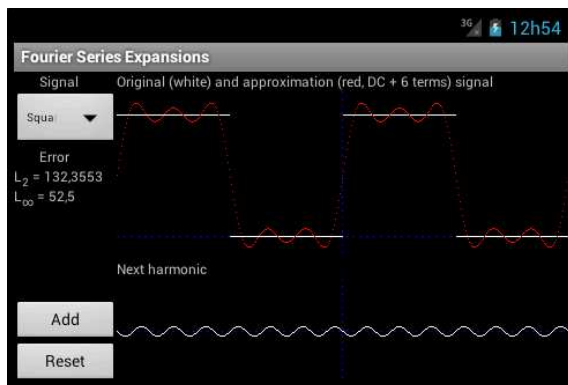


Figure 3: Fourier series expansion of a square-like wave: DC term plus 6 harmonics.

The “control” section of the interface is placed on the left side of the screen. As seen above, the spinner below the word “Signal” is used to choose the signal to approximate. Once a signal is chosen, more information and control buttons will be visible. The error, over a period, of the approximation is presented using two different metrics:

- $L_2 = \sqrt{\sum_{i=1}^N (\text{signal}_i - \text{approximation}_i)^2}$;
- $L_\infty = \max_i |\text{signal}_i - \text{approximation}_i|$.

The “Add” button is used to add the next term (the signal presented next to this button and below the text “Next harmonic”) to the approximation signal. The “Reset” button restarts the approximation.

The upper right side of the screen presents two signals plots and one text message:

- the original signal plot (in white);
- the current approximation signal plot (in red);
- the number of terms used in the current approximation (DC term, plus harmonics, if any).

Every time the “Add” button is pressed, the current harmonic plotted below the text “Next harmonic” is added to the current approximation, a new harmonic is presented here replacing the previous one, the approximation signal is re-plotted, the new errors are computed, and the number of terms used in the current approximation is updated in the text message next to the main graph.

Figure 3 shows an example of an approximation. In this case, the original signal is a square-like wave, plotted in white, and the result of the approximation, after summing the DC component plus six harmonics/terms of the Fourier series expansion, is plotted in red, over the same graph. On the graph below, it is presented the plot of the next harmonic/term to be added to the result of the approximation plotted in red, if the “Add” button is pressed. The current error of the approximation over a period is also shown ($L_2 = 132.3553$ and $L_\infty = 52.5$), and it will decrease if more harmonics/terms are added to the approximation.

Note that the scales (and axes) used for the signals and harmonic plots are different, and automatically adapt from expansion to expansion. However, these scales remain constant during a particular expansion, so the student can realize the different weights that successive harmonics will have in the approximation. For example, figure 4 shows the result of the Fourier series approximation to a square-like wave, after using the DC term plus 18 harmonics. When compared to figure 3 we may note the big difference in the weights of harmonic number 7 and harmonic number 19 (obviously, the same applies to the error

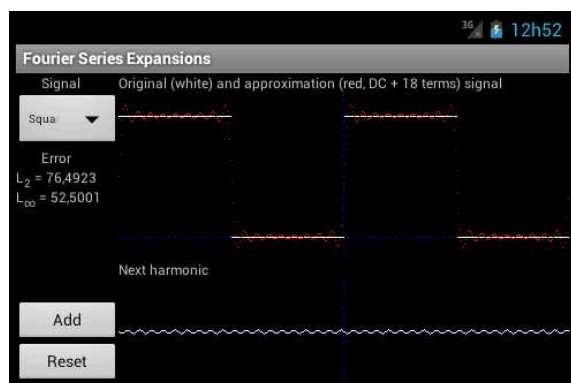


Figure 4: Fourier series expansion of a square-like wave: DC term plus 18 harmonics.

$L_2 = 76.4923$ and $L_\infty = 52.5001$). Note that, in this case, L_∞ error remains constant (52.5); this is due to the Gibbs phenomenon (see, for example, Jerri, 2011) for details).

Figures 5 and 6 show the Fourier series expansion approximation to a sawtooth-like wave. In figure 5 it is used the DC term only to the approximation (all the approximations start with the DC term only). It is well visible that, in this case, $DC = 0$ and so it overlaps the t (time) axis in the plot. Also note the first harmonic term was scaled to completely fill the plotting area reserved to the plot of the harmonics. Now, from figure 6 we can see that the fifth harmonic term will have a much lower weight in the series final result. Note also that the time axis is now well visible in the approximation plot; as stated above, when the approximation started the null DC term overlapped the time axis.

A help menu is also available, by pressing the “menu” button of the smartphone/tablet.

4 SOME RESULTS

We have distributed a copy of the application directly to our students, i.e., the students installed the application directly on their devices without the need of downloading it from “Google Play”, after the subject was taught in class.

We have anonymously surveyed our students with the following questions:

1. Gender.
2. Do you consider that the application has an ease to use interface?
3. Have you ever used or needed to use the help menu?

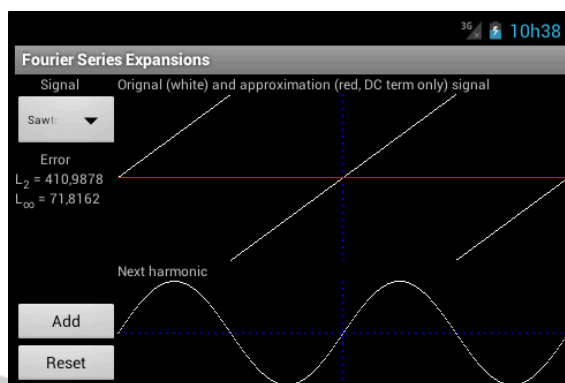


Figure 5: Fourier series expansion of a sawtooth-like wave: DC term only.

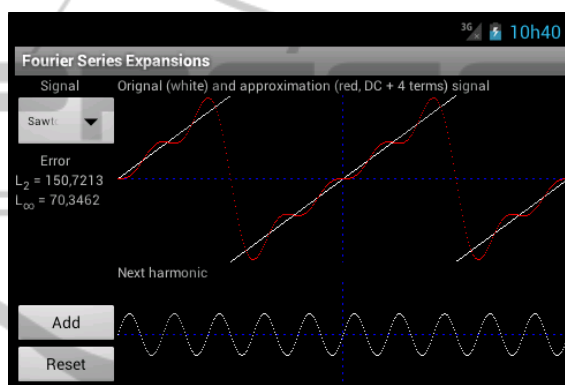


Figure 6: Fourier series expansion of a sawtooth-like wave: DC term plus four harmonics.

4. Do you prefer a web-based tool (applet) or this (android) version? Please tell us why.
5. Do you think that the application matches/covers the contents taught related to the Fourier series?
6. Do you think this application helped you in understanding the Fourier series (basics, how it works and its applicability)?
7. Do you think this application helped you in understanding other topics/subjects in this or other courses?
8. Do you think it would be beneficial to have similar tools in other subjects & courses?

In a total of 85 enrolled students, 55 completed the survey, from which 50 (91%) were males and 5 (9%) females. 51 (93%) of the surveyed students consider that the application has an ease to use interface, and only 4 (7%) found it somehow difficult or hard to use. However, only 2 (4%) of the total responding students have tried the help menu; curiously, these students are two of the four considering the application hard or difficult to use.

From the responding students, 47 (85%) prefer

this android version, when compared to other web-based applications (applet like) available on the Internet. When asked why, they answered that this application “is really portable (in the sense that we don’t need to be always connected; we can do simulations even if we do not have Internet access)” and “it seems that the interface was designed and developed to extract the maximum from my android device, in contrast to a web-based applet where we constantly need to adjust the screen size, resolution, among other”. The other 8 (15%) students consider the web-based (applet) version more suited to their needs: “don’t like to install apps” and “web-based applications are really platform independent” were the two main reasons presented for that. In this group we also counted the iOS (iPhone/iPad) users, who revealed that they are willing for an iOS version of the tool.

A total of 53 (96%) of the responding students consider that the tool matches/covers the subjects studied within the Fourier series topics, and 54 (98%) believe that it helped them in understanding the basics of the Fourier series, how it works and its applicability. In addition, 50 (91%) students consider that this tool helped them to understand other topics under this course and even in other courses. The referred topics include: some signals’ properties (like symmetry, frequency, etc.); Fourier transform; math series construction; and programming (the code of the application was made available to our students).

When asked if other courses/topics/subjects should benefit from similar tools/applications the answer was the answer was unanimous (100% (55) of the responding students said yes).

5 CONCLUSIONS

Fourier series expansions are among the problems that an electrical engineer most often faces. An undergraduate course in the subject not only provides the student with relevant know-how concerning the problems, but may also be of considerable value in understanding how a background in Fourier and numerical analysis can be applied in the field, to concrete engineering problems.

In this context, we felt a need for tools that could be easily used inside or outside the classroom, in formal and informal contexts, and at the same time were easy to update and maintain. The application that we have described allows the student to try several Fourier series expansions on a set of typical signals, and then compare the results in an easy-to-use environment, and using their own Android powered devices.

The application, named “FouSE—Fourier Series Expansions”, is freely available for download from “Google Play” (www.google.com), and can be used by the students in their daily work, inside or outside the classroom, and have proved to be a valuable tool for teaching and understanding Fourier series expansions.

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